

1 Crystal Plasticity Analysis of Change in Incompatibility and Activities of Slip Systems in
2 α -phase of Ti alloy under Cyclic Loading

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16 Highlights

- 17 • Changes in activities of slip systems in the α -phase of Ti-6Al-4V under cyclic loading
18 are investigated by crystal plasticity analysis of bicrystals.
- 19 • The trade-off relationship of activities of the basal slip system is observed in a crystal
20 grain and the prismatic $\langle a \rangle$ slip system in the other crystal grains.
- 21 • Incompatibility between the crystal grains due to elastic anisotropy and its changes by
22 work hardening exhibits the potential to lead the trade-off relationship.
- 23 • The mechanisms of the trade-off relationship are more easily explained by wedge
24 disclination type deformation fields.

1 Abstract

2 With respect to Ti-6Al-4V alloy (Ti-64) with duplex phases constructed by α -phase with
3 an hcp structure and β -phase with a bcc structure, basal slips in α -phase can potentially
4 lead to crack initiation under fatigue loading. Hence, it is necessary to quantitatively
5 evaluate the activity of basal slip systems. However, the critical resolved shear stress
6 (CRSS) for the basal slips in α -phase is equal to or slightly exceeds that for the prismatic
7 slips, and mechanisms for the activation of slip systems are not completely understood.
8 The present study involved conducting crystal plasticity analysis of α -phase subjected to
9 cyclic loading and uniaxial tensile loading employing bicrystal models and uniaxial
10 tensile analysis of tricrystal models. The results indicated that a trade-off relationship
11 exists between the activities of slip systems. Specifically, the inactivation of prismatic
12 $\langle a \rangle$ slip system in a crystal grain activates the basal slip system in the neighboring crystal
13 grains due to the changes of incompatibility by work hardening around the boundary triple
14 junction line. The changes in incompatibility can be explained by wedge disclination type
15 deformation field.

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17 Keywords: Crystal plasticity analysis, Activities of slip systems, α -phase, Cyclic loading,
18 incompatibility

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1. Introduction

Ti alloys are widely used in several fields, including aerospace, medical, and sports. Thus, improvements in the accuracy of the fatigue life predictions are important. Specifically, Ti-6Al-4V (Ti-64) is the most widely used Ti alloy [1], and the enhancement in the quality of predictions can lead to developments of products with higher reliability and performance. However, the fatigue mechanisms of Ti-64 are not fully understood.

Ti-64 with duplex phases is composed of α -phase with an hcp structure and β -phase with a bcc structure. The microstructures of Ti-64 change due to plastic forming and heat treatments [1,2] and include fully lamellar microstructures, equiaxed microstructures, and bimodal type microstructures. When a globular α -phase (primary α) exists in the microstructures, fatigue crack initiation is mainly observed within the basal slip bands in the primary α [3,4]. This suggests the high activity of the basal slip system, and the basal slips influence the fatigue failure in the primary stage [4-6].

Under fatigue loadings, the number of cycles required to nucleate a crack may constitute 5-20% of the total lifetime in the high cycle fatigue (HCF) regime [7,8] and higher percentages in the very high cycle fatigue (VHCF) when compared with those of HCF [8]. Thus, it is necessary to quantitatively evaluate the activity of a basal slip system to reveal initial fatigue mechanisms in Ti-64.

Active slip systems of primary α in Ti-64 are basal slip system, prismatic $\langle a \rangle$ slip system, first-order pyramidal $\langle a \rangle$ slip system, and first-order pyramidal $\langle c+a \rangle$ slip system [9,10].

1 The basal and prismatic $\langle a \rangle$ are mainly observed as active slip systems in Ti-64 under
2 fatigue loadings. However, previous studies [9,11-13] showed that critical resolved shear
3 stress (CRSS) on the basal slip system is equal to or slightly exceeds that for the prismatic
4 $\langle a \rangle$ slip system. Thus, mechanisms exist to actively operate the basal slip system. The
5 reasons for the high activity of basal slip system remain less well-defined while other
6 effects, such as elastic anisotropy [5] and work softening [9], can be considered as
7 plausible reasons.

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9 Incompatibility between crystal grains [14,15] potentially causes the high activity of a
10 basal slip system under fatigue loadings. A reason for the incompatibility corresponds to
11 the anisotropic elasto-plasticity in the level of crystal grains. In the deformation of
12 polycrystals, crystal grains with anisotropic elasto-plasticity deform in different shapes
13 from others to reflect their crystal orientations. However, their deformations are mutually
14 constrained by the grain boundaries, and the stresses and strains become inhomogeneous.
15 Under cyclic loadings, inactivation of slip systems by work hardening also affects
16 incompatibility. The work hardening rates of α -Ti alloy change based on interstitial
17 elements such as oxygen [16]. Thus, it is important to reveal the mechanisms of fatigue
18 failure in the primary stage to understand the relationship between changes of
19 incompatibility and activities of slip systems.

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21 Mechanical interactions between crystal grains in polycrystals are considerably complex,
22 and analysis employing bicrystals were performed to obtain simpler understandable
23 results. Previous studies indicated that localized stresses occurred near the grain
24 boundaries due to the incompatibility, and the stresses inhibit or enhance the activities of

1 slip systems [17,18]. Specifically, reports suggest that crack initiation occurs in the
2 regions far from the grain boundary of bicrystals with high compatibility while cracks
3 nucleate from the grain boundary of bicrystals with incompatibility [14]. However, results
4 obtained even by the analysis employing bicrystals are complex, and cannot not be easily
5 understood only by experimental methods. Thus, numerical analysis of bicrystals has
6 been conducted by molecular dynamics (MD) [19,20], finite element method (FEM)
7 [21,22]. Furthermore, FEM analyses dealing with continuum cannot track the behavior
8 of dislocations on an atomic scale but can reproduce interactions between them in the
9 crystal grain scale.

10

11 Extant studies performed the analysis of bicrystals by employing crystal plasticity finite
12 element methods (CPFEM). The effects of elasticity and plasticity are considered
13 separately by CPFEM. Ohashi [21] performed uniaxial tensile analysis of a plastically
14 incompatible type bicrystal in which crystal grains were arranged in the loading direction
15 and reproduced inhomogeneous deformation near the grain boundary. Kondou & Ohashi
16 [23] performed uniaxial tensile analysis of a compatible type bicrystal in which crystal
17 grains were arranged in a direction perpendicular to the loading direction. The results
18 indicated that the bicrystal was inhomogeneously deformed by the effect of changes in
19 the shapes of grains due to plastic anisotropy and the displacement constraints and that
20 dislocation bands were formed by the inhomogeneous deformation. Mayama et al. [24]
21 conducted a cyclic loading plasticity analysis employing a bicrystal model similar to that
22 used by Kondou & Ohashi [23], and investigated the relationship between the dislocation
23 patterns and amplitudes of strains. Kawano et al. [25] investigated the relationship
24 between inhomogeneous deformations and changes in incompatibility due to inclinations

1 of the grain boundary in bicrystal models. However, there is a paucity of studies
2 examining anisotropic elasticity with respect to changes in incompatibility under cyclic
3 loadings.

4

5 The fore-mentioned studies show that the combination of changes in shapes of crystal
6 grains by plastic deformations and the boundary conditions induce inhomogeneous
7 deformation even in compatible-type bicrystals in which crystal grains are arranged in a
8 direction perpendicular to the loading direction. In contrast, interactions do not occur
9 between crystal grains by plastic deformation when the angles of slip plane normal
10 vectors and slip directions correspond to 45° in the loading direction and plastic strains in
11 the loading direction are equal between the crystal grains. This is because the shearing
12 components of plastic deformations correspond to zero under the conditions (as shown in
13 Fig. 1).

14

15 As previously stated, crystal plasticity analysis employing bicrystals simplifies the
16 phenomena and aids in understanding the results. However, it is necessary to focus on the
17 difference in the displacement constraints between bicrystals and actual materials. With
18 respect to the above analysis employing bicrystals, displacement constraints exist on the
19 bottom surfaces, and uniform displacements are applied to the top surfaces. Thus, the
20 shapes of bottom and top surfaces remain flat throughout the deformation process
21 independent of nonuniformity and anisotropy in the materials. The constraint surfaces are
22 considered as the constraint due to “pseudo” crystal grains [26]. However, the constraint
23 condition is strict when compared to the actual condition. Crystal grains in structural
24 materials commonly deform without maintaining the flat shapes of grain boundaries. It is

1 necessary to investigate the effects of the constraint conditions on the results.

2

3 In this study, crystal plasticity analysis of α -phase is performed to reveal the effects of
4 elastic anisotropy and work hardening on the activity of basal slip system by employing
5 bi- and tricrystal models. First, cyclic loading analysis using a bicrystal model is
6 performed to survey the activity changes in basal and prismatic $\langle a \rangle$ slip systems. Second,
7 the reasons for the activity changes are investigated by using analyses employing bicrystal
8 and tricrystal models. The effects of boundary conditions of bicrystal models on the
9 activities of slip systems are also investigated using tricrystal models.

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1 2. Procedure for crystal plasticity analysis

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3 2.1 Constitutive equations

4 A three-dimensional crystal plasticity finite element analysis code developed by Ohashi
5 [21,23-27] was employed. The constitutive equations employed in the study are shown
6 below. It is assumed that Schmid's law is used, and an elasto-plastic constitutive equation
7 for slip deformation is given as follows:

$$\dot{\epsilon}_{ij} = \left[S_{ijkl}^e + \sum_n \sum_m \{h^{(nm)}\}^{-1} P_{ij}^{(n)} P_{kl}^{(m)} \right] \dot{\sigma}_{kl}, \quad (1)$$

8 where $\dot{\epsilon}_{ij}$, $\dot{\sigma}_{kl}$, S_{ijkl}^e , $h^{(nm)}$ and $P_{ij}^{(n)}$ denote the increment in the strain, the increment
9 in the Cauchy's stress tensor, elastic compliance, coefficient of work hardening, and the
10 Schmid tensor, respectively [28]. The superscripts m and n indicate slip systems. The
11 CRSS is given by a modified Bailey-Hirsch model as follows:

$$\theta^{(n)} = \theta_0(T) + \sum_m a\mu\tilde{b}\Omega^{(nm)}\sqrt{\rho_s^{(m)}}, \quad (2)$$

12 where θ_0 , $\Omega^{(nm)}$, a , μ , \tilde{b} and $\rho_s^{(m)}$ denote the lattice friction stress, interaction
13 matrix between slip systems, a numerical factor of the order of 0.1, elastic shear modulus,
14 magnitude of Burgers vector, and density of statistically stored dislocations (SSDs),
15 respectively [27]. The increment in the SSD density is given as follows:

$$\dot{\rho}_s^{(n)} = \frac{c\dot{\gamma}^{(n)}}{\tilde{b}L^{(n)}}, \quad (3)$$

16 where c is a numerical coefficient of the order of 1 and $\dot{\gamma}^{(n)}$ denotes the increment in
17 the plastic strain. Additionally, $L^{(n)}$ is the mean free path of dislocations that is defined
18 as follows:

$$L^{(m)} = \frac{c^*}{\sqrt{\sum_k \omega^{(km)} (\rho_s^{(k)} + \|\rho_G^{(k)}\|)}}, \quad (4)$$

1 where c^* and $\omega^{(km)}$ denote a numerical constant and the weight matrix. $\|\rho_G^{(k)}\|$ is the
 2 norm of geometrically necessary dislocation (GND) density defined as follows:

$$\|\rho_G^{(m)}\| = \sqrt{(\rho_{G,edge}^{(m)})^2 + (\rho_{G,screw}^{(m)})^2}, \quad (5)$$

$$\rho_{G,edge}^{(m)} = -\frac{1}{\tilde{b}} \frac{\partial \gamma^{(m)}}{\partial \xi^{(m)}}, \quad \rho_{G,screw}^{(m)} = \frac{1}{\tilde{b}} \frac{\partial \gamma^{(m)}}{\partial \zeta^{(m)}}, \quad (6)$$

3 where $\rho_{G,edge}^{(m)}$ and $\rho_{G,screw}^{(m)}$ are densities of edge and screw components of GNDs,
 4 respectively. Here $\xi^{(m)}$ and $\zeta^{(m)}$ denote directions parallel and perpendicular to the
 5 slip direction on the slip plane, respectively. The increment of the CRSS is given as
 6 follows:

$$\dot{\theta}^{(n)} = h^{(nm)} \dot{\gamma}^{(m)}. \quad (7)$$

7 The strain hardening coefficient $h^{(nm)}$ is obtained from Eqs. 2-4 and Eq. 7 as follows:

$$h^{(nm)} = \frac{ac\mu\Omega^{(nm)}}{2L^{(m)}\sqrt{\rho_s^{(m)}}}. \quad (8)$$

8
 9 When GN dislocations accumulate in materials under cyclic loading, internal stress field
 10 will develop and this will affect the subsequent activities of dislocations and result in the
 11 Bauschinger effect under cyclic loading. Recently, this effect of internal stress is being
 12 introduced in cyclic loading analyses [29]. However, the full understandings of GN
 13 dislocations near grain boundaries as a source of back stress are still controversial. Thus,
 14 the effect of internal stress is not taken into account in the current study although the

1 higher-order theory [30] is one possibility.

2

3 2.2 Finite element analysis models

4 2.2.1 Bicrystal model

5 Crystal plasticity analysis for bicrystals is conducted with changing loading conditions,
6 crystal orientations, and CRSSs. The geometries are shown in Figs. 2 and 3. Two types of
7 loading conditions are employed, namely a cyclic loading condition (Fig. 2) and a uniaxial
8 tensile loading condition (Fig. 3). The tensile or compressive loading is given by the
9 enforced displacement in the Y-direction of all the nodes on the top surface. Additionally,
10 all the nodes on the bottom surface are fixed in the Y-direction. Elastic compliances for
11 Ti with a hcp structure employed in the analyses are described in Table 1 [31].

12

13 The cyclic loading analysis (Fig. 2) is conducted employing four sets of c^* shown in
14 Table 2. Work hardening rates increase with decreases in c^* since the mean free path of
15 moving dislocations becomes shorter and accumulation rates of SSDs increase on the
16 condition of higher c^* . In Table 2, EQ-hardening is the condition in which work
17 hardening rates are identical between basal and prismatic $\langle a \rangle$ slip systems, and No-basal
18 hardening is the condition in which work hardening does not occur on the basal slip
19 system.

20

21 In α -phase in Ti-64, relative CRSSs for the prismatic $\langle a \rangle$ slip system normalized to CRSS
22 for the basal slips correspond to 0.93-1.30 as reported by Medina Perilla & Gil Sevillano
23 [11], 1.43 by Seminatin & Beiler [12], and 1.135 by Bridier et al. [9]. Gil Sevillano [11]
24 experimentally estimated that the CRSSs for the basal and prismatic $\langle a \rangle$ slip systems

1 correspond to 395 MPa and approximately 500 MPa, respectively. Bridier et al. [13]
2 estimated 420MPa for the basal slips and 370 MPa for the prismatic $\langle a \rangle$ slips by the data
3 from Bridier et al. [9]. In the PRI-BSL cyclic analysis (Fig. 2) as shown in Table 3, 425.5
4 MPa and 370 MPa are employed as the CRSSs for the basal slips and prismatic $\langle a \rangle$ slips,
5 respectively. The relative CRSS of the basal slip system normalized to the CRSS for the
6 prismatic $\langle a \rangle$ slips corresponds to 1.15. The crystal orientations are set such that the
7 prismatic $\langle a \rangle$ and basal slip systems are the primary slip systems in Grains 1 and 2,
8 respectively.

9

10 The uniaxial tensile analysis is conducted with two types of crystal orientations, namely
11 PRI-BSL (Fig. 3 (a)) and PRI-PRI (Fig. 3 (b)). In the PRI-PRI model, both c-axes of
12 crystal grains correspond to the Z-axis, and prismatic $\langle a \rangle$ constitutes the primary slip
13 system in both Grains 1 and 2. Crystal grains with the hcp structure exhibit anisotropic
14 elasticity albeit with isotropic elasticity within the basal plane. An analysis of PRI-PRI
15 tension is performed to compare the results of PRI-BSL analysis. In order to investigate
16 the relationship between activities of slip systems and CRSSs, analyses are conducted
17 several times with changing sets of CRSSs as shown in Tables 3 and 4. Several CRSSs
18 are employed for the prismatic $\langle a \rangle$ slip system while CRSS for the basal slip system is
19 fixed.

20

21 2.2.2 Tricrystal model

22 A tricrystal plasticity analysis is conducted to investigate effects of the forced
23 displacement on the bicrystal models, and the geometries are shown in Fig. 4. The
24 tricrystal models are structured by adding the third crystal grain on the top (Fig. 4 (a)) or

1 bottom of the bicrystal model (Fig. 4 (b)). The boundary conditions are the same as those
2 used in the bicrystal models. The bottom surface is fixed in the Y-direction, and the top
3 surface is subjected to forced tensile displacement. Crystal orientations in Grains 1 and 2
4 are identical to those in the PRI-BSL model (Fig. 3 (a)), and it is assumed that the c-axis
5 in Grain 3 corresponds to the loading direction. The elastic deformation in α -phase is the
6 most difficult in the c-axis direction [32] and the Schmid factors on the basal and
7 prismatic $\langle a \rangle$ slip systems correspond to zero in the crystal orientation of Grain 3. Thus,
8 Grain 3 exhibits the most difficult orientation in the loading direction.

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3. Results and discussion

3.1 Cyclic loading analysis

Fig. 5 shows the distributions of slip strains on the basal and prismatic $\langle a \rangle$ slip systems at the peak of tension ($\bar{\varepsilon}_{yy} = 0.7\%$) and compression ($\bar{\varepsilon}_{yy} = 0.7\%$) under the No-basal hardening condition with $c^*_{prismatic\langle a \rangle} = 15.0$. First, distributions of slip strains at the 1st cycle are observed. Slip systems operate within the regions near the corner at the bottom of the bicrystal and the grain boundary near the top surface. Schmid factors correspond to 0.5 in both Grains 1 and 2, and the primary slip systems correspond to the prismatic $\langle a \rangle$ and basal slip systems in Grain 1 and Grain 2, respectively. The active slip systems correspond to prismatic $\langle a \rangle$ system in Grain 1 and basal slip system in Grain 2. Thus, the basal slip system is activated despite CRSS for the basal slip system is higher than that of prismatic $\langle a \rangle$ slip system. This is because the α -phase is difficult to deform in the c-axis, and crystal grains with the c-axis more close to the loading direction tend to receive higher stresses [32].

Fig. 6 shows distributions of stresses in the elastic stage. The stresses are higher in Grain 2 when compared to that in Grain 1. A stress concentration is observed near the grain boundary in Grain 2. Kondou & Ohashi [33] conducted uniaxial tensile analysis employing fcc bicrystal models with tilt angle grain boundary and isotropic elasticity. The results showed that inhomogeneous deformation occurred due to plastic anisotropy and restraint of displacement on the top and bottom surfaces of the bicrystal. Additionally, the distributions of slip strains are similar to those obtained by the current study. However, a

1 difference also exists between the results. The current results show slip systems activated
2 along the grain boundary in Grain 2 due to the stress concentration caused by the
3 anisotropic elasticity. In contrast, a similar activation of slip system along the grain
4 boundary was not observed in the study by Kondou & Ohashi [33] since the isotropic
5 elasticity model was employed. Kondou & Ohashi [36] conducted a uniaxial tensile
6 loading analysis of a bicrystal with anisotropic elasticity, and the results indicated that
7 slip systems were activated along the grain boundary.

8
9 Second, changes in the distributions of slip strains are described below (Fig. 5). The slip
10 strains are positive at the peak of tension ($\bar{\varepsilon}_{yy} = 0.7\%$) and negative in the compression
11 ($\bar{\varepsilon}_{yy} = -0.7\%$) in the early stage of cyclic loading. The distributions of the absolute values
12 of slip strains at the tension and compression are almost identical. In Grain 1, work
13 hardening occurs during cyclic loading. Furthermore, the maximum slip strain decreases
14 and positive and negative residual strains increase with cycles progress. Conversely, basal
15 slip strains in Grain 2 exhibit a slightly increase at the corner of the bottom of the bicrystal.

16 Other results obtained in the No-basal hardening condition with different $c^*_{prismatic\langle a \rangle}$
17 also indicate the same tendency. Fig. 7 shows dependency of $c^*_{prismatic\langle a \rangle}$ on the
18 distributions of slip strains at the peak of tension in the 25th cycle. The above tendency is
19 observed in the No-basal hardening condition while both slip strains of basal and
20 prismatic $\langle a \rangle$ decrease with respect to the EQ-hardening condition.

21
22 This is followed by discussing changes in the activities of basal and prismatic $\langle a \rangle$ slip
23 systems. Figs 7 and 8 show that the changes in cumulative slip strains on an average with

1 respect to the basal and prismatic $\langle a \rangle$ slip systems within each crystal grain. Fig. 8 shows
2 the rates of increase in both cumulative basal and prismatic $\langle a \rangle$ slip strains decrease with
3 respect to the cycles for the EQ-hardening condition. In contrast, the cumulative slip
4 strains exhibit a linear increase on the basal slip system for the No-basal hardening
5 condition, and the activity of the basal slip system does not decrease. Fig. 9 shows the
6 dependency of $c^*_{prismatic\langle a \rangle}$ on the activities of slip systems for the No-basal hardening
7 condition. Increases in the rates of cumulative slip strains on the prismatic $\langle a \rangle$ slip
8 systems decrease earlier with respect to higher $c^*_{prismatic\langle a \rangle}$. This is because SSDs are
9 more easily accumulated with higher $c^*_{prismatic\langle a \rangle}$ as shown in Fig. 10. In contrast, the
10 activity of the basal slip system is higher when $c^*_{prismatic\langle a \rangle}$ is higher (i.e., the activity
11 of the basal slip system is higher in Grain 2 when the work hardening occurs more easily
12 in Grain 1).

13

14 In the current simulations, bicrystals are deformed by the forced displacement. The
15 maximum and minimum displacements in the Y-axis remain unchanged throughout the
16 cyclic loading. Additionally, work hardening does not occur in Grain 2 (on the basal slip
17 system) for the No-basal hardening condition. Thus, it is necessary for the activity of the
18 slip system in Grain 2 to remain unchanged throughout the cyclic loading when the Grain
19 2 is not affected by Grain 1. That is, work hardening of the prismatic $\langle a \rangle$ slip system in
20 Grain 1 affects the activity of the basal slip system in Grain 2. Therefore, the results
21 indicate that inactivation of the prismatic $\langle a \rangle$ slip system in a crystal grain potentially
22 activates the slip systems in the neighboring crystal grains.

23

1 The activity of the basal slip system decreases for the condition when work hardening
2 occurs in the basal slip system with respect to the EQ-hardening condition, and the trade-
3 off relationship between the activities of slip systems is hidden and could not be
4 recognized. However, the trade-off relationship may result in the scattering of fatigue life,
5 and thus should not be ignored. The mechanisms are discussed in the next section.

6 7 8 3.2 Investigation of effect of incompatibility

9 Crystal plasticity analysis of unidirectional tensions of bicrystals is performed by
10 employing several CRSSs for the prismatic $\langle a \rangle$ slip system. Increases in CRSSs for the
11 prismatic $\langle a \rangle$ slip system stagnate the activity of the prismatic $\langle a \rangle$. Thus, work hardening
12 is approximately expressed by increases in CRSSs, and the study investigates the manner
13 in which the work hardening on the prismatic $\langle a \rangle$ slip system in Grain 1 influences the
14 activity of the basal slip system in Grain 2. The results are shown in Fig. 11.

15
16 Fig. 11 shows the distributions of slip strains obtained on the condition PRI-BSL tension.
17 The activity of the prismatic $\langle a \rangle$ slip system decreases with increases in CRSS for the
18 prismatic $\langle a \rangle$ slip system while the activity of basal slip system increases. The slip strains
19 are observed within the region near the corner of the bottom in both Grain 1 and 2 and
20 near the grain boundary on the upper side in Grain 2. These results show a tendency
21 similar to those obtained by the current cyclic loading analysis and indicate that similar
22 phenomena are reproduced by the increase in CRSS for the prismatic $\langle a \rangle$ slip system.

23
24 Next, the study examines the results obtained in the condition of PRI-PRI tension in which

1 the effect of anisotropic elasticity does not appear in the crystal grains in Fig. 12. The
2 prismatic $\langle a \rangle$ slip system is the primary system in both Grains 1 and 2 for the PRI-PRI
3 tension condition. Two sets of CRSSs are employed, namely one in which the CRSSs of
4 Grain 1 and Grain 2 are equal (EQ-CRSS) and the other in which the CRSSs of Grain 1
5 exceeds that in Grain 2 (NonEQ-CRSS). In the condition corresponding to EQ-CRSS, the
6 bicrystal deforms homogeneously, and the stresses and strains are also homogeneous (Fig.
7 12 (a), (c)). In condition NonEQ-CRSS, normal stresses in Grain 1 exceed those in Grain
8 2 (Fig. 12 (b)) while normal strains in the Y-axis are homogeneous (Fig. 12 (c)).

9
10 The above homogeneous deformations in the Y-axis indicates that bicrystals deform while
11 maintaining compatibility between Grains 1 and 2. Three reasons are identified with
12 respect to maintaining the compatibility in the condition PRI-PRI tension. First, isotropic
13 elasticity exists within the basal plane. Second, angles of slip plane normal vector and
14 slip direction are 45° with respect to the loading direction (as shown in Fig. 1). The crystal
15 grains maintain the rectangular shapes in both elastic and plastic deformations under the
16 conditions. Third, the bicrystal model is subjected to the forced displacement that is equal
17 between Grains 1 and 2. The difference in the CRSSs between Grains 1 and 2 causes a
18 difference in the elastic and plastic strains in the Y-axis between them. However, the
19 displacements in Grains 1 and 2 are the same in the Y-axis, and the effect of the difference
20 in elastic and plastic strains on inhomogeneous deformation is negligible under the
21 current condition. The elastically compatible-type bicrystals (on the condition
22 corresponding to PRI-PRI tension) deform homogeneously, and thus the elastic
23 incompatibility is mainly attributed to the inhomogeneous deformation on the condition
24 corresponding to PRI-BSL tension and PRI-BSL cyclic conditions.

1

2 3.3 Effect of work hardening on activation of slip systems of neighboring crystal grains

3 It is assumed that the deformation shapes of crystal grains cause the incompatibility. Fig.
4 13 shows the schematics of deformation shapes of crystal grains when the crystal grains
5 are separated at the grain boundary. In the PRI-BSL condition, Grain 1 with the c-axis
6 corresponding to the Z-axis deforms while maintaining a rectangular shape while Grain
7 2 with an inclined c-axis deforms with shapes containing shearing (Fig. 13 (a)). Therefore,
8 the gap occurs between Grains 1 and 2 due to the anisotropic elasticity, and Grains 1 and
9 2 elasto-plastically deform to close the gap. When work hardening occurs in Grain 1, the
10 activity of slip system and deformation volume decrease in Grain 2. The decrease in the
11 deformation volume in Grain 1 must be filled by that in Grain 2. This results in activation
12 of the basal slip system in Grain 2.

13

14 Incompatibility in the bicrystal model in the PRI-BSL condition shown in Fig. 13 (a) is
15 treated as a problem of incompatibility around the grain boundary triple junction line
16 when the constrain surface on the top or bottom in the bicrystal model is assumed as the
17 “pseudo” grain boundary. The deformation field formed by the incompatibility around
18 the grain boundary triple junction line contains wedge disclination [33-38]. The
19 deformation field formed by wedge disclination is reproduced by the following procedure
20 shown in Fig. 14: First, a discoidal field is assumed and a wedge-shaped piece is removed
21 from it (Fig. 14 (a)). Second, the discoidal field is deformed such that a newly surface
22 formed by the removal of the piece corresponds to the other surfaces (Fig. 14 (b)).

23

24 The deformation field of the wedge disclination is employed to understand

1 inhomogeneous deformations around grain boundary triple junction lines. Kondou &
2 Ohashi [33,35] conducted uniaxial tensile analysis using symmetric and asymmetric
3 bicrystal models. They suggested that inhomogeneous deformation fields in the bicrystals
4 induced by the boundary conditions and plastic anisotropy corresponded to those by the
5 disclination. The factors contributing to developments of geometrically necessary
6 dislocation (GND) bands formed in the bicrystal [33,35] and tricrystal models [36] were
7 explained using the disclination-type deformation field. Hosoya et al. [37] and Hosoya
8 [38] proposed finite element methods in solids with disclination and applied them to
9 elastic incompatibility problems in tricrystals. This included investigating the accuracy.
10 The results showed that stress fields around the grain boundary triple junction lines were
11 approximately reproduced by using crystal grains with isotropic elasticity and disclination.

12
13 The trade-off relationship of slip systems between Grains 1 and 2 in the PRI-BSL
14 condition is explained (Fig. 11) by employing a field of the wedge disclination-type
15 deformation. Slip systems are activated within larger regions near the bottom than those
16 near the top in the bicrystal model. The work hardening around the grain boundary triple
17 junction line on the bottom surface mainly affects the deformation field when compared
18 to that around the grain boundary triple junction line on the top surface. Thus, the changes
19 in deformation field around the grain boundary triple junction line on the bottom are
20 investigated. The deformation field around the grain boundary triple junction line on the
21 bottom is expressed by the wedge disclination-type deformation as shown in Fig. 15. The
22 new surfaces that are produced by the removal of the wedge piece from the disc
23 correspond to the grain boundary and the region of left-hand side of the disc (denoted as
24 Grain 1) and the right-hand side (denoted as Grain 2). The bottom interface of Grains 1

1 and 2 is subjected to the restraint of deformation in the Y-direction.

2
3 The gap resulting from anisotropic elasticity (Fig. 15 (a)) is closed by the elasto-plastic
4 deformations of Grains 1 and 2 (Fig. 15 (b)). In the No-basal hardening condition, work
5 hardening occurs only in Grain 1 under a cyclic loading, and the volume of plastic
6 deformation decreases in Grain 1. In order to ensure the boundary in Grain 1 conforms to
7 that in Grain 2, it is necessary to compensate the decrease in deformation volume in Grain
8 1 by the increase in that in Grain 2 (Fig. 15 (c)). Thus, the inactivation of slip system by
9 work hardening in Grain 1 leads to the increase of deformation volume (i.e., the
10 activation of slip systems) in Grain 2. The results suggest the possibility that the activity
11 of basal slip system increases in the neighboring crystal grains arranged even in the
12 direction perpendicular to the loading direction when the activity of prismatic $\langle a \rangle$ slip
13 system increases in the crystal grains.

14 15 16 3.4 Effect of boundary condition on the activities of slip systems

17 Effects of the boundary conditions in the bicrystals are investigated by using tricrystal
18 models (Fig. 4) in which the third crystal grain sets are on the top (Tricrystal A) or bottom
19 of the bicrystal (Tricrystal B). The conditions of the analysis are similar to those of the
20 bicrystal model with the BSL-PRI tension condition, and several CRSSs for the prismatic
21 $\langle a \rangle$ slip system in Grain 1 are employed as shown in Table 3. The results are shown in
22 Figs. 16 and 17.

23
24 Fig. 16 shows the results obtained by employing model Tricrystal A (Fig. 4 (a)). The

1 distributions of slip strains on prismatic $\langle a \rangle$ and basal slip systems indicate that the
2 prismatic $\langle a \rangle$ slip system in Grain 1 and the basal slip system in Grain 2 are activated
3 depending on their crystal orientations. All slip systems are not activated in Grain 3 since
4 Schmid factors for the basal and prismatic $\langle a \rangle$ slip systems correspond to zero. The
5 localized slip strains occur within the region near the corner in the bottom side in Grain
6 1 and near the grain boundary in Grain 2. The tendency is similar to the results obtained
7 by the bicrystal model with the BSL-PRI tension condition while the slip systems are
8 activated in the regions larger than those of the bicrystal model.

9

10 Next, the dependency of activity of the basal slip system on the CRSSs for the prismatic
11 $\langle a \rangle$ slip system is observed as shown in Fig. 16. The tendency corresponds with the result
12 obtained in the BSL-PRI tension condition in Fig. 11. The trade-off in the relationship of
13 activities between basal and prismatic $\langle a \rangle$ slip systems is observed. Thus, the boundary
14 condition does not significantly influence on the activity of slip systems in the Tricrystal
15 A condition.

16

17 Fig. 17 shows the results obtained in the Tricrystal B condition, and the regions where the
18 basal slip system activates near the grain boundary and the corner in the bottom side in
19 Grain 2. The tendency is the same as that of Tricrystal A. However, distributions of the
20 slip strain and the dependency of the activities of slip systems on CRSSs for the prismatic
21 $\langle a \rangle$ in Tricrystal A condition differ substantially from those for Tricrystal B condition.
22 The prismatic $\langle a \rangle$ slip system that activates within the whole region in Grain 1 does not
23 include the region around the boundary triple junction line, and it is not possible to
24 directly observe localized slip strains at the corner in Grain 1. The dependency of

1 activities of slip systems on the CRSSs for the prismatic $\langle a \rangle$ does not suggest a trade-off
2 relationship. Both activities of prismatic $\langle a \rangle$ and basal slip systems decrease with
3 increasing in the CRSSs for the prismatic $\langle a \rangle$.

4
5 We examine the reason for the differences between the results obtained in the Tricrystal
6 A and B conditions by employing the deformation field of disclination as shown in Fig.
7 18. In contrast to the results obtained for the Tricrystal A condition, the gap between Grain
8 1 and 2 is closed by the deformation of Grain 3 as well as those of Grains 1 and 2 in the
9 Tricrystal B condition. The deformation of Grain 3 decreases the deformation volumes of
10 Grains 1 and 2, and the plastic deformation does not exist around the grain boundary triple
11 junction line in the current condition. Within the elastic regions, changes in CRSSs for
12 the prismatic $\langle a \rangle$ slip system do not influence the activities of slip systems. Therefore,
13 an increase of CRSSs for the prismatic $\langle a \rangle$ slip system does not affect the deformation
14 field of disclination on the current condition since the elastic deformation occurs around
15 the grain boundary triple junction line without plastic deformation. The activity of the
16 basal slip system is not changed by changes of CRSSs for the prismatic $\langle a \rangle$. These results
17 indicate that a potential exists for changes in which slip systems activate and work
18 hardening exists depending on the boundary conditions, and the effect of the work
19 hardening on the activation of slip systems also changes based on the locations in which
20 work hardening exists.

21 22 23 3.5 Effects of work hardening with respect to the regions

24 The trade-off in the relationship of activities of basal and prismatic $\langle a \rangle$ slip systems is

1 examined by considering the wedge disclination-type deformation field: the prismatic
2 $\langle a \rangle$ slip system is first activated and then work hardening occurs around the grain
3 boundary triple junction line. The decrease in the prismatic $\langle a \rangle$ slip affects the
4 disclination-type deformation field. This may result in activation of basal slip systems.
5 However, the disclination-type deformation cannot be used to explain the decrease of the
6 activity of basal slip system with decreases in that of prismatic $\langle a \rangle$ slip (Fig. 17).

7

8 Wedge disclination-type deformation fields are formed by wedge-shaped incompatibility
9 around the grain boundary triple junction lines. Thus, incompatibility of normal strains in
10 the loading direction is not considered in the wedge disclination. As shown in Fig. 19,
11 incompatibility exists between Grains 1 and 2 in the loading direction due to anisotropic
12 elasticity and difference in CRSSs for the primary slip systems in Grains 1 and 2 when a
13 difference in the displacements is allowed in the loading direction between Grains 1 and
14 2. The occurrence of work hardening and reduction of deformation volume in Grain 1
15 under cyclic loading prevents deformation of Grain 2 in the Y-axis. This is due to Grain
16 1 since Grain 2 shares the grain boundary with Grain 1. Hence, it is necessary for Grains
17 1 and 2 to deform to conform the grain boundary with the other. Therefore, the effect of
18 work hardening in Grain 1 is stronger in the region nearer the grain boundary. Thus, a
19 decrease in activity in prismatic $\langle a \rangle$ slip system in Grain 1 can decrease the activity of
20 the basal slip system near the grain boundary in Grain 2.

21

22 Macroscopic displacements are unchanged with respect to the condition of forced
23 displacements, and it is necessary to directly offset a decline in the deformation volumes
24 of crystal grains in the loading direction with respect to increasing volumes of the other

1 crystal grains arranged in the loading direction. Specifically, normal strains in Y-axis in
2 Grain 3 increase with decreases in those in Grain 1 as shown in Figs. 16 and 17. Given
3 the condition that Grain 3 has a crystal orientation in which slip systems are activated, the
4 activities of slip systems in Grain 3 increase with decreasing the deformation volumes in
5 Grains 1 and 2. However, sufficient results were obtained in the study to explain this type
6 of a relationship between strain redistribution and activation of slip systems in the loading
7 direction, and thus additional investigations are required.

8
9 In this study, the results indicate that incompatibility derived from anisotropic elasticity
10 and its changes by work hardening on the prismatic $\langle a \rangle$ slip system activate the basal slip
11 system in the arranged neighboring crystal grains even in the direction perpendicular to
12 the loading direction. The result reveals the basal slip system in the neighboring crystal
13 grains is activated with decreases in the activity of prismatic $\langle a \rangle$ slips by the work
14 hardening under a fatigue loading although the prismatic $\langle a \rangle$ slips initially operate in a
15 crystal grain.

16
17 It is barely possible to detect the effect of wedge disclination-type incompatibility and
18 work hardening on activities of slip systems especially in the condition that crystal grains
19 arranged in a direction perpendicular to the loading direction. This is because the effect
20 did not appear on the condition in which work hardening existed on the basal slip system
21 (EQ-hardening condition) as shown in Fig. 8 and in the condition that work hardening did
22 not occur around the triple junction line (i.e., on the condition that work hardening did
23 not occur within the regions that are directly relevant to the formation of the disclination-
24 type deformation fields) as shown in Fig. 17. However, factors, such as the minute

1 changes of activities of slip systems may affect the fatigue mechanisms and result in
2 scattering with respect to the fatigue life in the HCF or VHCF regime.

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4 Conclusions

In this study, the following analyses were performed: (i) cyclic loading plasticity analysis employing a bicrystal model, and (ii) uniaxial tensile loading analysis employing bicrystal and tricrystal models. Additionally, the study investigated the effect of changes in incompatibility by work hardening on the activity of basal slip system in the α phase of Ti-6Al-4V alloy. The results can be summarized as follows;

1) Incompatibility between crystal grains derived from anisotropic elasticity was weakened by the deformations of elasto-plastic deformations. Thus, the occurrence of work hardening in a crystal grain decreases the volume of deformation and increase those of elasto-plastic deformations increases in neighboring crystal grains. The phenomenon leads to activation of slip systems in neighboring crystal grains. Thus, although the prismatic $\langle a \rangle$ slips initially operates under cyclic loading, the basal slip system in the neighboring crystal grains displays a potential for activation by work hardening on the prismatic $\langle a \rangle$ slip system.

2) It is assumed that the surfaces subjected to the displacement restraint in the bicrystal model correspond to pseudo grain boundaries. Therefore, the lines in which the actual grain boundary and pseudo grain boundary intersect are considered as boundary triple junction lines. The deformation field around the boundary triple junction line is easily determined by using a wedge disclination-type deformation field and this also helps in understanding changes in incompatibility with work hardening around the triple junction line.

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3) The effects of work hardening on the activities of slip systems were changed depending on the locations where the work hardening occurred. Activities of slip systems of neighboring crystal grains were increased when work hardening occurred within the regions around the triple junction line involved in formation of wedge disclination-type deformation field. Conversely, the activities of slip systems were decreased in the neighboring crystal grains arranged in the direction perpendicular to the loading direction when work hardening occurred to prevent the deformation in the loading direction in a crystal grain.

Acknowledgement

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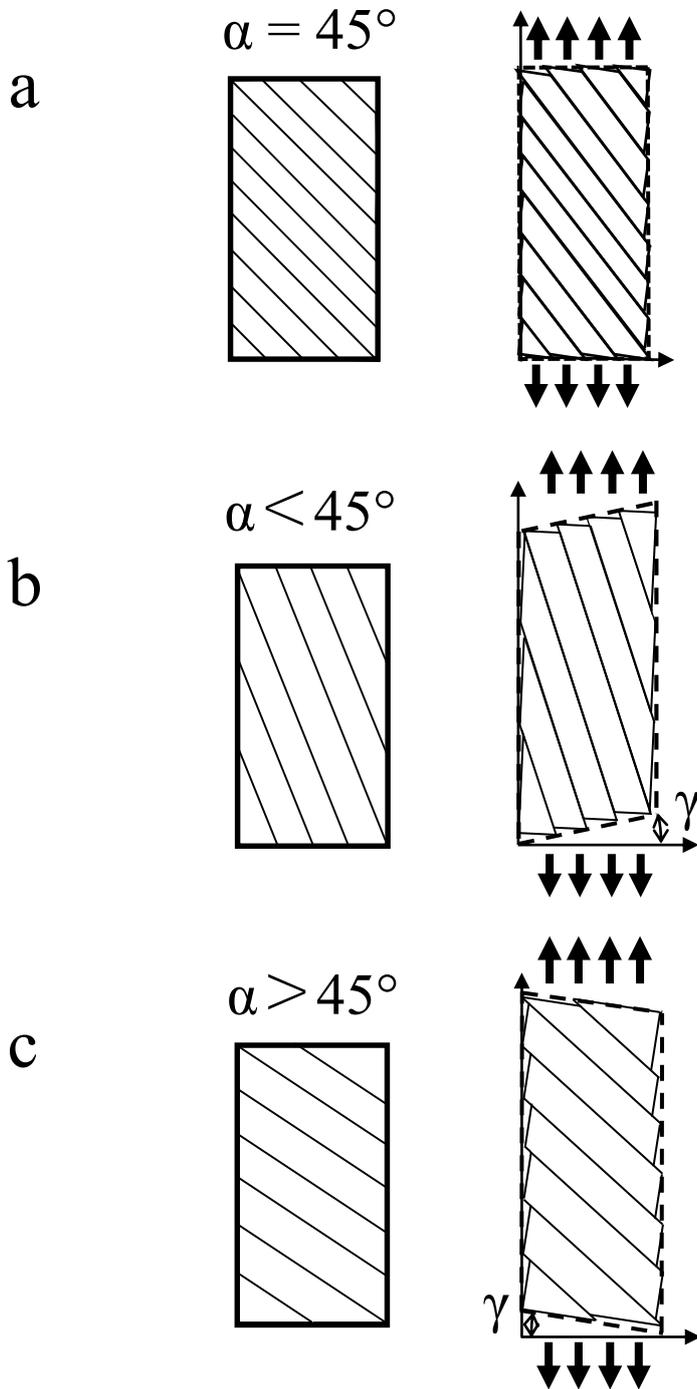
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1 **Figures**

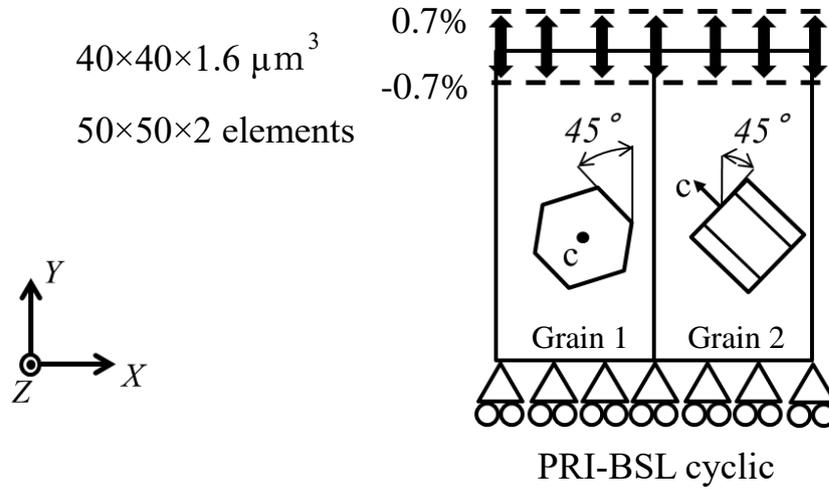


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3 Fig. 1 Deformation shapes of single crystals based on angles of slip planes to the loading

4 direction where α denotes the angle between the slip plane and loading direction.

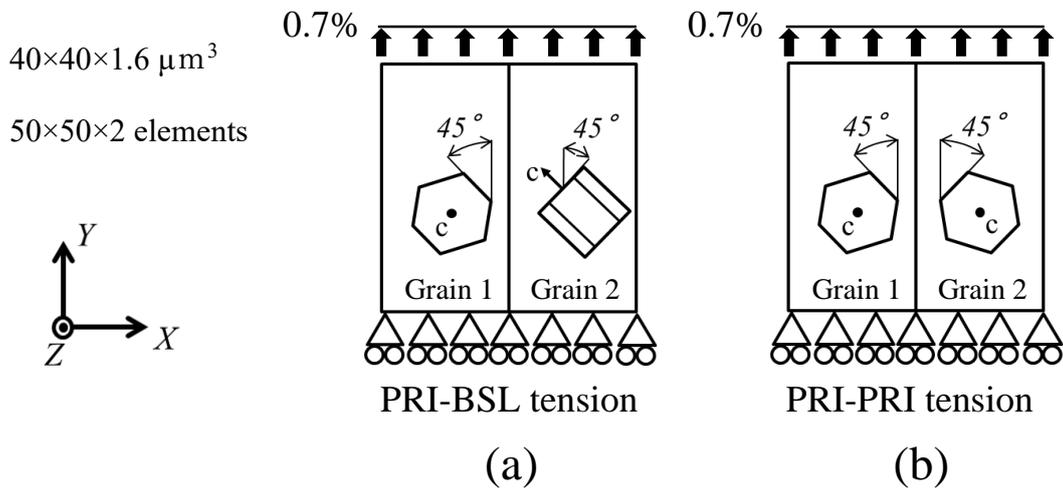
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3 Fig. 2 Schematic of the analysis model and boundary conditions for cyclic loading
4 analysis.

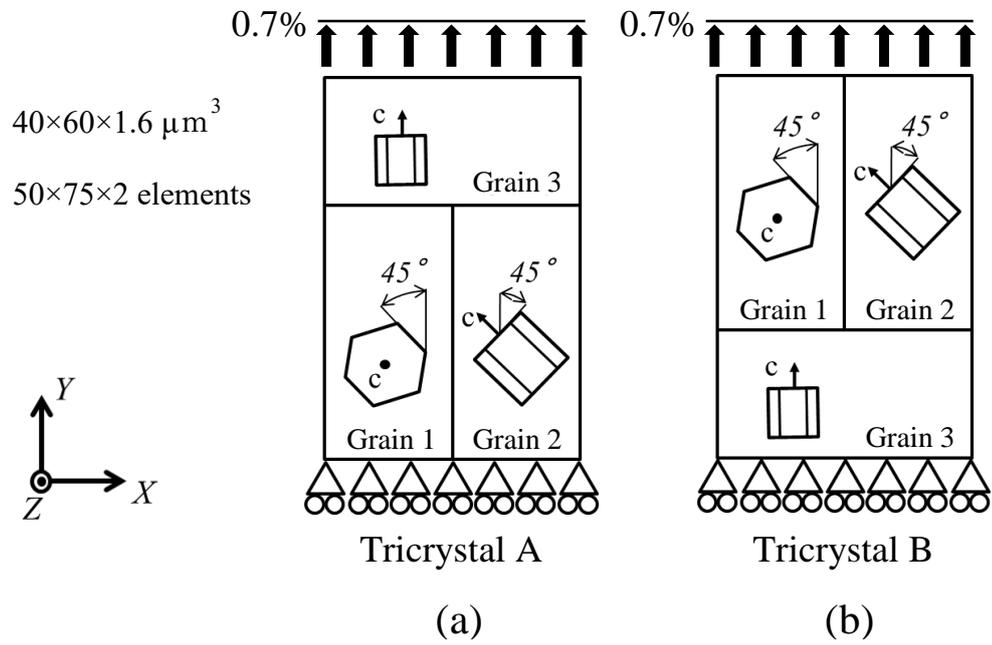
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7 Fig. 3 Schematic of the analysis model and boundary conditions for uniaxial loading analysis of
8 bicrystals.

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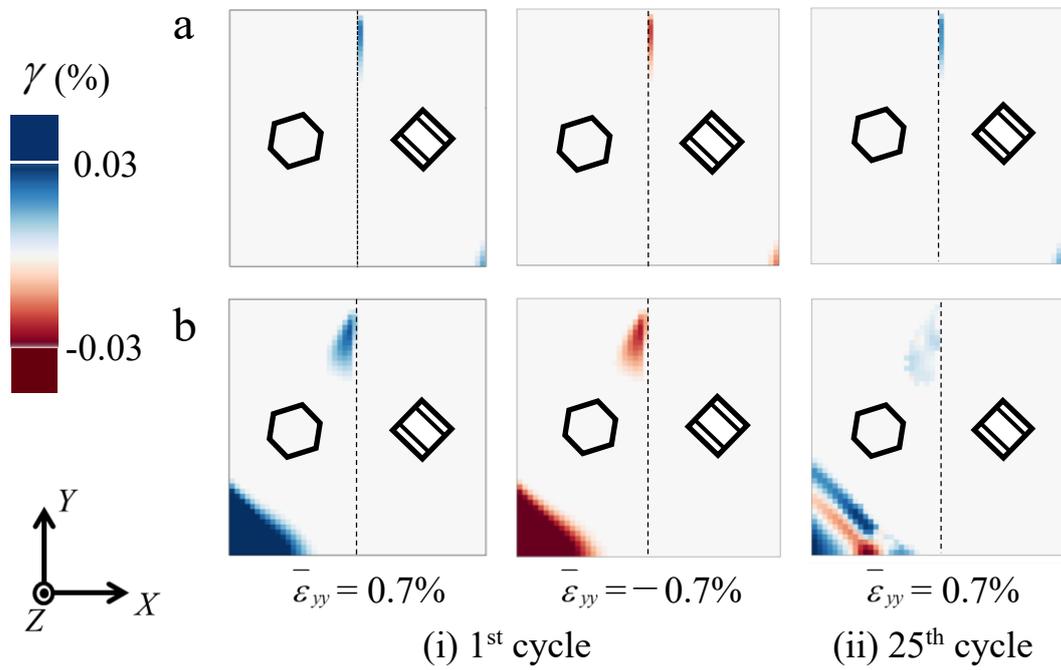
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2 Fig. 4 Schematic of the analysis model and boundary conditions for uniaxial loading analysis of
 3 tricrystals.

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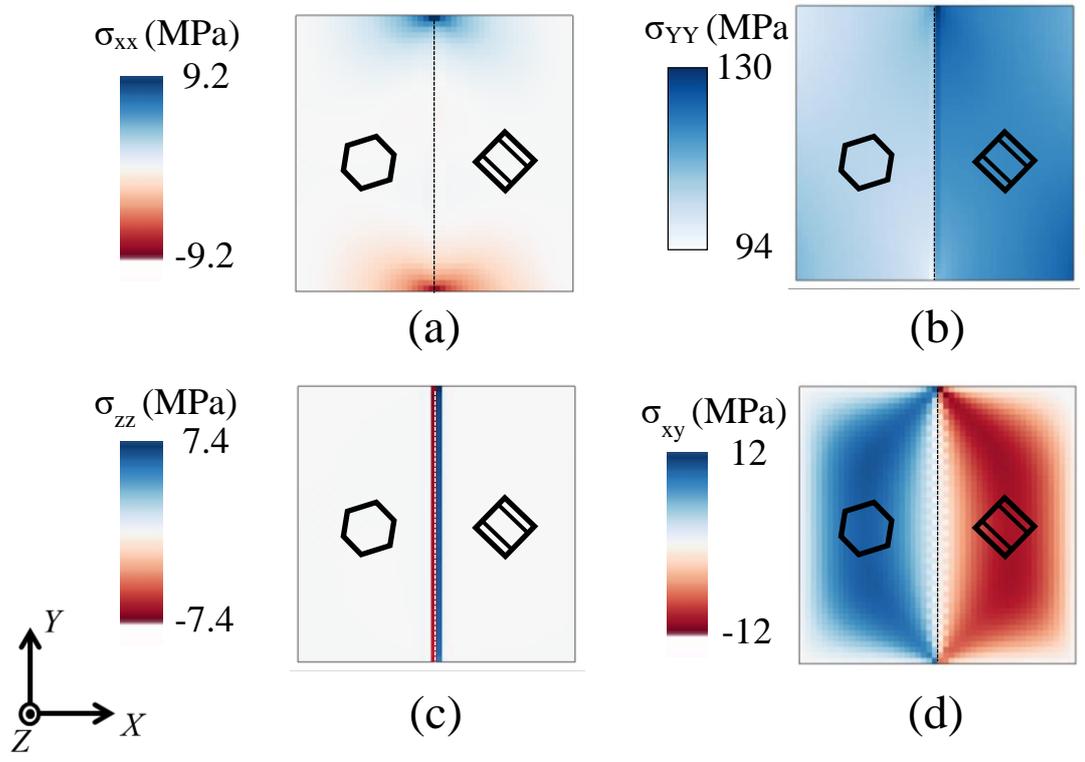
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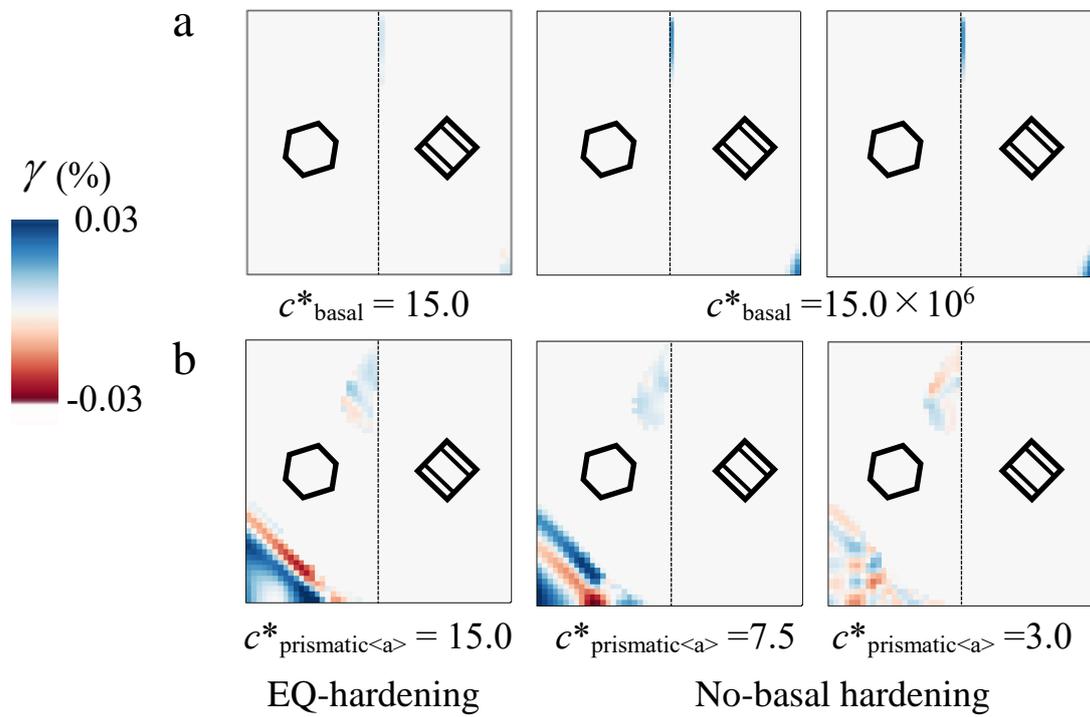
2 Fig. 5 Distributions of slip strains on the basal (a) and prismatic $\langle a \rangle$ slip system (b) during
 3 cyclic loading at (i) the 1st peak strain of tension and compression and (ii) the 25th peak
 4 strain of tension. The No-basal hardening condition is employed and $c^*_{prismatic\langle a \rangle} = 15.0$.



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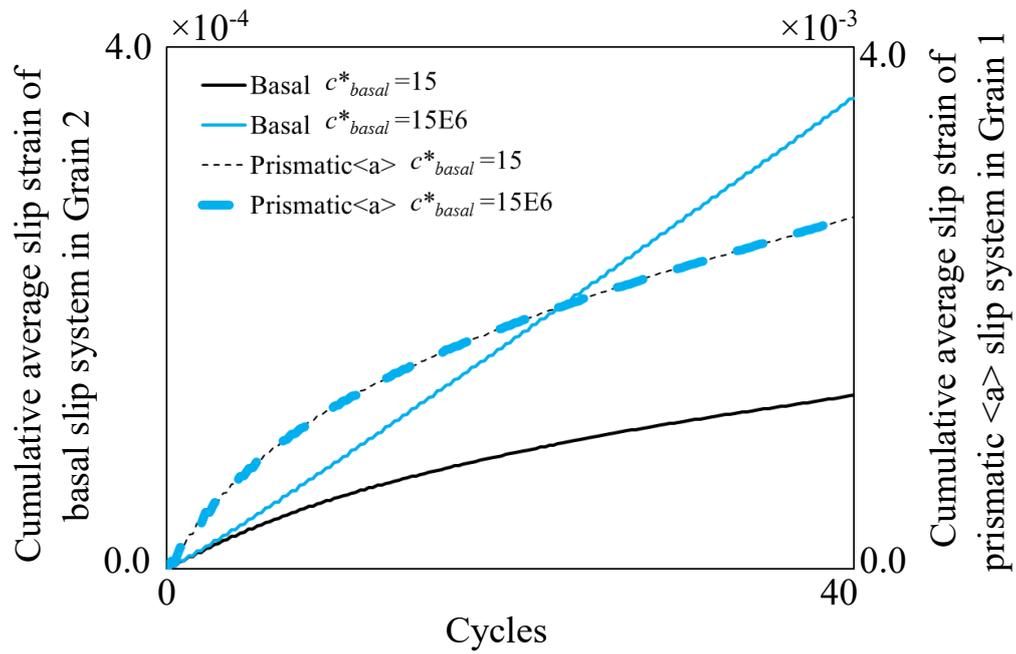
2 Fig. 6 Distributions of stresses at the elastic stage ($\bar{\varepsilon}_{YY} = 0.1\%$): (a) normal stress in the
 3 X-axis, (b) in the Y-axis, and (c) in the Z-axis, and (d) X-Y component of shear stress.

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 2 Fig. 7 Effect of c^* on the distributions of slip strains on the basal (a) and prismatic $\langle a \rangle$
 3 slip system (b) at the 25th peak strain of tension.

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2 Fig. 8 Difference in cumulative slip strains between EQ-hardening and No-basal
 3 hardening conditions in which $c^*_{prismatic\langle a \rangle}$ corresponds to 15.0 throughout the series of
 4 simulations.

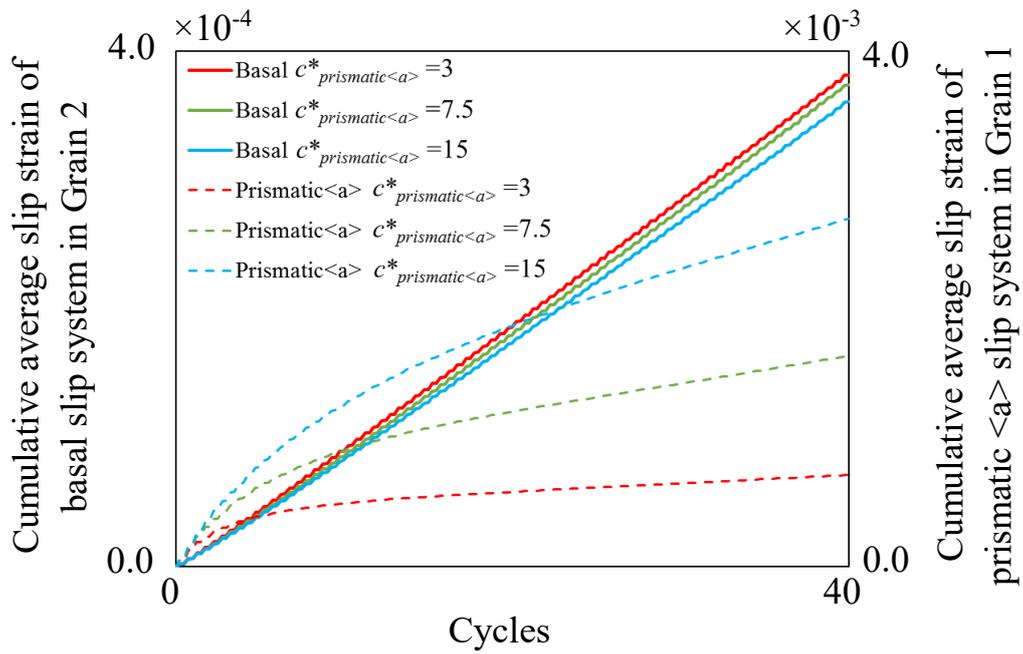
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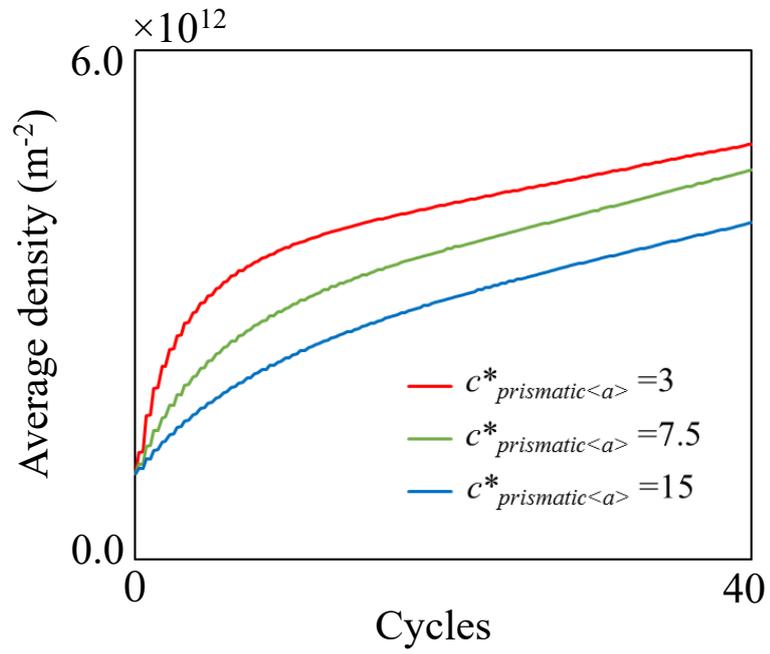
2 Fig. 9 Activity changes in slip systems based on $c^*_{prismatic\langle a \rangle}$ with respect to the No-basal

3 hardening condition where c^*_{basal} corresponds to 15.0×10^6 throughout the series of

4 simulations.

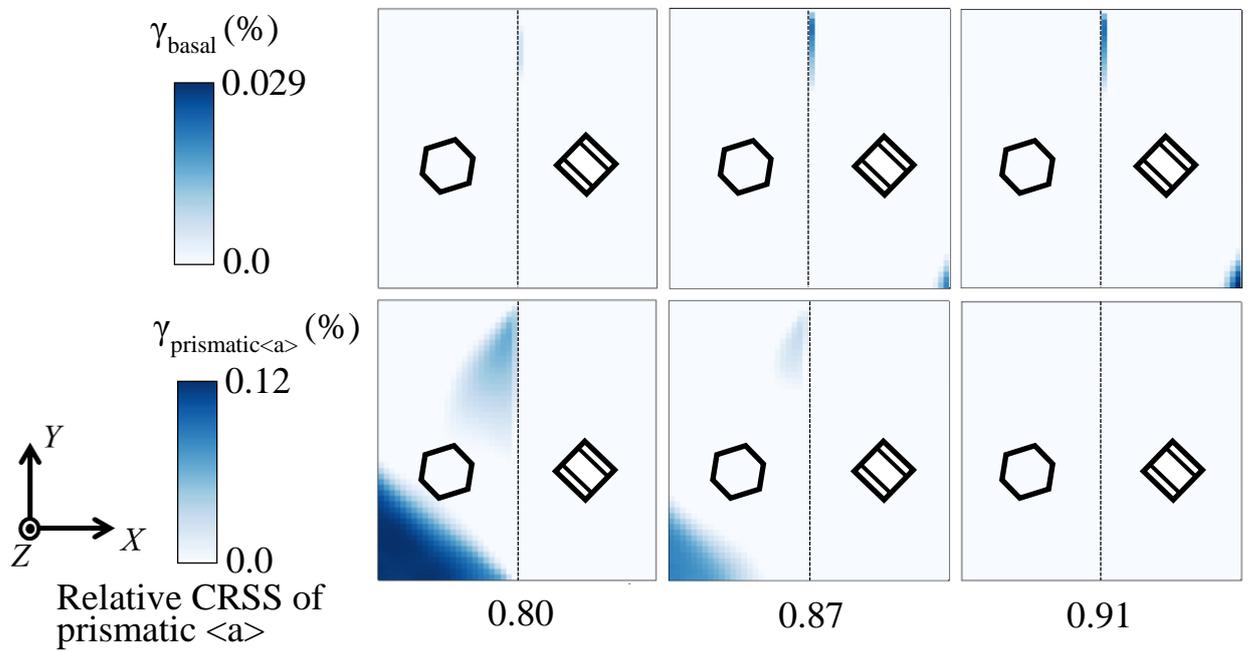
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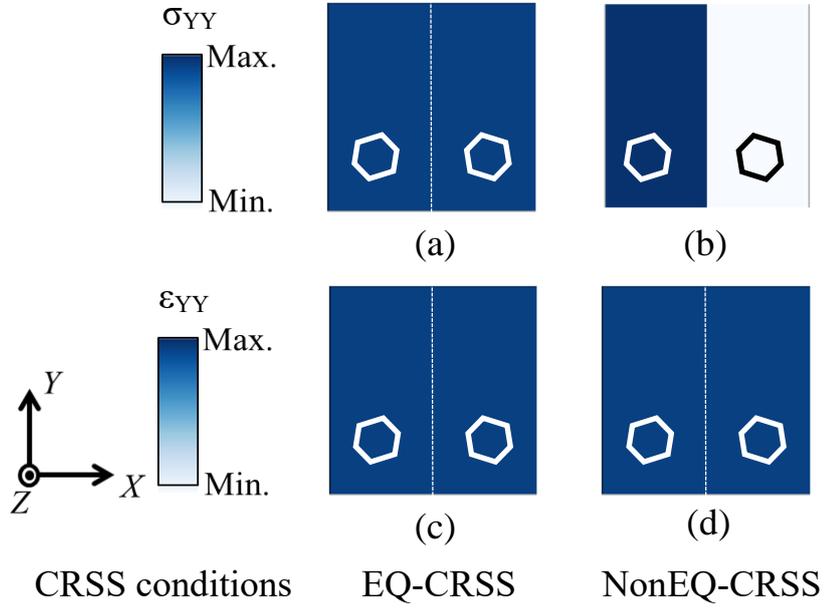
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 2 Fig. 10 Average SSD densities of the prismatic slip system in Grain 1 based on
 3 $c^*_{prismatic\langle a \rangle}$.

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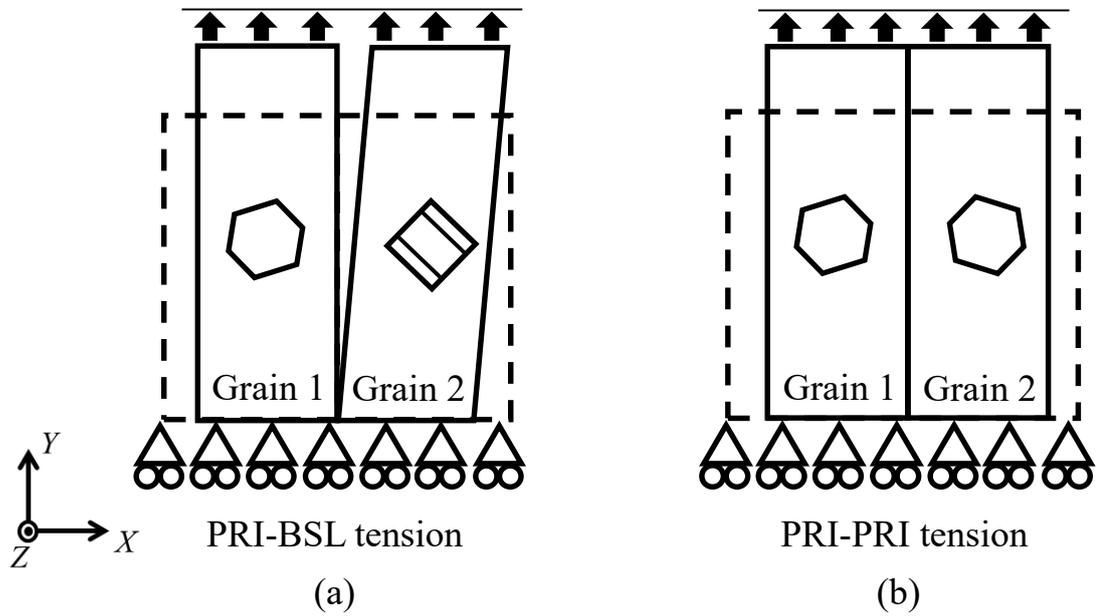
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 2 Fig. 11 Distributions of slip strains based on relative CRSSs for the prismatic $\langle a \rangle$ slip
 3 system normalized to those for the basal slip system.

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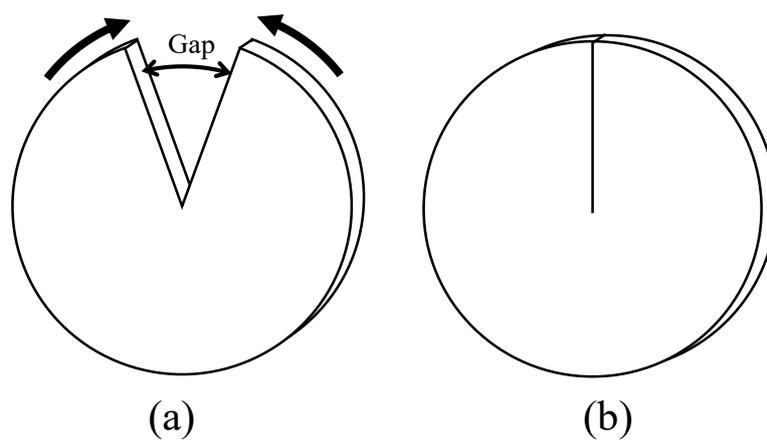
1 Fig. 12 Distributions of normal stress σ_{YY} and normal strain ϵ_{YY} in the model PRI-PRI
 2 tension. (a) σ_{YY} in the EQ-CRSS condition, (b) σ_{YY} in the NonEQ-CRSS condition, (c)
 3 ϵ_{YY} in the EQ-CRSS condition, and (d) ϵ_{YY} in the NonEQ-CRSS condition.

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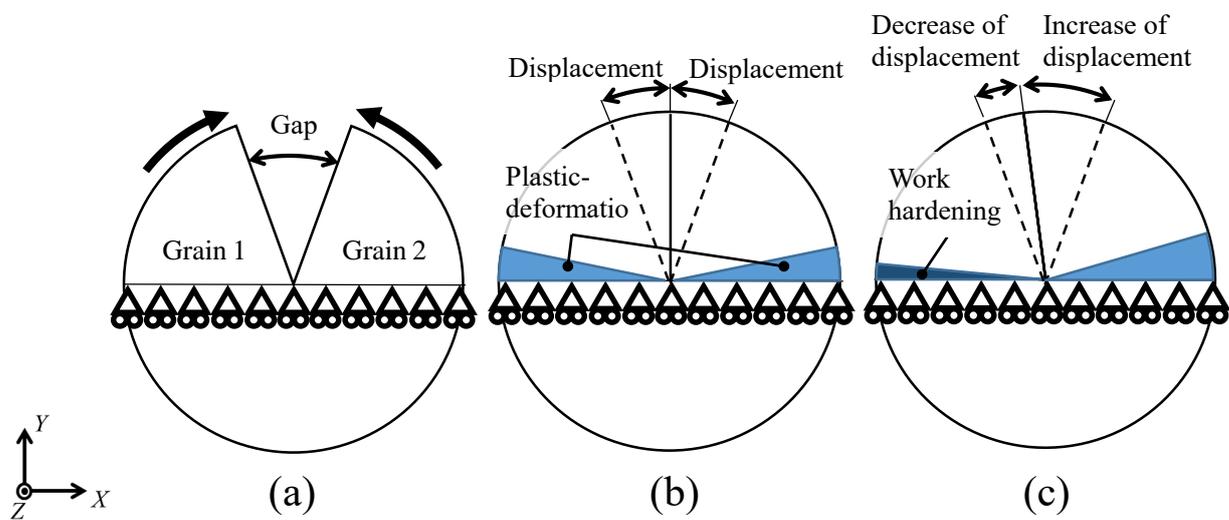
1 Fig. 13 Schematic of deformed shapes of crystal grains in which the crystal grains are separated at the
 2 grain boundaries.

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7 Fig. 14 Wedge disclination type deformation. States (a) before and (b) after jointing the
 8 boundary.



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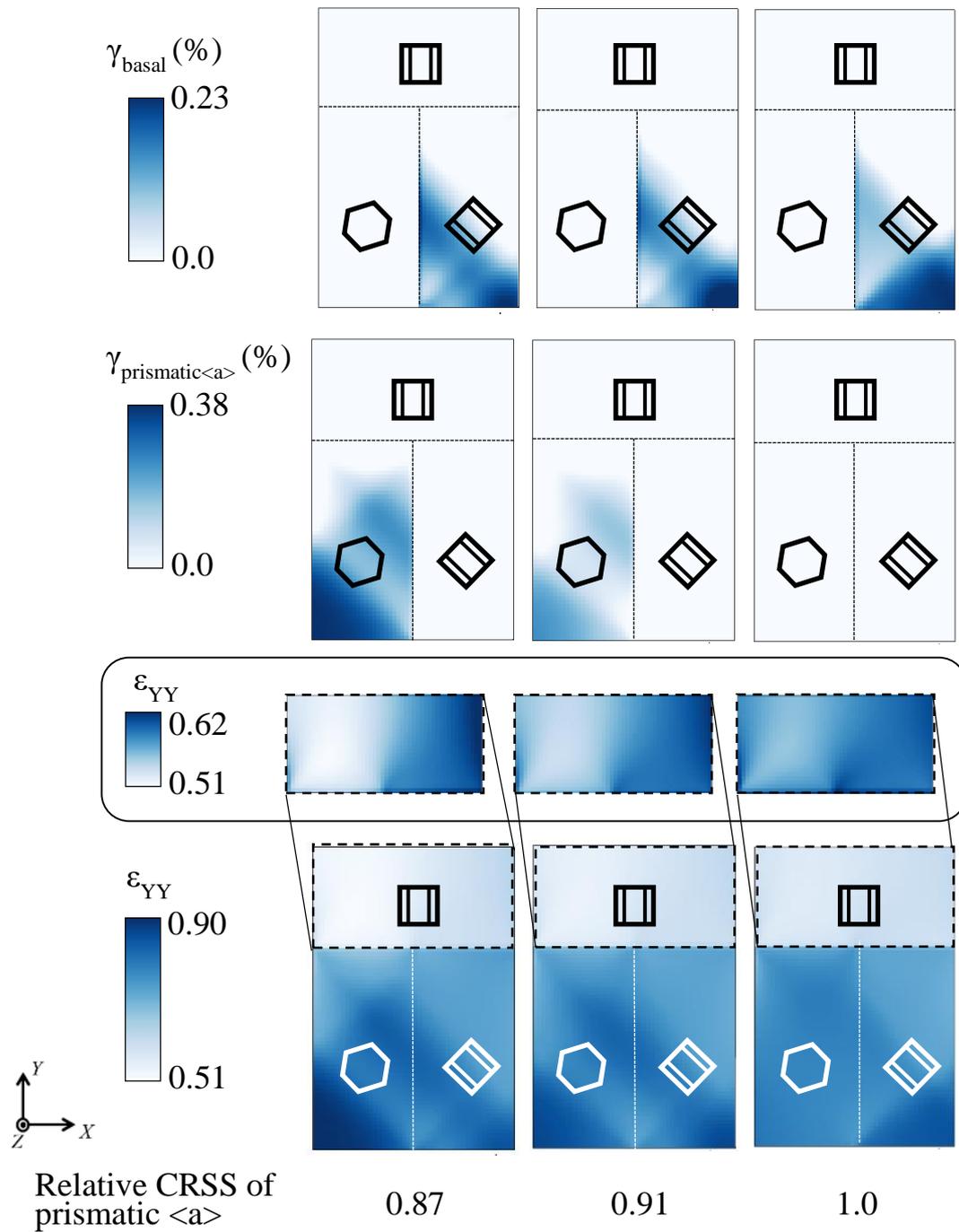
2 Fig. 15 Schematics showing the (a) initial condition and (b) wedge disclination type

3 deformation before and (c) after work hardening.

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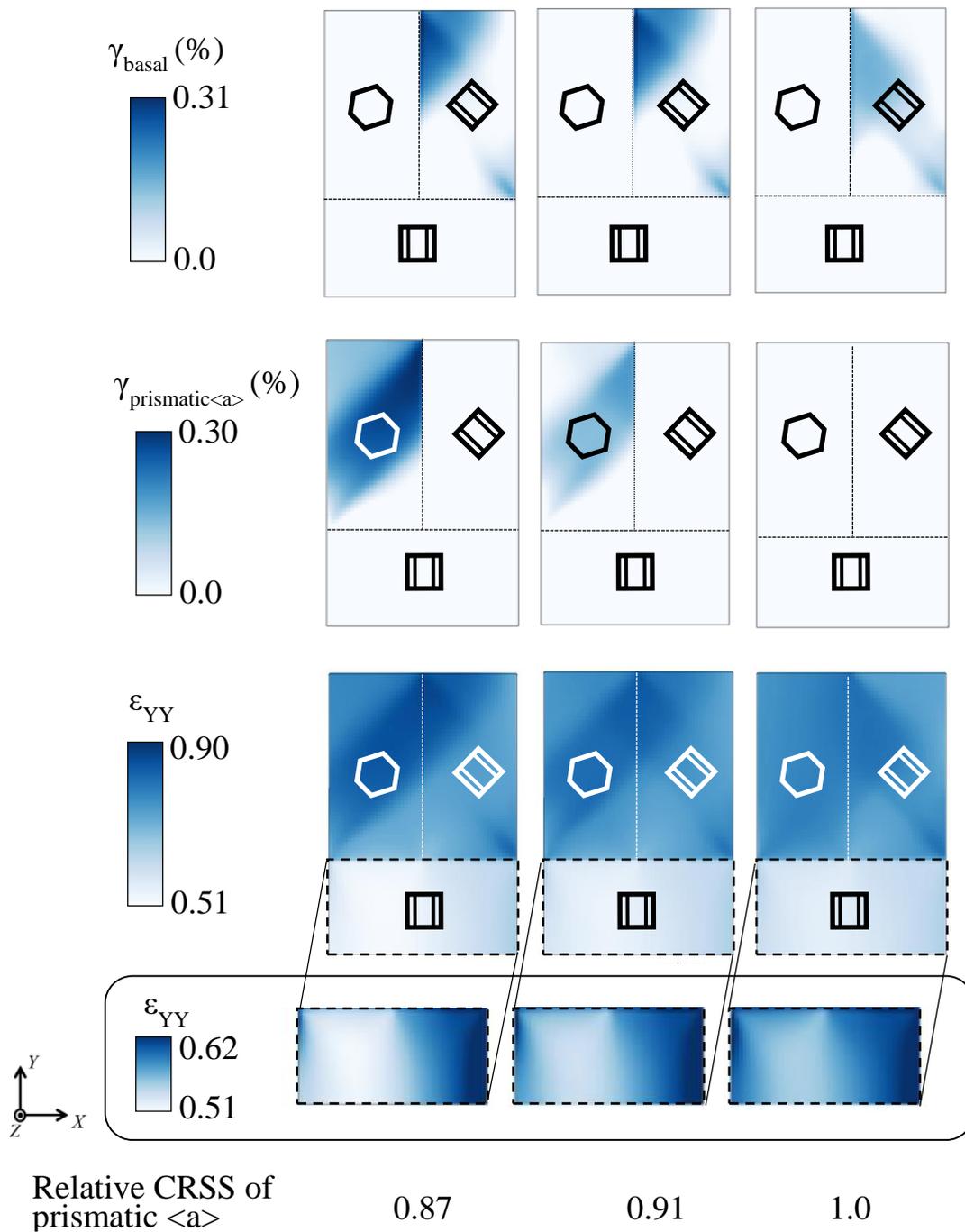
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2 Fig. 16 Distributions of slip strains on the basal and prismatic $\langle a \rangle$ slip systems and normal
 3 strains in the Y-axis when another crystal grain arranged on the top of the bicrystal model.

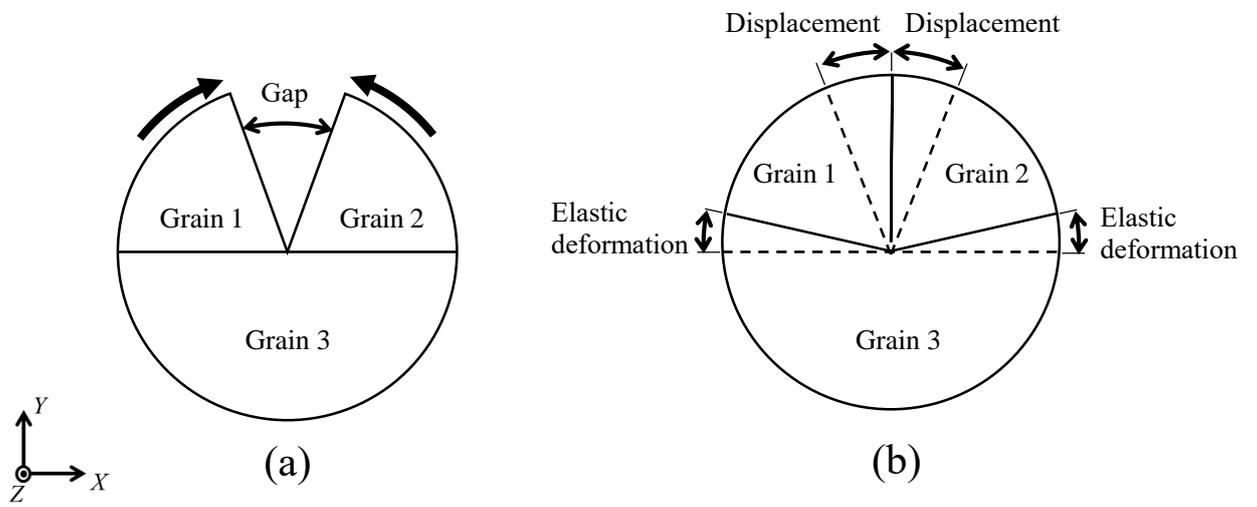
4 $\bar{\varepsilon}_{yy} = 0.7 \%$.



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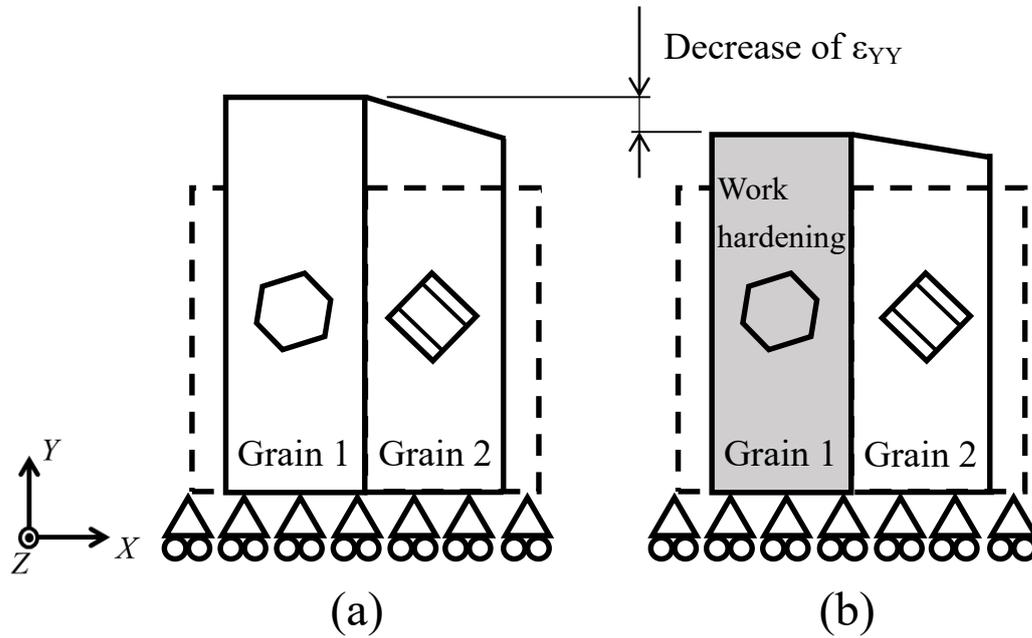
2 Fig. 17 Distributions of slip strains on the basal and prismatic $\langle a \rangle$ slip systems and normal
 3 strains in the Y-axis when another crystal grain arranged on the bottom of the bicrystal

4 model. $\bar{\epsilon}_{yy} = 0.7\%$.



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 2 Fig. 18 Schematics showing (a) incompatibility between crystal grains and (b) closing of
 3 the gap due to wedge disclination-type deformation.

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Fig. 19 Schematics showing the decrease in the deformation volume of the bicrystal in Y-axis due to work hardening under the condition in which uniform stress is virtually applied to the top surface. The states (a) before and (b) after work hardening.

1 Tables

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3 Table 1 Elastic compliances for Ti with an hcp structure [(TPa)⁻¹] [31].

S ₁₁	S ₁₂	S ₁₃	S ₃₃	S ₄₄
9.581	-4.623	-1.893	6.980	21.413

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6 Table 2 c^* for cyclic loading analysis.

Slip systems	EQ-hardening	No-basal hardening
c^*_{basal}	15.0	15.0×10 ⁶
$c^*_{prismatic<a>}$		3.0 7.5 15.0

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12 Table 3 The initial CRSS for each slip system, relative CRSS of basal slip system

13 normalized to CRSS for the prismatic <a> slips, and relative CRSS of prismatic <a> slip

14 system normalized to CRSS for the basal slips.

	PRI-BSL cyclic	PRI-BSL tension	Tricrystal A Tricrystal B
CRSS of basal	425.5 MPa		
CRSS of prismatic <a>	370.0 MPa	340.4 – 444.0 MPa	370.0 – 444.0 MPa
Relative CRSS of basal	1.15	0.96 – 1.25	0.96 – 1.15
Relative CRSS of prismatic <a>	0.870	0.800 – 1.04	0.870 – 1.04

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1 Table 4 The initial CRSSs for the prismatic <a> slip system employed in the PRI-PRI
2 tension analysis [MPa].

	EQ-CRSS	NonEQ-CRSS
Grain 1	370.0	370.0
Grain 2	370.0	444.0

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