

**Quantitative evaluation of slip activity in polycrystalline α -titanium considering
non-local interactions between crystal grains**

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Abstract

An indicator to predict the slip operation at the crystal-grain level, namely slip operation factor (SOF), was established as a function of the Schmid factor (SF) and critical resolved shear stress (CRSS). Plastically “soft” and “hard” regions were estimated by the Schmid factor values normalized by the CRSS—the normalized Schmid factor (NSF). The effect of the interaction among the regions was incorporated into SOF. A microstructural map of α -titanium (α -Ti) was obtained by the electron backscatter diffraction patterns. Several spatial distributions of SOF were calculated based on the map by changing the interaction range among the regions. The distributions were compared with those of the strains obtained by the crystal plasticity finite element (CPFE) analysis. Good agreement between the distributions was found near the macroscopic yield point when the interaction range was appropriate, although some significant differences between the distributions were also noticed after the yielding point. The prediction accuracy by SOF was higher than that by SF and NSF. The reasons for the high accuracy revealed by the SOF analysis and the differences between the distributions indicated by the CPFE analysis were also investigated.

Keywords:

Slip operation factor; Schmid factor; Crystal plasticity analysis; Image-based simulation; α -titanium

1. Introduction

1.1 Need for efficient environment for evaluation of non-uniform deformation

The inhomogeneous deformation of metal materials is closely related to the fracture phenomenon, and a lot of studies have been conducted to reveal the relevant mechanisms.

Asaro and Rice (1977) reported that in single crystals, work softening for slip systems is not necessary to form strain localization. Peirce et al. (1982) published that geometric softening induced by the crystal lattice rotation with the deformation is an important factor in forming slip bands. As for the polycrystalline materials, the deformation resistance in individual crystal grains varies, while the deformation is inhomogeneous at the beginning. Most metals used as structural materials are polycrystals. As such, understanding the mechanisms of inhomogeneous deformation in the polycrystalline materials leads to safety enhancement and improvements in the performance of many products. The mechanism of inhomogeneous deformation in the polycrystalline materials has been investigated using experimental and numerical methods.

During the initial stages of deformation in the polycrystalline materials, the anisotropic elasticity of crystal grains induces inhomogeneous deformation (Hook and Hirth, 1967a).

When the deformation progresses, the plastic deformation occurs, and the spatial inhomogeneity in strains develops owing to the difference in the deformation resistance and deformation incompatibility among the grains (Bache, 2003; Evans, 1998; Kondou et al., 2008; Kawano et al., 2015; Kawano et al., 2018a). The spatial distributions of crystallographic orientations are required to investigate the mechanisms of inhomogeneous deformation in the elastoplastic ranges at the crystal grain level.

Both experimental and numerical studies have been conducted to investigate the inhomogeneous deformation of polycrystalline materials depending on their crystallographic orientations. For example, in a number of experimental studies, the relationship between the microstructures with respect to the crystal orientation and slip deformation has been investigated by the X-ray and electron backscatter diffraction (EBSD) patterns (Bridier et al., 2005; Wang et al., 2017; Zhang et al., 2018). In numerical studies, on the other hand, continuum frame work including dislocation theory has been established (Ohashi, 1994; Le and Stumpf, 1996; Ohashi, 1997; Berdichevsky, 2006; Dunne et al., 2007a; Beyerlein and Tomé, 2008; Le and Sembiring, 2008; Langer et al., 2010; Le and Nguyen, 2012; Knezevic et al., 2013; Le and Nguyen, 2013; Le and Günther, 2014; Koster et al., 2015; Langer, 2015; Le et al., 2017; Le, 2018). In the conducted numerical studies, crystal plasticity analysis considering such continuum dislocation theory, the slip and twinning deformation modes, or both of them was employed, and the relationship between the microstructures and deformation mechanisms was investigated (Dunne et al., 2007b; Mayama et al., 2011; Roters et al., 2019; Kotha et al., 2019; Kawano et al., 2019a, 2019b). Currently, the crystal plasticity analysis can reproduce various phenomena at the crystal grain level, such as the size effect (Kuroda and Tvergaard, 2006; Ohashi et al., 2007; Le and Sembiring, 2008; Aoyagi et al., 2014; Lewandowski and Stupkiewicz, 2018; Sun et al., 2019), non-Schmid effects (El Kadiri et al., 2013; Cereceda et al., 2016;), temperature dependency and strain rate sensitivity (Lee et al., 2010; Langer, 2015; Zhang et al., 2015a; Zhang et al., 2016; Ardeljan et al., 2016; Langer, 2016; Cereceda et al., 2016; Le, 2018; Waheed et al., 2019), Bauschinger effect (Yoshida and Uemori, 2002; Le and Sembiring, 2008; Barlat et al., 2011), development of density in

1 atomic vacancies (Ohashi, 2018; Ohashi and Okuyama, 2019), and growth of voids in the
2 crystal materials (Asim et al., 2019).

3
4 Grain morphology as well as constitutive models affects the deformation mechanism in
5 polycrystalline materials (Zhang et al., 2015b). The crystal plasticity analysis can employ
6 the geometric models built from real microstructures obtained by methods such as EBSD.
7 The spatial distribution of strains obtained by the analysis agrees well with the
8 experimental results (Zhao et al., 2008; Weinberger et al., 2014; Guery et al., 2016a;
9 Zhang et al., 2018; Kawano et al., 2019b). As such, the deformation of polycrystals can
10 be accurately predicted by the crystal plasticity analysis. However, the analysis requires
11 the identification of various physical parameters, and the process of their determination
12 is laborious. Therefore, the development of methods that facilitate the investigation of
13 inhomogeneous deformation mechanisms will lead to the progress of its study.

15 **1.2 Significance of slip deformation indicator and the object of this study**

16
17 The Schmid factor (SF) has often been used as a simple method to predict the slip
18 operation. SF not only characterizes the slip operation, but also represents the ratio of the
19 contribution of the uniaxial stress to the resolved shear stress for each slip system. Thus,
20 when the isotropic elasticity and critical resolved shear stresses (CRSSs) are the same
21 among all of the slip systems, SF can predict the strain distribution at the initial
22 deformation stage with a certain degree of accuracy. However, if the materials possess
23 different CRSSs among the slip systems and anisotropic elasticity (e.g., hexagonal close-
24 packed (HCP) materials with strong anisotropic elasticity), these properties as well as SF

1 affect the mechanism of slip deformation. Moreover, the deformability near the regions
2 of interest affects the deformation in the regions of interest.

3
4 Normalized Schmid factor (NSF) is a type of the Schmid factor normalized by minimum
5 CRSS among the slip systems (Capolungo et al., 2009; Bantounas et al., 2009; Bao et al.,
6 2011). NSF can predict a strain operation more accurately than SF even for materials with
7 different CRSSs among the slip systems. However, the effects of anisotropic elasticity
8 and plastic deformability in the volumes close to the regions of interest are not considered
9 in NSF. Thus, NSF does not fully predict the strain distribution in polycrystalline
10 materials. The aim of this study is to develop an indicator, namely, a slip operation factor
11 (SOF), which accurately and efficiently predicts the strain distribution in such HCP
12 materials with different CRSSs among the slip systems.

13
14 Previous studies have indicated the importance of mechanical interactions among the
15 adjacent “soft” and “hard” grains for the inhomogeneous deformation in HCP materials
16 (Evans, 1998; Bache, 2003; Dunne et al., 2007b; Bieler et al., 2014; Abdolvand et al.,
17 2018). For example, Dunne et al. (2007b) revealed the worst neighbor combination of
18 crystallographic orientations to incur damage under fatigue loading in α -titanium (α -Ti)
19 with the HCP structure. In the inhomogeneous deformation mechanisms, the slip transfer
20 across the grain boundaries is essential (Bieler et al., 2014; Tsuru et al., 2016; Zheng et
21 al., 2017). As such, several indicators to examine the slip transfer were developed (Bieler
22 et al., 2014; Tsuru et al., 2016). Furthermore, an indicator, which is a function of Schmid
23 factor and crystal orientations in adjacent crystal grains, was also developed, and it
24 successfully predicted the strain in Mg alloy (Sun et al., in press). These indicators are

the functions of slip strains or values (e.g., the Schmid factor) determined by the crystal orientations. These studies also indicated that the inhomogeneous deformation could be estimated from the distribution of crystal orientations. Mechanical interactions can occur among distant and adjacent grains (Kawano et al., 2019b), and there is a possibility that the prediction accuracy of slip operation increases by considering the effect of the interactions among the distant grains.

In this study, SOF is formulated and validated. First, we show the formulation of SOF, where a range of the mechanical interactions among the regions is decided arbitrarily. Next, using SOF, we predict the strain distribution of polycrystalline α -Ti, where its microstructure is obtained by EBSD under uniaxial tensile deformation. The strain distribution obtained by the crystal plasticity finite element (CPFE) method is compared with that calculated by SOF. Finally, the accuracy of the prediction of strain distribution is discussed.

2. Formulation of SOF

Let us assume a polycrystalline material under uniaxial tension (Fig. 1). The deformability in region i is determined not only by the softness of the region itself, but also by that of the region surrounding it (Evans, 1998; Bache, 2003; Dunne et al., 2007a; Dunne and Rugg, 2008; Kawano et al., 2018a; Abdolvand et al., 2018). Herein, a novel numerical indicator (SOF) is presented and considers the deformability in the surrounding region to predict the ease of slip operation.

First, normal stress σ in a continuum body under uniaxial tension is applied (Fig. 2). When the plastic deformation occurs in the continuum body by the activation of slip system k , the plastic deformability is determined by the resolved shear stress (RSS) of the slip system. That is, the deformability can be evaluated by SF, which represents the contribution of σ to RSS in slip system k ,

$$m_{SF}^{(k)} = \cos \varphi_1^{(k)} \cos \varphi_2^{(k)}, \quad (1)$$

where $\varphi_1^{(k)}$ is the angle between the loading direction and the vector normal to the slip plane, whereas $\varphi_2^{(k)}$ corresponds to the angle between the loading direction and the vector in the slip direction. When CRSSs among the slip systems are different (e.g., HCP materials), the difference in NSF to evaluate the ease of slip operation has to be considered and is defined as (Capolungo et al., 2009; Bantounas et al., 2009; Bao et al., 2011)

$$m^{(k)} = m_{SF}^{(k)} \cdot \frac{\tau_{CRSS}^{\min}}{\tau_{CRSS}^{(k)}}, \quad (2)$$

2

3 where $\tau_{CRSS}^{(k)}$ is CRSS for slip system k , and τ_{CRSS}^{\min} is minimum CRSS in all of the slip
 4 systems. If the Schmid law is assumed to be proven, the slip deformation operates easier
 5 with higher NSF in the homogeneous stress fields.

6

7 Next, the effect of deformability in the volumes surrounding region i is considered. “Soft”
 8 regions, existing in the loading direction (LD) (Fig. 1b), impede the deformation of region
 9 i because the “soft” regions preferentially deform rather than region i . However, region i
 10 is easier to deform when the “soft” regions exist in the transverse direction (TD) to LD
 11 (Fig. 1c). Indicators \hat{m}_i^{LD} and \hat{m}_i^{TD} need to be defined to consider the effects of relative
 12 softness between regions i and j in LD and TD to predict the ease of slip system operation
 13 as follows:

14

$$\hat{m}_i^{LD} = \sum_{j \neq i} \left[w(\|r_{ij}\|, r_e^{LD}) \cdot \min \left(\frac{m_i^{\max}}{m_j^{\max}}, R_{\max}^{LD} \right) \cdot |\cos \theta_{ij}| \right], \quad (3)$$

16

$$\hat{m}_i^{TD} = \sum_{j \neq i} \left[w(\|r_{ij}\|, r_e^{TD}) \cdot \min \left(\frac{m_j^{\max}}{m_i^{\max}}, R_{\max}^{TD} \right) \cdot |\sin \theta_{ij}| \right], \quad (4)$$

18

19 where m_i^{\max} is the maximum value of m' in all of the slip systems, and m_i^{\max} / m_j^{\max}
 20 in Eq. (3) represents the relative ease of slip deformation in region i , in which the value

m_i^{\max}/m_j^{\max} is lower when region j is softer. Although m_j^{\max}/m_i^{\max} in Eq. (4) also represents the relative ease of slip deformation in region i , the value m_j^{\max}/m_i^{\max} is higher when region j is softer. R_{\max}^{LD} and R_{\max}^{TD} are the upper limits of these relative values. They prevent the relative values from being excessively large as there should be a limit to the degree of the relative ease of slip operation. θ_{ij} corresponds to the angle that LD makes with direction vector \mathbf{r}_{ij} from region i to j (See Fig. 3a). $|\cos \theta_{ij}|$ and $|\sin \theta_{ij}|$ provide the component of the value in the directions parallel and perpendicular to LD, respectively. w is the weight function that assumes a larger value with closer distance $\|\mathbf{r}_{ij}\|$ between regions i and j . Parameters r_e^{LD} and r_e^{TD} are the effective interaction range between regions i and j in LD and TD, respectively. That is, Eqs. (3) and (4) are calculated by the summation of the relative ease of plastic deformation in LD within the object volume in materials, considering the mechanical interaction among the regions.

Assuming that the plastic deformability in region i is evaluated by the summation of NSF in region i , and the relative ease of slip operation between regions i and j takes the effect of TD and LD into consideration, the ease of slip operation in region i can be evaluated by

$$\hat{m}_i = m_i^{\max} + \hat{m}_i^{LD} + \hat{m}_i^{TD}. \quad (5)$$

Next, \hat{m}_i^{LD} and \hat{m}_i^{TD} in Eq. (5) are normalized using their maximum values to

quantitatively evaluate their effect on the ease of slip operation as follows:

$$\hat{m}_i^{LD} = \frac{\hat{m}_i^{LD}}{\max_{j \in J} [\hat{m}_j^{LD}]}, \quad (6)$$

$$\hat{m}_i^{TD} = \frac{\hat{m}_i^{TD}}{\max_{j \in J} [\hat{m}_j^{TD}]}, \quad (7)$$

where J represents the whole regions in the specimen, and \hat{m}_i^{LD} and \hat{m}_i^{TD} are the normalized indicators that consider the effects in LD and TD, respectively, to predict the ease of slip operation. Coefficients A^{LD} and A^{TD} are multiplied by \hat{m}_i^{LD} and \hat{m}_i^{TD} , respectively, and the resulting values are added to yield SOF as follows:

$$\hat{m}_i' = m_i^{\max} + A^{LD} \hat{m}_i^{LD} + A^{TD} \hat{m}_i^{TD}. \quad (8)$$

In Eq. (8), the degree of interaction in LD and TD change with coefficients A^{LD} and A^{TD} , respectively, and the interaction range can be also changed by $w(\|\mathbf{r}_{ij}\|, r_e)$.

In this study, we employ the following weight function that is typically used in smooth particle hydrodynamics (SPH) which is a type of the particle method (Monaghan, 1988; Müller et al., 2003):

$$w(r_{ij}, r_e) = \begin{cases} \frac{315}{64\pi r_e^9} (r_e^2 - \|r_{ij}\|^2)^3, & 0 \leq \|r_{ij}\| \leq r_e \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

As shown in Fig. 3b, the interaction between the regions is stronger with a closer distance between i and j owing to the effect of w .

In SOF, the ease of slip operation is evaluated by NSF, assuming that the Schmid law is proven. The “relative” ease of slip operation in individual regions is evaluated by the summation of the CRSSs ratio between the regions using Eqs. (3) and (4). As such, the relative ease of slip operation is evaluated based on physics in SOF, and slip operation is likely to occur with higher SOF (although the SOF values do not form a direct connection with the physical values).

3. Prediction of strain distribution by CPFE analysis

In this study, we compare the SOF spatial distributions and those of strain obtained by the CPFE analysis to validate SOF. In this section, we present a numerical simulation of uniaxial deformation of polycrystalline α -Ti. A microstructure of the α -Ti specimen was obtained by EBSD. First, we present the CPFE method. Then, we describe the α -Ti specimen that was employed, the geometric model, and conditions for the analysis. Finally, we reveal the study results.

3.1 Methods of crystal plasticity analysis

A rate-dependent finite-strain crystal plasticity model (Peirce et al., 1983; Asaro and Needleman, 1985) was implemented in this study. The model is described in detail in Appendix. The shear slip rate $\dot{\gamma}$ for slip system k was calculated by

$$\dot{\gamma}^{(k)} = \dot{\gamma}_0 \operatorname{sgn}(\tau^{(k)}) \left| \frac{\tau^{(k)}}{\hat{\tau}^{(k)}} \right|^{\frac{1}{m}}, \quad (10)$$

where $\dot{\gamma}_0$ is the reference shear strain rate for plastic slip, τ is the resolved shear stress (RSS), $\hat{\tau}$ is the reference shear stress, m is the slip rate sensitivity parameter, and $\hat{\tau}$ for slip system k is provided by the Voce hardening law (Voce, 1955; Kocks, 1976) as follows:

$$\hat{\tau}^{(k)} = \tau_0^{(k)} + \left(\tau_1^{(k)} + \theta_1^{(k)} \gamma \right) \left\{ 1 - \exp \left(- \frac{\theta_0^{(k)} \gamma}{\tau_1^{(\alpha)}} \right) \right\}, \quad (11)$$

2

3 where τ_0 , τ_1 , θ_0 , and θ_1 are the parameters to represent the relationship between the
4 shear stress and shear strain.

5

6 **3.2 Sample material and geometric model**

7

8 The chemical composition of the test material employed in this study is summarized in
9 Table 1. The processing sequence used for the test material was hot and cold rolling with
10 heat treatment for 2 h at 650 °C for the microstructure homogenization. The EBSD crystal
11 orientation map was obtained from the test material (Fig. 4a). The geometric model for
12 the numerical analyses (Fig. 4b) was built from the map using the data conversion system
13 developed by Kawano et al. (2018b).

14

15 **3.3 Conditions for CPFE analysis**

16

17 The uniaxial loading was applied to the geometric model. Fig. 4b demonstrates the surface
18 on the left side that is fixed in the X-direction with the forced displacement applied to the
19 surface on the right side. Zero stress was applied to other surfaces. In this study, we
20 analyze the inhomogeneous strain distribution at the initial deformation stage. The
21 important physical parameters at the initial deformation stage typically constitute the
22 elastic compliance and CRSS. The elastic compliances of pure Ti (Fisher and Renken,
23 1964) were employed (Table 2). To investigate the effect of anisotropic elasticity on the

strain distribution, the isotropic elasticity condition with the Poisson's ratio ($E = 106.3$ GPa) and Young's modulus ($\nu = 0.34$) from polycrystalline CP-Ti was also applied (Hook and Hirth, 1967a; Kondou et al., 2008; Mayama et al., 2009; Kawano et al., 2015; Tada and Uemori, 2018; Kawano et al., 2018a). The slip systems are assumed to be basal, prismatic $\langle a \rangle$, pyramidal $\langle a \rangle$, and 1st- and 2nd-pyramidal $\langle c+a \rangle$ (Fig. 5). The CRSSs are determined by parameter $\hat{\tau}^{(\alpha)}$ in Eq. 11. The parameters used in Eq. (11) are shown in Table 3, which are determined based on CRSS of α -Ti as estimated by Pagan et al. (2017), although the CRSS values of α -Ti differ depending on the study (Conrad, 1981; Philippe et al., 1995; Wu et al., 2007; Warwick et al., 2012; Benmhenni et al., 2013; Hama et al., 2017; Pagan et al., 2017). The $\hat{\tau}^{(\alpha)}$ value changes as a function of slip strain (Fig. 6). Work softening on each slip system in α -Ti (Pagan et al., 2017; Wang et al., 2017) was not taken into consideration. The material employed in this study was polycrystalline α -Ti with a single phase and large grain size (average grain size $D_{ave} = 69.2 \mu\text{m}$). The effects of size (Okazaki and Conrad, 1972; Sergueeva et al., 2001) and the second phase (Radecka et al., 2016) were ignored in the current CPFEM analysis. The occurrence of twinning commonly decreases with increasing concentrations of oxygen (O) and aluminum (Al) (Hanada, 1990; Lütjering and Williams, 2007; Fitzner et al., 2016). Although the O and Al contents were low in the test material employed in this study (Table 1), twinning was ignored for the sake of simplicity. As shown in τ_0 in Table 3, the magnitude relation of initial CRSSs among the slip systems is as follows: prismatic $\langle a \rangle < \text{basal} < 1^{\text{st}} \text{ pyramidal} \langle a \rangle < 1^{\text{st}} \text{ and } 2^{\text{nd}} \text{ pyramidal} \langle c+a \rangle$. The strain rate was assumed to be 1.0×10^{-3} . The slip rate sensitivity parameter $m = 0.02$ was employed (Hasija et al., 2003; Hama et al., 2017).

3.4 Strain distribution predicted by CPFE analysis

The spatial distributions of normal strain ε_{XX} in the X (loading)-direction are presented in Fig. 7a, and the relationship between the average normal stress and strain are shown in Fig. 7b. The distribution of ε_{XX} is inhomogeneous around the yield point ($\overline{\varepsilon_{XX}} = 0.006$), and the inhomogeneity increases with the deformation ($\overline{\varepsilon_{XX}} = 0.031$). The distributions of slip strain (γ) obtained during the same analysis are displayed in Fig. 8. The distributions of slip strain (γ_{total}) correspond well with those of ε_{XX} . That is, the slip strain determines the distribution of ε_{XX} after the yielding in this analysis. In addition, the slip strain in the primary slip system (γ_{basal}) is dominant around the yielding point, and the distribution is similar to that of ε_{XX} due to the small elastic deformation. In contrast, the activation of the secondary slip systems makes a difference between the distributions of γ_{basal} and ε_{XX} as the deformation progresses. The inhomogeneous strain distributions observed in the current analysis reflect the low work-hardening rate of the primary slip system in α -Ti. When the work-hardening rate is higher due to the effect of the second phase (Radecka et al., 2016), the strain distributions become more homogeneous, and the secondary slip systems may activate during the initial stages of deformation.

4. Prediction of the ease of slip operation by SOF

4.1 Conditions for SOF calculation

The geometric model was subjected to tensile loading (Fig. 4b). The model was assumed to be the same as that for the current CPFE analysis, and the spatial distributions of the ease of slip operation were evaluated by SOF. The slip systems similar to those used in the CPFE analysis were assumed (Fig. 5). The parameters for the calculation of SOF are shown in Table 4. CRSSs were the same as those for the CPFE analysis, and the values were determined in accordance with the experimental results by Pagan et al. (2017). The SOF spatial distributions were calculated with the changes in the influence range, as shown in Table 4 (I)–(V).

4.2 Distributions of SOF and comparison with CPFE analysis results

Figs. 9a–e represent the SOF distributions under the conditions of Table 4. The distribution of slip strain obtained by the CPFE analysis is shown in Fig. 9f. SOF defined by Eq. (8) represents the ease of slip operation considering the non-local interaction among the regions. The dependency of the influence ranges (r_e^{LD} and r_e^{TD}) on the distributions of SOF is as follows. When the influence ranges equal the average grain size ($r_e^{LD} = r_e^{TD} = D_{ave}$), the distribution of SOF (Fig. 9a) has a good correspondence with that of γ_{total} (Fig. 9f). When $r_e^{LD} = 2D_{ave}$ and $r_e^{TD} = D_{ave}$ (Fig. 9b), the difference between the values among the regions becomes more distinct. SOF in region “A” becomes low, and the distribution in the region becomes similar to that of γ_{total} , although the

distributions in Figs. 9a-b do not differ substantially. When $r_e^{LD} = D_{ave}$ and $r_e^{TD} = 2D_{ave}$ (Fig. 9c), SOF becomes higher in the regions closer to the grain boundary, and the distribution does not correspond to that of γ_{total} . When $r_e^{LD} = r_e^{TD} = 2D_{ave}$ (Fig. 9d), the distribution is similar to that in Fig. 9a. When $r_e^{LD} = 3D_{ave}$ and $r_e^{TD} = 2D_{ave}$ (Fig. 9e), the high slip strain in region “B” cannot be predicted, whereas high SOF extended in the Y-direction appears in region “C”, which captures the feature in the distribution of γ_{total} in the grain.

The above results indicate: i) the strain distribution at the yielding point can be predicted by SOF when appropriate r_e^{LD} and r_e^{TD} are employed; ii) r_e^{LD} , r_e^{TD} , and the ratio between them strongly affect the prediction accuracy in the strain distribution; and iii) appropriate r_e^{LD} and r_e^{TD} differ depending on the region as the prediction accuracy of strain distribution in each region is different when r_e^{LD} and r_e^{TD} change. However, the prediction accuracy of strain distribution by SOF decreases with the deformation. One of the reasons for that is the activation of the secondary slip systems. However, the effect of secondary slip systems is not taken into consideration in SOF.

5. Discussion

Here, we discuss why SOF could accurately predict the strain distributions and the existence of the partial difference in the distributions obtained by the SOF and CPFE analysis. Initially, we compare the distributions of SOF, SF, and NSF to investigate the effect of interactions among the regions on the strain distributions. Then, we investigate the effect of anisotropic elasticity and lattice rotation with plastic deformation on the strain distribution to reveal the reasons for the difference.

5.1 Comparison of SF and NSF distributions

Herein, we compare the distributions of SOF with those of SF and NSF. Fig. 10 shows the SF distributions. The distributions were evaluated using the maximum SF value among all of the slip systems (Fig. 10a) and in individual slip systems (Fig. 10b). Fig. 11 shows the NSF distributions. The maximum values among the slip systems (m^{\max}) were employed to reveal the distribution.

The SF distributions do not predict the strain distribution (Fig. 7a), and the NSF distributions also do not show a good correspondence with the strain distribution in the regions indicated by arrows in Fig. 11. The inhomogeneous distribution within each crystal grain (Fig. 11) cannot be represented by SF and NSF, although NSF performs the predictions more accurately than SF does by considering CRSSs. That is, SOF predicts the strain distributions more accurately than SF and NSF do by considering the interactions among the “soft” and “hard” regions, which can be distant or adjacent to each

other.

5.2 Effect of anisotropic elasticity on strain distribution

Anisotropic elasticity contributes to the inhomogeneous deformation in polycrystals at the early stages of deformation (Hook and Hirth, 1967a; Kondou et al., 2008; Mayama et al., 2009; Kawano et al., 2015; Tada and Uemori, 2018; Kawano et al., 2018a). In this study, we investigate the effect of elastic anisotropy on the inhomogeneous deformation in polycrystalline α -Ti. The results were obtained under the conditions of anisotropic or isotropic elasticity and can be seen in Fig. 12. Other conditions were the same as those mentioned in Figs. 7 and 8, except for the elasticity. The Poisson's ratio ($\nu = 0.34$) and Young's modulus ($E = 106.3$ GPa) from polycrystalline CP-Ti were employed for the isotropic elasticity. As for the isotropic elasticity condition, the elasticity was assumed to be homogeneous for the whole geometric model.

The obtained macroscopic stress-strain relationship values were similar (Fig. 12c). However, the distributions of the slip strain were different at the initial deformation stages (Figs. 12a and b). When the slip deformation started to occur, the distribution within each crystal was homogeneous on the condition of isotropic elasticity (Fig. 12a) with $\overline{\varepsilon_{xx}} = 0.004$. In contrast, the distribution within each crystal was inhomogeneous on the condition of anisotropic elasticity due to the localized slip operation (Fig. 12b) with $\overline{\varepsilon_{xx}} = 0.004$. The difference between the distributions immediately disappeared with the deformation (Figs. 12a $\overline{\varepsilon_{xx}} = 0.005$ and 0.006). As a result, the effect of elastic

anisotropy on the distribution of slip strain is negligible near the yielding point of the current analysis.

5.3 Effect of lattice rotation on strain distribution

Herein, we investigate the effect of lattice rotation with the deformation on the inhomogeneous deformation. Figs. 13a–d show the pole figures, and Figs. 13e–f represent the changes in the angles of vectors normal to (0001) and $\{10\bar{1}0\}$ planes from the initial states when $\overline{\varepsilon_{xx}} = 0.031$.

The changes in the pole figures were negligible even when $\overline{\varepsilon_{xx}} = 0.031$ (Figs. 13a–d). The changes in the angles of vectors normal to the planes were insignificant, and the average angle change was $< 2^\circ$, although the localized, substantial changes in the angles existed. The changes near the yielding point ($\overline{\varepsilon_{xx}} \approx 0.006$) were much smaller than those at $\overline{\varepsilon_{xx}} = 0.031$. Consequently, although the lattice rotation is an important factor for the understanding of deformation mechanisms, its effect on the inhomogeneous deformation is negligible near the yielding point in the current analysis.

5.4 Other effects and future studies

A number of effects might cause the difference in the distributions of ε_{xx} , as predicted by CPFE and SOF. In this study, the following effects were discussed: the activation of the secondary slip systems, anisotropic elasticity, and lattice rotation. SOF predicted the

1 strain distribution accurately near the yielding point; nevertheless, the effects of the
2 secondary slip systems and lattice rotation tend to increase with deformation, and it is
3 difficult to predict the strain distributions in the larger deformation stage by the current
4 SOF. Other effects, such as deformation incompatibility between the crystal grains, also
5 may contribute to the inhomogeneous deformation (Hook and Hirth, 1967b; Kondou et
6 al., 2008; Kawano et al., 2015; Kawano et al., 2018a).

7
8 As stated above, SOF can predict the distributions of normal strain along the tensile
9 direction obtained by the crystal plasticity analysis. The crystal plasticity analysis is a
10 method that allows to reproduce the distributions of strain obtained by the experiments
11 (Raabe et al., 2001; Zhao et al., 2008; Weinberger et al., 2014; Tasan et al., 2014; Guery
12 et al., 2016b; Zhang et al., 2018b; Kawano et al., 2019b). Therefore, it is possible that
13 SOF can reproduce the strain distributions obtained by the experiments (e.g., DIC). In
14 addition, SOF was evaluated using the influence ranges (r_e^{LD} and r_e^{TD}), i.e., the
15 interaction ranges among the regions can be investigated by SOF. The calculation cost of
16 SOF is much lower than that of FEM. Thus, it is an effective indicator that can be used
17 for the understanding of the deformation mechanisms in the microstructures of crystalline
18 materials. Furthermore, it is possible that the effects of size (Okazaki and Conrad, 1972;
19 Sergueeva et al., 2001) and the second phase (Radecka et al., 2016) will be taken into
20 account in SOF by the change in CRSSs, depending on their effects. When SOF
21 considering these effects is developed, the strain distributions in materials with the second
22 phase and finer grains will be predicted. SOF can be used in future studies to reveal the
23 deformation mechanisms at the crystal grain level.

6. Conclusions

In this study, we developed an indicator, namely, a slip operation factor (SOF), to represent the ease of slip operation at a lower computational cost. We also considered the relative plastic deformability among the regions, which is a function of the Schmid factor and CRSS. SOF was adapted to a microstructural map of polycrystalline α -Ti, obtained by EBSD, and several distributions of SOF were acquired by changing the influence ranges (r_e^{LD} and r_e^{TD}) under the condition of uniaxial tension. The obtained distributions were compared with those of the strains predicted by the crystal plasticity finite element (CPFE) analysis. The results can be summarized as follows:

- (1) SOF can predict the spatial distributions of slip strain more accurately than the Schmid factor (SF) or normalized Schmid factor (NSF) when the appropriate influence ranges (r_e^{LD} and r_e^{TD}) are employed.
- (2) In its current form, SOF can predict the normal strain in the tensile direction near the yielding point. This is because the effects of anisotropic elasticity, activation of the secondary slip systems, lattice rotation, and deformation incompatibility have negligible impacts on the inhomogeneous deformation near the yielding point, whereas the relative deformability among the plastically “soft” and “hard” regions strongly affects the strain distributions.
- (3) The difference in the distributions of SOF and normal strains in the tensile

direction increases with the deformation after the yielding point due to the activation of secondary slip systems. However, the impacts of anisotropic elasticity and lattice rotation were minor on the inhomogeneous deformation when $\varepsilon \approx 0.03$.

(4) The prediction accuracy of strain distributions differs depending on the regions and changes with r_e^{LD} and r_e^{TD} . Therefore, the mechanical interaction range may vary depending on the region.

(5) The mechanical interactions among the adjacent crystal grains in inhomogeneous deformation in α -Ti have been studied, while little attention has been paid to longer-range interactions. The current analysis using SOF showed the importance of the longer-range interaction for the inhomogeneous deformation as well as the interactions among the adjacent grains.

Appendix: Constitutive model for CPFE analysis

The rate-dependent finite strain crystal plasticity model proposed by Peirce et al. (1983) was employed in this study. The velocity gradient L was calculated as

$$L = L^* + L^p, \quad (A1)$$

where L^* and L^p are the elastic and plastic velocity gradients, respectively. The deformation rate tensor D and spin tensor W were calculated as follows:

$$D = \frac{L + L^T}{2}, \quad (A2)$$

$$W = \frac{L - L^T}{2}, \quad (A3)$$

where L^T is the transposed matrix of L .

The slip contribution of slip system α to the plastic velocity gradient was defined as

$$L^{p(k)} = \dot{\gamma}^{(k)} \left(s^{(k)} \otimes m^{(k)} \right), \quad (A4)$$

and the plastic velocity gradient was calculated as the sum of the contributions of all the slip systems

$$\mathbf{L}^p = \sum_{k=1}^N \dot{\gamma}^{(k)} \left(\mathbf{s}^{(k)} \otimes \mathbf{m}^{(k)} \right), \quad (\text{A5})$$

2

3 where N is the number of slip systems. The plastic deformation rate tensor and the
4 plastic spin tensor were defined as

5

$$\mathbf{D}^p = \frac{\mathbf{L}^p + \mathbf{L}^{pT}}{2} = \sum_{\alpha=1}^N \dot{\gamma}^{(\alpha)} \left[\frac{\mathbf{s}^{(k)} \otimes \mathbf{m}^{(k)} + \mathbf{m}^{(k)} \otimes \mathbf{s}^{(k)}}{2} \right] \equiv \sum_{\alpha=1}^N \dot{\gamma}^{(k)} \mathbf{p}^{(k)}, \quad (\text{A6})$$

$$\mathbf{W}^p = \frac{\mathbf{L}^p - \mathbf{L}^{pT}}{2} = \sum_{\alpha=1}^N \dot{\gamma}^{(\alpha)} \left[\frac{\mathbf{s}^{(k)} \otimes \mathbf{m}^{(k)} - \mathbf{m}^{(k)} \otimes \mathbf{s}^{(k)}}{2} \right] \equiv \sum_{\alpha=1}^N \dot{\gamma}^{(k)} \mathbf{w}^{(k)}. \quad (\text{A7})$$

8

9 The elastic deformation rate tensor \mathbf{D}^* and elastic spin tensor \mathbf{W}^* were calculated as

10

$$\mathbf{D}^* = \frac{\mathbf{L}^* + \mathbf{L}^{*T}}{2}, \quad (\text{A8})$$

$$\mathbf{W}^* = \frac{\mathbf{L}^* - \mathbf{L}^{*T}}{2}. \quad (\text{A9})$$

13

14 The following elastic constitutive equation was employed:

15

$$\overset{\nabla}{\boldsymbol{\sigma}}^* = \dot{\boldsymbol{\sigma}} - \mathbf{W}^* \boldsymbol{\sigma} + \boldsymbol{\sigma} \mathbf{W}^* = \mathbf{C} : \mathbf{D}^*, \quad (\text{A10})$$

17

18 where \mathbf{C} is the fourth-order elastic stiffness tensor.

19 The elastoplastic deformation tensor was assumed to be

1

$$\mathbf{D} = \mathbf{D}^* + \mathbf{D}^p . \quad (\text{A11})$$

3

4 The relationship between the elastic stress rate and \mathbf{D} was given by

5

$$\overset{\nabla}{\boldsymbol{\sigma}}^* = \mathbf{C} : \mathbf{D} - \sum_{\alpha=1}^N \dot{\gamma}^{(\alpha)} \mathbf{C} : \mathbf{p}^{(\alpha)} . \quad (\text{A12})$$

7

8 The Joumann stress rate was calculated as

9

$$\overset{\nabla}{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}} - \mathbf{W} \boldsymbol{\sigma} + \boldsymbol{\sigma} \mathbf{W} , \quad (\text{A13})$$

11

12 and the following relationship was obtained from Eqs. (A7), (A10), (A12), and elastic

13 and plastic decomposition of spin tensor \mathbf{W} as

14

$$\overset{\nabla}{\boldsymbol{\sigma}} = \mathbf{C} : \mathbf{D} - \sum_{\alpha=1}^N \dot{\gamma}^{(\alpha)} \left[\mathbf{C} : \mathbf{p}^{(\alpha)} + \mathbf{w}^{(\alpha)} \boldsymbol{\sigma} - \boldsymbol{\sigma} \mathbf{w}^{(\alpha)} \right] . \quad (\text{A14})$$

16

17 Changes in the directions of \mathbf{s} and \mathbf{m} according to the rotation of crystal lattice were

18 determined as

19

$$\dot{\mathbf{m}}^{(\alpha)} = \mathbf{W}^* \mathbf{m}^{(\alpha)} , \quad (\text{A15})$$

$$\dot{\mathbf{s}}^{(\alpha)} = \mathbf{W}^* \mathbf{s}^{(\alpha)} . \quad (\text{A16})$$

22

1

2

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4

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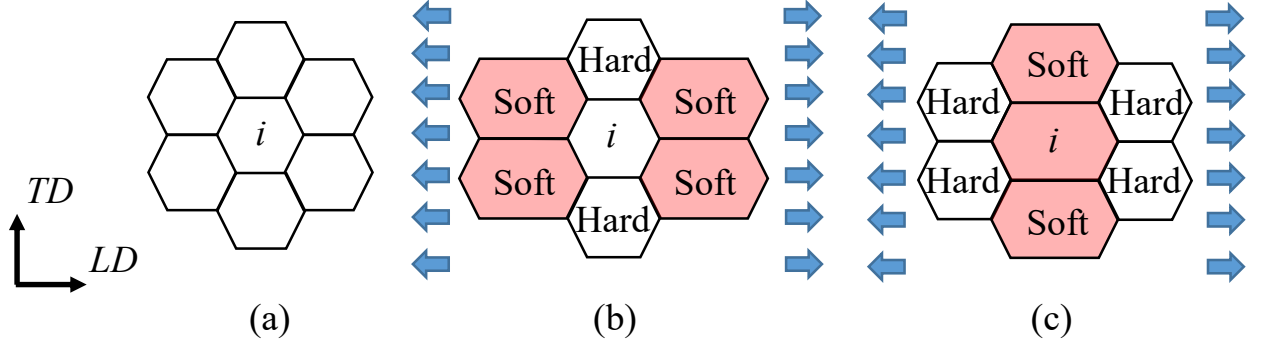
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1 **Figures**



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 3 Fig. 1 A schematic showing the difference in the deformability of region i depending on
 4 the softness of the surrounding regions. (a) The initial state and states after the
 5 deformations by tensile loading (b) and (c). (b) The condition, in which region i is
 6 connected to the “soft” regions in series and “hard” regions in parallel. (c) The condition,
 7 in which region i is connected to the “soft” regions in parallel and “hard” regions in series.

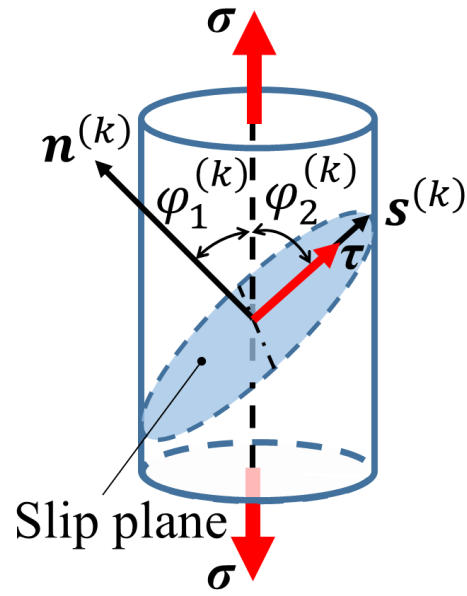


Fig. 2 A schematic showing the geometry of a slip system under uniaxial tensile loading. \mathbf{n} is the vector normal to the slip plane and \mathbf{s} is the slip direction vector. τ is the resolved shear stress (RSS) of the slip system k .

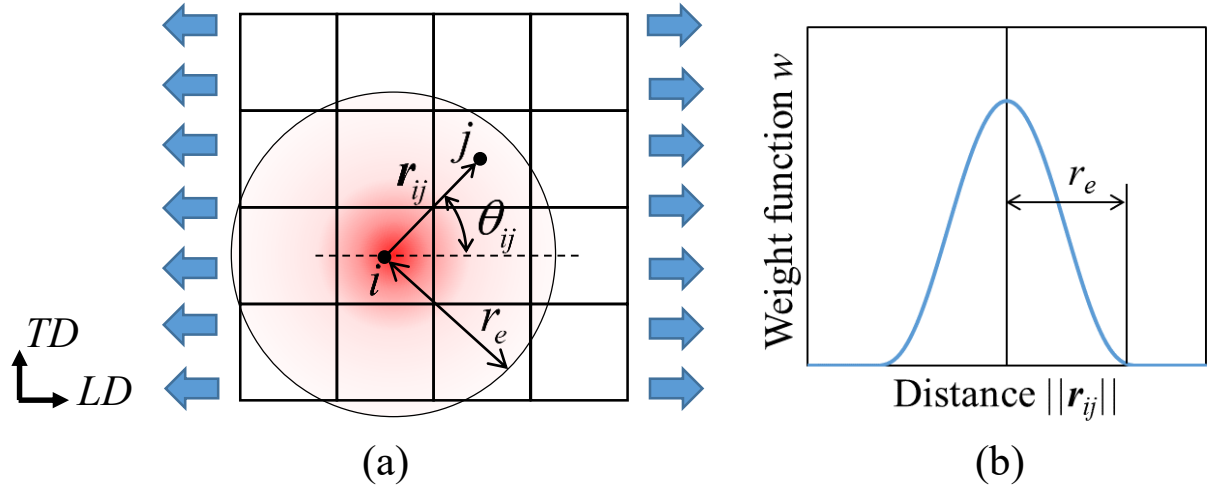


Fig. 3 (a) A schematic diagram showing the geometry of regions i and j . (b) A profile of the weight function to determine the interaction range.

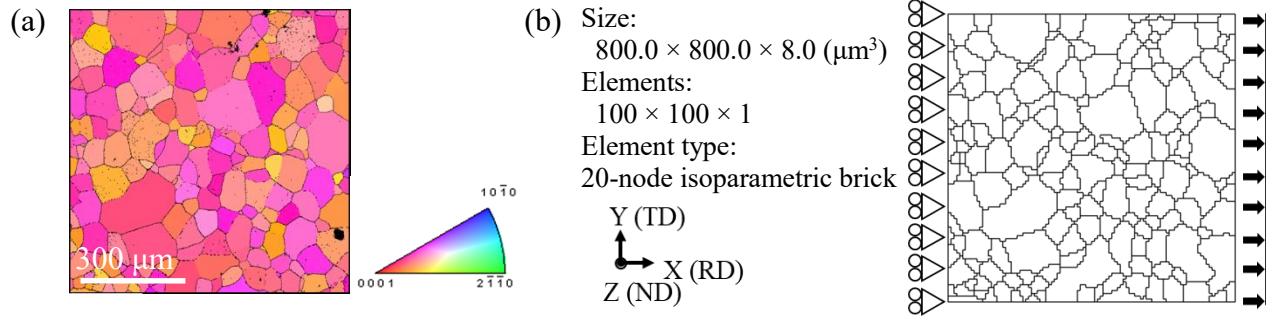


Fig. 4 (a) EBSD crystal orientation map of the studied sample. (b) The geometry model used for the uniaxial tensile loading analysis. The model (b) was built from the map (a) using a procedure developed by Kawano et al. (2018b).

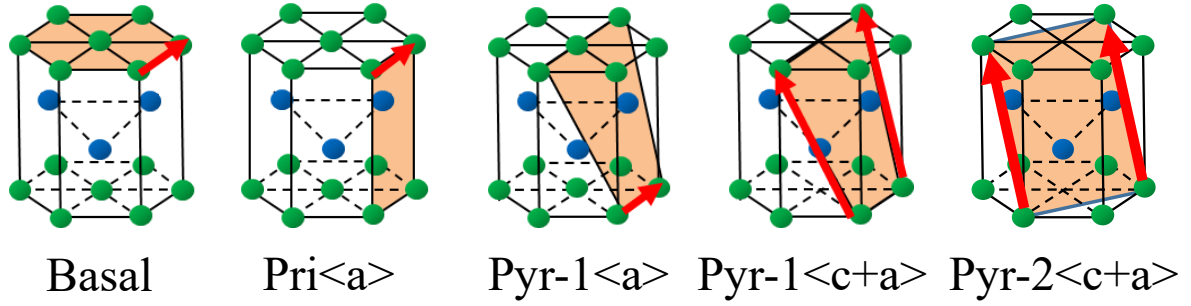
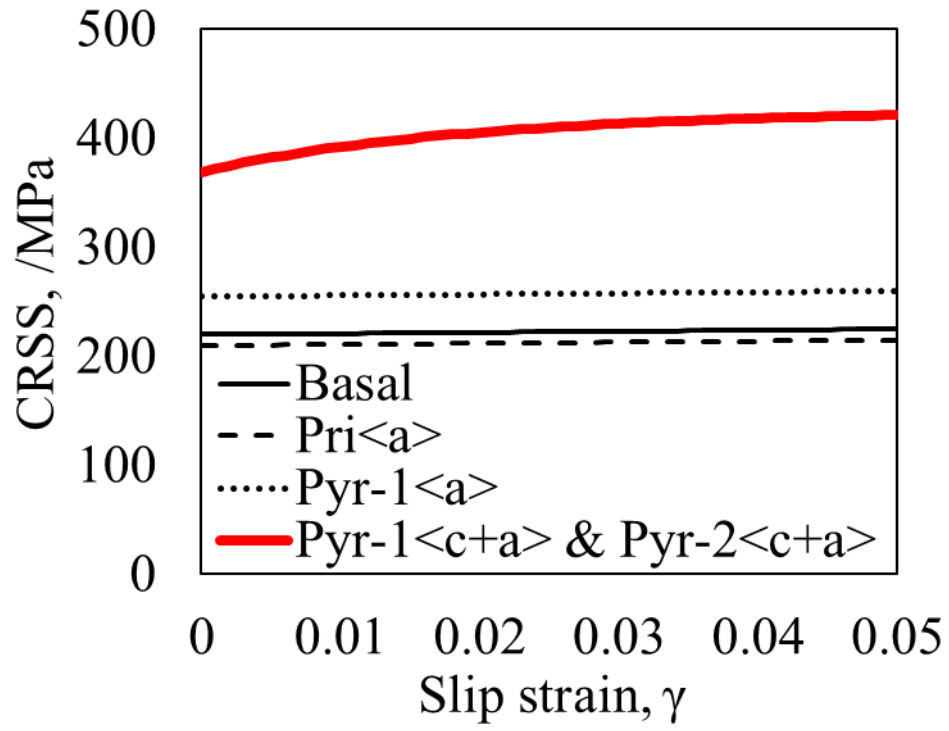


Fig. 5 Schematics showing slip systems of α -Ti. The symbols of “Basal”, “Pri<a>”, “Pyr-1<a>”, “Pyr-1<c+a>”, and “Pyr-2<c+a>” correspond to the basal slip system, prismatic <a> slip system, 1st pyramidal <a> slip system, and 1st and 2nd pyramidal <c+a> slip systems, respectively.

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3 Fig. 6 CRSS for each slip system as a function of slip strain.

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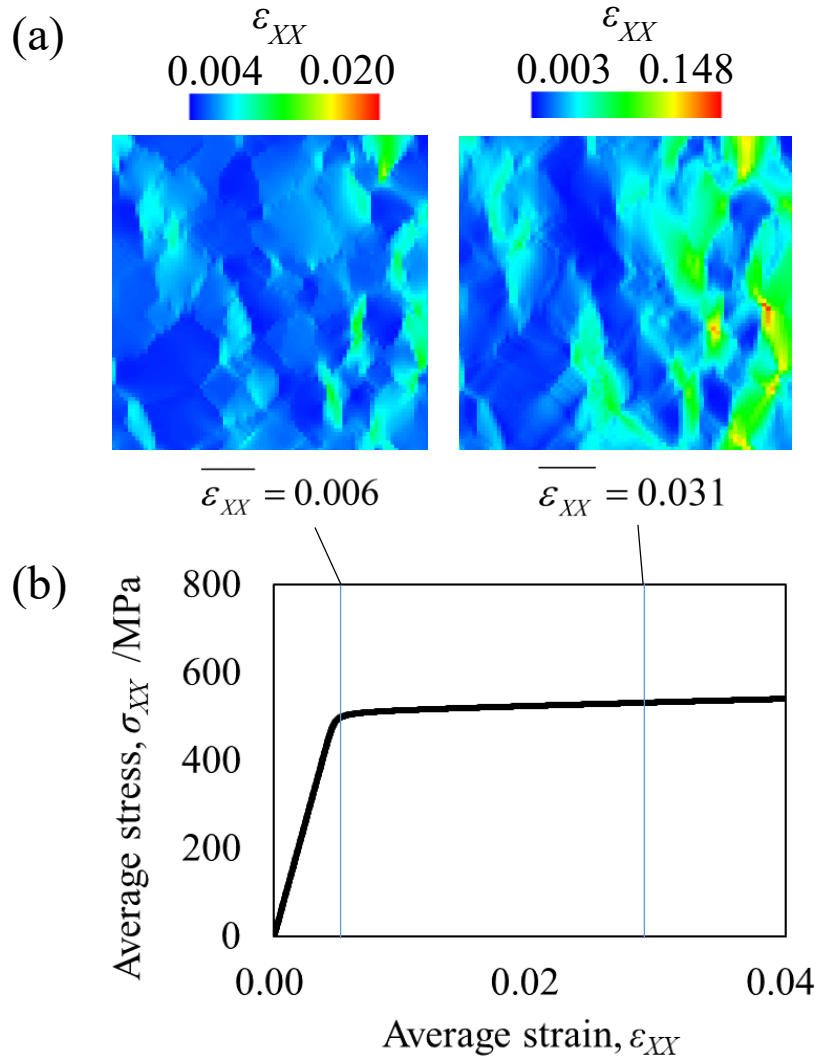


Fig. 7 (a) Distributions of normal strain in the X-direction. The regions 10 μm in size were removed from each side of the model as they were strongly affected by boundary conditions. (b) Profile of the stress-strain relationship.

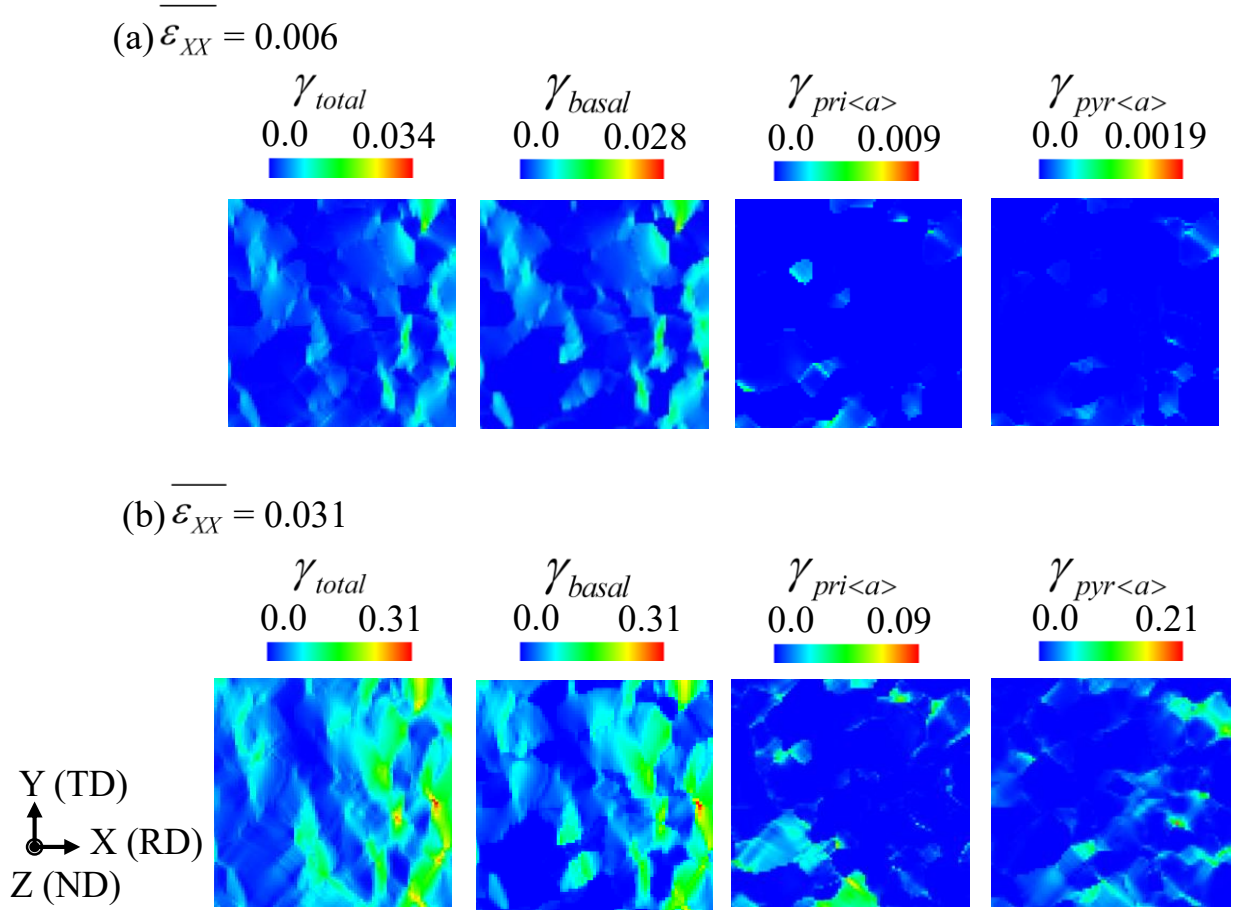


Fig. 8 Distributions of slip strains γ when (a) $\overline{\varepsilon_{xx}} = 0.006$ and (b) $\overline{\varepsilon_{xx}} = 0.031$.

γ_{basal} , $\gamma_{pri< a >}$, and $\gamma_{pyr< a >}$ are the summations of the absolute values of slip strains for the individual slip system families. The subscripts for γ refer to basal, prismatic $<a>$, and 1st pyramidal $<a>$ slip systems, and γ_{total} is the summation of γ_{basal} , $\gamma_{pri< a >}$, and $\gamma_{pyr< a >}$. The regions 10 μm in size were removed from each side of the model as they were strongly affected by boundary conditions.

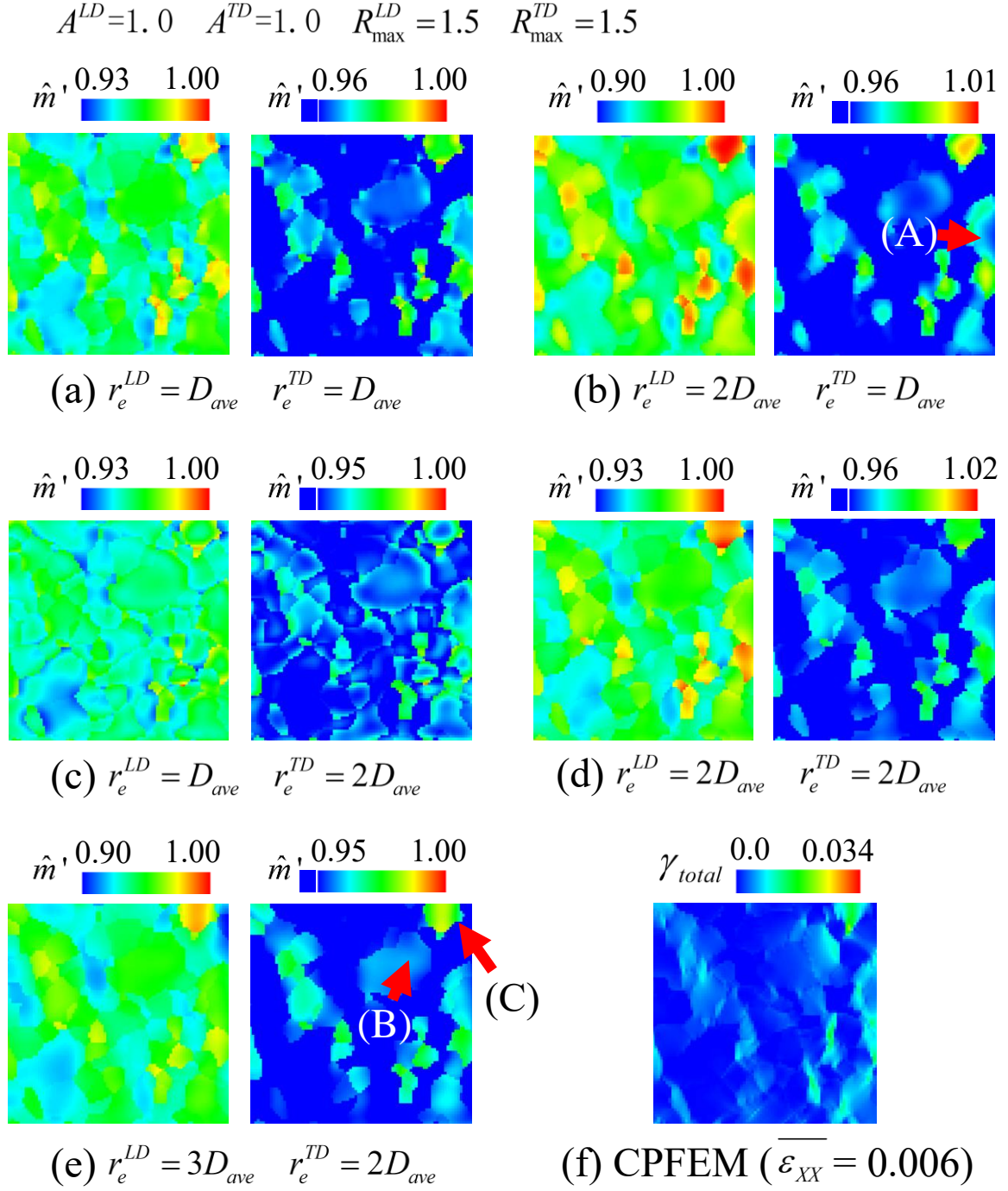


Fig. 9 Distributions of SOF (\hat{m}') and slip strain (γ_{total}) at the yield point obtained by the CPFE analysis. (a)–(e) Changes in the distributions of \hat{m}' with the influence range r_e , whereas r_e^{LD} and r_e^{TD} represent the influence range in the loading and transverse

directions, respectively. The SOF distributions were obtained from the conditions (I)-(V) in Table 4. D_{ave} (69.2 μm) is an average grain size. The regions 10 μm in size were removed from each side of the model as they were strongly affected by boundary conditions. Right-hand side figures contain the adjusted color contours to be easily compared with the distribution of Fig. 9f. (f) Distributions of slip strains γ_{total} when $\overline{\varepsilon_{xx}} = 0.006$.

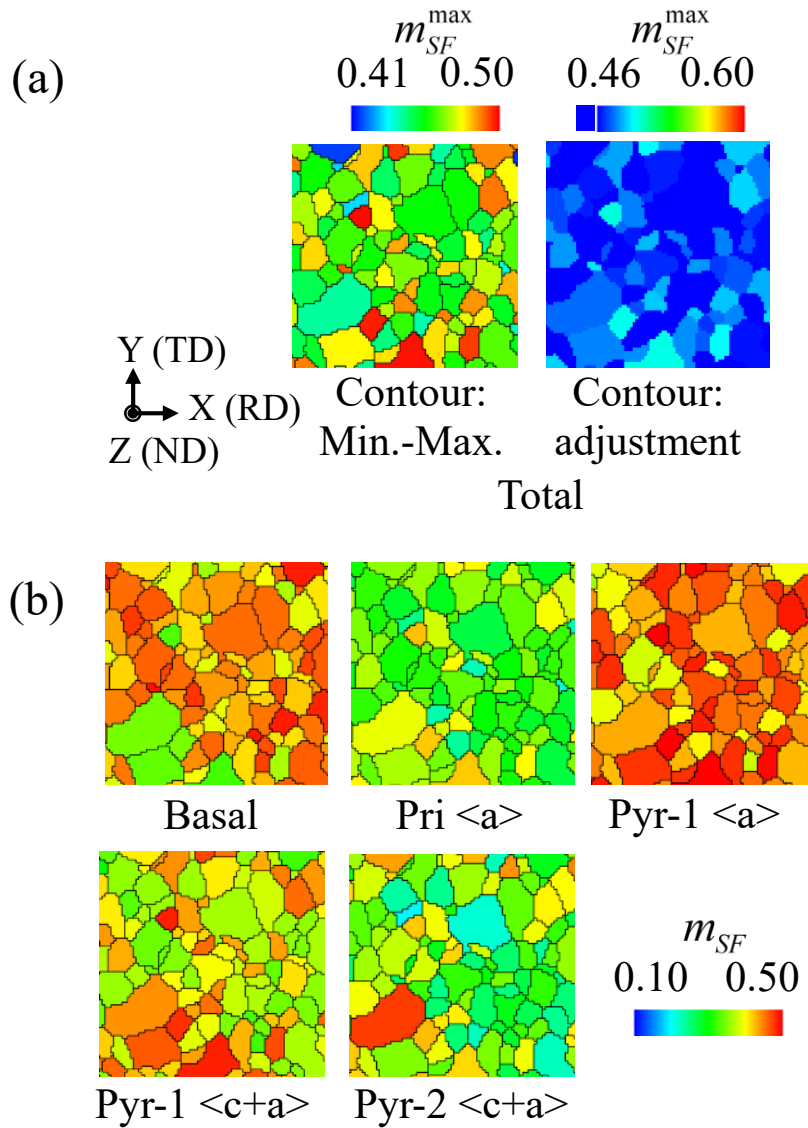


Fig. 10 Spatial distributions of the Schmid factor (SF) in the X-direction. (a) Spatial distributions for all of the slip systems. The right-hand side figure depicts the adjusted color contours to be easily compared with the distribution of Fig. 9(f). (b) Spatial distributions for each slip system. The regions 10 μm in size were removed from each side of the model as they were strongly affected by boundary conditions.

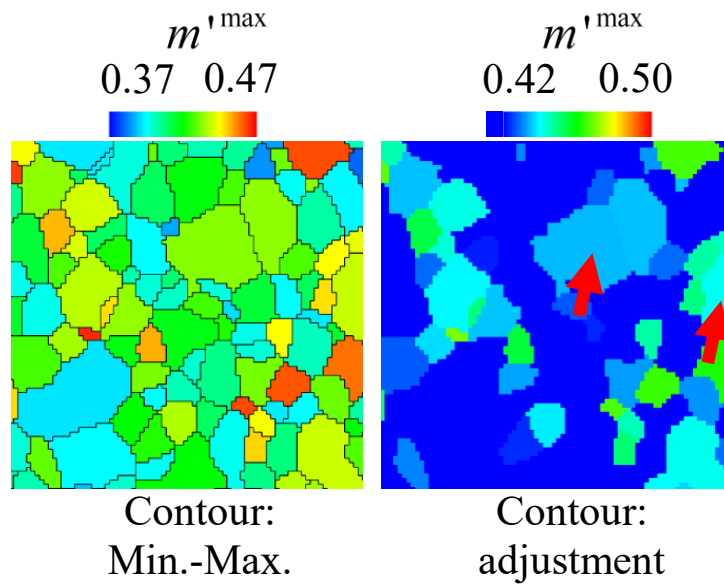
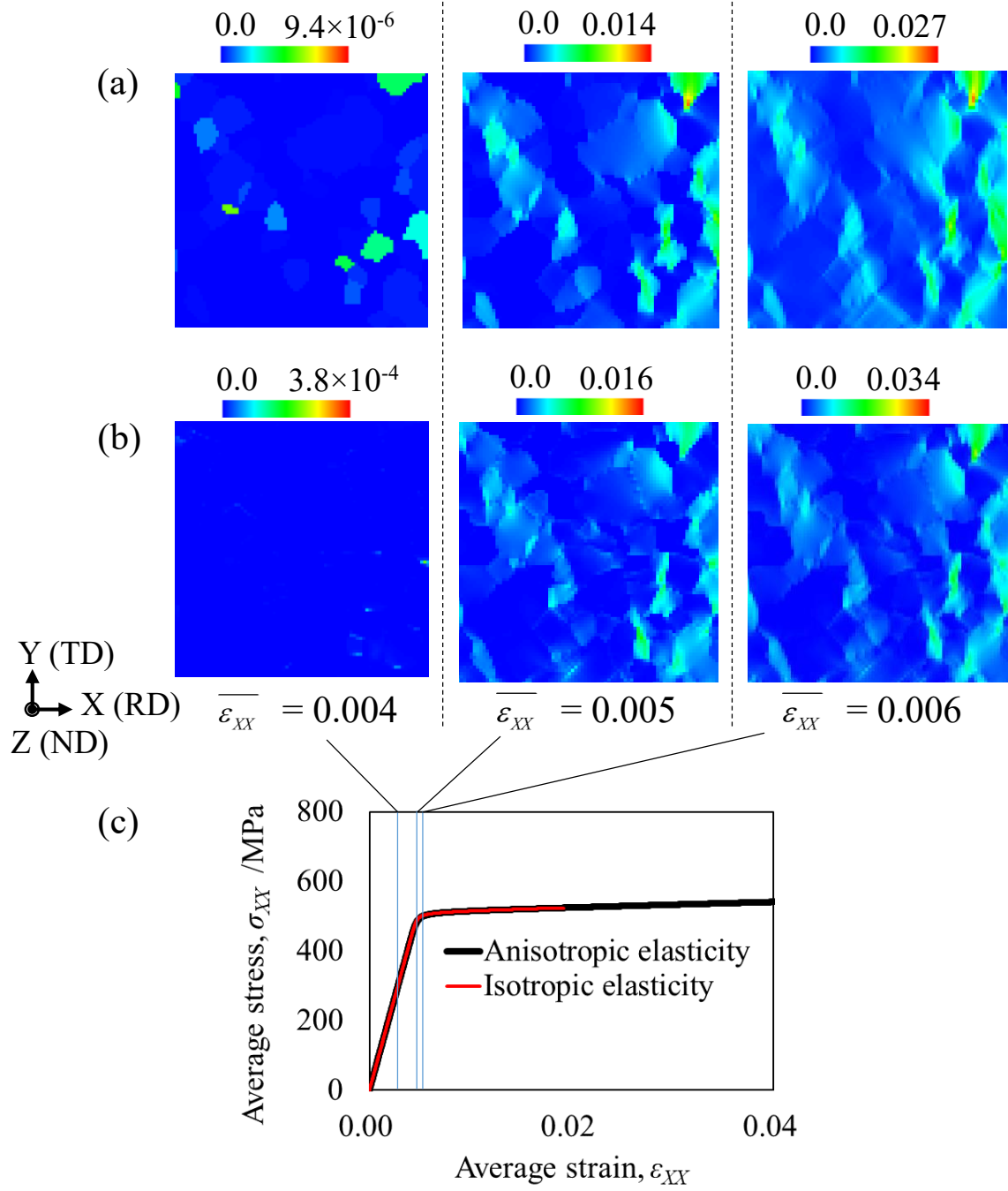


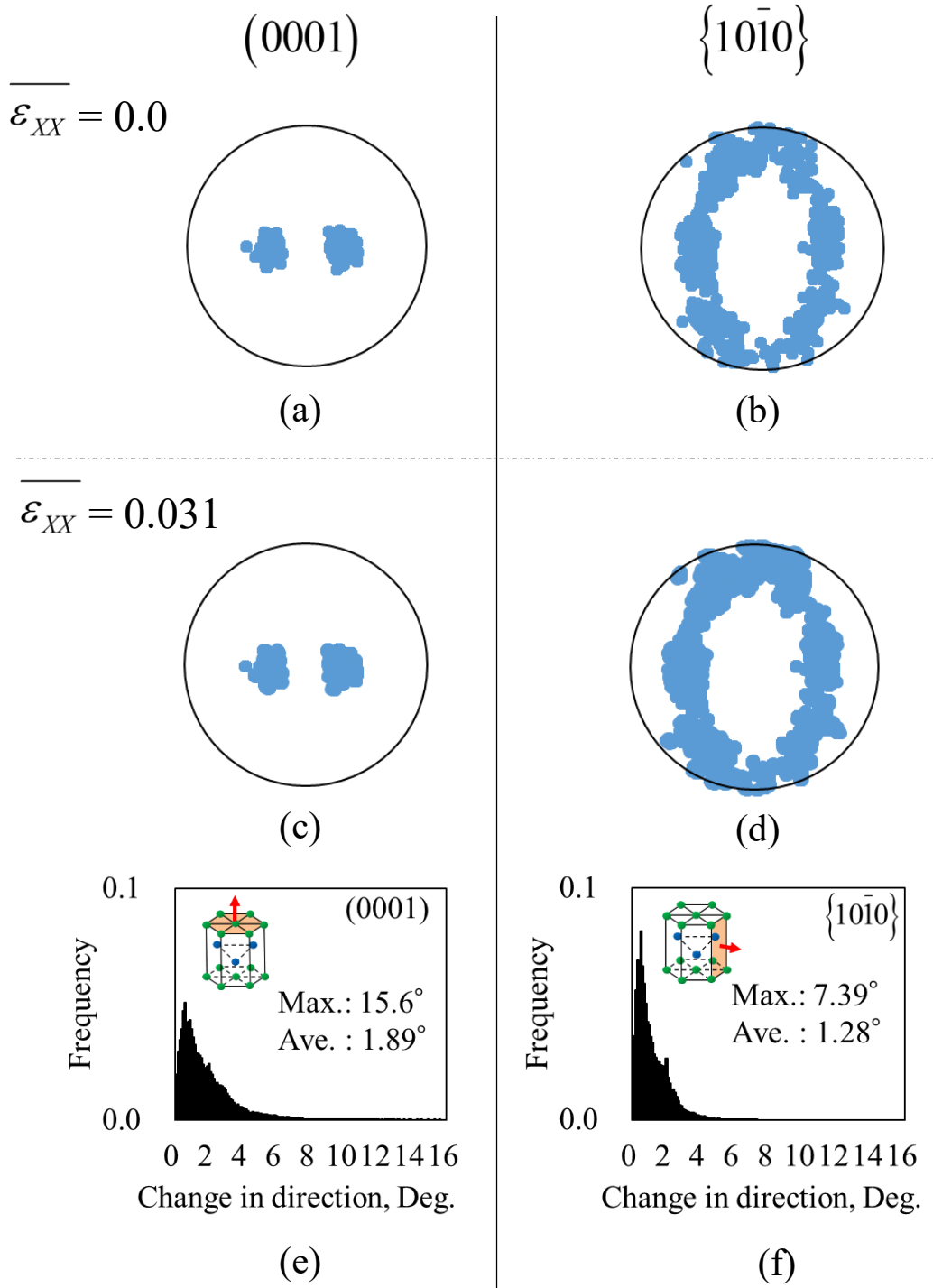
Fig. 11 Spatial distributions of the normalized Schmid factor (NSF) in the X-direction. The right-hand side figure depicts the adjusted color contours to be easily compared with the distribution of Fig. 9(f). The regions 10 μm in size were removed from each side of the model as they were strongly affected by boundary conditions.



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2 Fig. 12 (a)-(b) The distributions of γ_{total} . For (a), the γ_{total} distributions were obtained
 3 under the condition of isotropic elasticity when $E = 106.3$ GPa and $\nu = 0.34$. For (b), the
 4 γ_{total} distributions were calculated under the condition of anisotropic elasticity, a

condition similar to that in Fig. 9(f). The regions 10 μm in size were removed from each side of the model as they were strongly affected by boundary conditions. (c) The stress–strain relationship.



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3 Fig. 13 (a) and (c) correspond to (0001) pole figures and (b) and (d) are $\{10\bar{1}0\}$ pole

4 figures when $\overline{\epsilon_{XX}} = 0.0$ and 0.031 . (e) and (f) show the relative frequency distributions

1 of change in directions of the vectors normal to the (0001) and $\{10\bar{1}1\}$ planes when

2 $\overline{\varepsilon_{xx}}$ changes from 0.0 to 0.031.

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Tables

Table 1 Chemical composition of the sample material (wt.%).

Ti	O	Fe
Bal.	0.047	0.027

Table 2 Elastic compliance of pure titanium $[(\text{TPa})^{-1}]$ (Fisher and Renken, 1964).

S_{11}	S_{12}	S_{13}	S_{33}	S_{44}
9.581	-4.623	-1.893	6.980	21.413

Table 3 Values used for the Voce equation [MPa].

Slip systems	Basal	Pri<a>	Pyr-1<a>	Pyr-1<c+a> Pyr-2<c+a>
τ_0	220.0	210.0	255.0	369.0
τ_1	400.0	400.0	400.0	50.0
θ_0	100.0	100.0	100.0	3000.0
θ_1	200.0	200.0	200.0	100.0

1 Table 4 CRSS and other parameters for SOF for each condition. D_{ave} (69.2 μm) is an
 2 average grain size in the test specimen.

Conditions	(I)	(II)	(III)	(IV)	(V)
CRSS	Same as τ_0 in Table 3				
A^{LD}, A^{TD}	$A^{LD} = A^{TD} = 1.0$				
$R_{\max}^{LD}, R_{\max}^{TD}$	$R_{\max}^{LD} = R_{\max}^{TD} = 1.5$				
r_e^{LD}	D_{ave}	$2D_{ave}$	D_{ave}	$2D_{ave}$	$3D_{ave}$
r_e^{TD}	D_{ave}	D_{ave}	$2D_{ave}$	$2D_{ave}$	$2D_{ave}$

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