Generalized Internal Model Architecture for Gain Scheduled Control

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Abstract

A two-degree-of-freedom controller architecture and its design strategy for Linear Parameter Varying (LPV) systems, where the dependent parameters are assumed to be measurable, are proposed in the Generalized Internal Model Control (GIMC) framework. First, coprime factorisation and Youla Parameterisation for LPV systems are introduced based on a parameter-dependent Lyapunov function. Then, the GIMC architecture for Linear Time Invariant (LTI) systems is extended to LPV systems with these descriptions. Based on this architecture, good tracking performance and good robustness (disturbance rejection) are compatibly achieved by a nominal controller and a conditional controller, respectively. Furthermore, each controller design problem is formulated in terms of Linear Matrix Inequalities (LMIs) related to each L2-gain performance. Finally, a simple design example is illustrated.

Keywords: Internal model control; Gain scheduled control; Two-degree-of-freedom; Linear parameter varying systems; Youla-Parameterisation.

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1. Introduction

Linear parameter-varying (LPV) systems are formalized from a certain type of nonlinear systems and/or linear time-varying (LTV) systems. Several theoretical frameworks have been presented for systematic control of LPV systems based on advanced gain scheduled methodology (Shamma and Athans 1990, 1991, Packard 1994, Apkarian *et al.* 1995, Apkarian and Gahinet 1995, Apkarian and Adams 1998, Leith and Leithead 2000, Rugh and Shamma 2000, Scherer 2001, Xie and Eisaka 2004). In particular, a systematic design method of output feedback LPV controllers that guarantees L2-gain performance is given in (Apkarian and Adams 1998). To the author's knowledge, however, the general two-degree-of-freedom (TDOF) control scheme treating both feedback and tracking issues of LPV systems has not been explored rigorously.

For linear time invariant (LTI) systems, TDOF control framework is considered in variety of control schemes (Lang and Ham 1953, Hara and Sugie 1988, Morari and Zafiriou 1989, Limebeer *et al*. 1993, Tay *et al*. 1998, Yali and Eisaka 2000, Zhou and Ren 2001, Eisaka and Xie 2004). In particular, the parallel-model-and-plant paradigm referred to as 'Internal Model Control (IMC)' is a natural and tractable approach to the design and analysis of control systems (Morari and Zafiriou 1989). Furthermore, the internal structure of the IMC controller is described directly with Youla Parameterisation and called ‘Generalized Internal Model Control (GIMC)’ (Zhou and Ren 2001). The GIMC has the potential to overcome the conflict between performance and robustness practically.

In the present paper, we propose a TDOF controller architecture and its design strategy for LPV systems based on the GIMC framework. Since transfer functions and eigenvalues of state matrices are not available for describing LPV systems, the above mentioned 2DOF methodology for LTI systems cannot be applied in a straightforward manner to LPV systems. Thus, coprime factorisation and Youla Parameterisation described in the state space formulas are newly introduced for LPV systems based on parameter-dependent Lyapunov function. Then, according to these descriptions, the GIMC for LTI systems is extended to LPV systems. Based on this architecture, the LPV controller is designed in a two-step procedure. In the first step, a nominal LPV controller is considered to obtain tracking performance with L2-gain approximation between the feedback control system and its reference model. In the second step, a conditional LPV controller, that is only active when there are model uncertainties or external disturbances, is designed to eliminate the influence of them with another L2-gain performance. Each controller design problem is formulated in terms of linear matrix inequality (LMI) expression.

This note is organized as follows. In Section 2, after some notations and a lemma for LPV systems are introduced, the Youla Parameterisation is extended to LPV systems. Then, the TDOF controller design procedure is considered based on the GIMC architecture in Section 3. A design example and its simulation result are presented in order to illustrate the proposed approach in Section 4. Finally, a concluding remark is given in Section 5.

2. Youla Parameterisation for LPV systems

In this Section, we introduce a way of obtaining all parameter-dependent quadratically stabilizing controllers construction for LPV systems.

Suppose a stabilizable and detectable LPV plant $G(\theta(t))$ has the following realization:

$$G(\theta(t)) \triangleq \left[\begin{array}{c|c} A(\theta(t)) & B(\theta(t)) \\ \hline C(\theta(t)) & 0_{g\times q} \end{array}\right] \tag{2.1}$$

Signals are also notated that $x(t)\in\Re^n$ is the state vector, $u(t)\in\Re^q$ is control input vector and $y(t)\in\Re^g$ is measurable output vector. The state-space matrices $A(\theta(t)), B(\theta(t)), C(\theta(t))$ have compatible dimensions with related signals. Furthermore, the matrices depend only on bounded and continuous time varying parameters

$\theta(t) = [\theta_1(t),\ \theta_2(t),\ \cdots\ ,\theta_r(t)]^T$ which are assumed to measurable on-line. (Hereafter, the time variable *t* will be dropped)

Next, definitions about stability and control performance of LPV systems are noted.

Definition 1: An LPV system (2.1) is said to be parameter-dependent quadratically stable if and only of there exists a scalar function $V(x,\theta) = x^T(t)P(\theta)x(t) > 0$ and $dV(x,\theta)/dt < 0$ along all admissible parameter trajectories and all initial conditions. That is, a necessary and sufficient condition for parameter-dependent quadratic stability of an LPV system (2.1) is that there exists a parameter-dependent positive definite matrix $P(\theta)$ such that

$$A^T(\theta)P(\theta) + P(\theta)A(\theta) + \frac{dP(\theta)}{dt} < 0\,. \tag{2.2}$$

Definition 2: The L2-gain of the LPV system having input w and output z is defined as $\sup_{w \neq 0, \|w\|_2 < \infty} \frac{\|z\|_2}{\|w\|_2}$, where $\|z\|_2 = (\int_0^\infty z^T(t)z(t)dt)^{1/2}$ is the L2-norm of the signal z. The control system satisfying the condition: $\sup_{w \neq 0, \|w\|_2 < \infty} \frac{\|z\|_2}{\|w\|_2} < \gamma$ is said to have L2-gain performance with the bound γ related to w and z.

Next, we will introduce a lemma to consider parameter-dependent quadratic stability of the LPV control systems.

Lemma 1: Suppose matrices $A_{11}(\theta)$ and $A_{22}(\theta)$ are parameter-dependent quadratically stable. Then, every continuous and bounded block triangle matrix whose diagonal matrices consist of $A_{11}(\theta)$ and $A_{22}(\theta)$ is also parameter-dependent quadratically stable.

Proof: Consider a lower triangle matrix:

$$A(\theta)=\begin{bmatrix} A_{11}(\theta) & 0 \\ A_{21}(\theta) & A_{22}(\theta) \end{bmatrix},$$

and a bounded parameter-dependent matrix

$$P(\theta)=\begin{bmatrix} P_1(\theta) & 0 \\ 0 & \lambda P_2(\theta) \end{bmatrix}>0 \text{, with a positive real number } \lambda .$$

According to the Definition 1, $A(\theta)$ is said to be parameter-dependent quadratically stable if there exists a positive real number λ such that the following inequality holds.

$$\begin{aligned} & A^T(\theta)P(\theta)+P(\theta)A(\theta)+\frac{dP(\theta)}{dt} \\ &= \begin{bmatrix} A_{11}^T(\theta)P_1(\theta)+P_1(\theta)A_{11}(\theta)+\frac{dP_1(\theta)}{dt} & \lambda A_{21}^T(\theta)P_2(\theta) \\ \lambda P_2(\theta)A_{21}(\theta) & \lambda\left(A_{22}^T(\theta)P_2(\theta)+P_2(\theta)A_{22}(\theta)+\frac{dP_2(\theta)}{dt}\right) \end{bmatrix}<0 \end{aligned} \tag{2.3}$$

Using the Schur Complement , inequality (2.3) is equivalent to the following inequalities as

$$\lambda\left(P_2(\theta)A_{22}(\theta)+A_{22}^T(\theta)P_2(\theta)+\frac{dP_2(\theta)}{dt}\right)<0 \tag{2.4}$$

$$\begin{aligned}\Theta &= A_{11}^T(\theta)P_1(\theta)+P_1(\theta)A_{11}(\theta)+\frac{dP_1(\theta)}{dt}\\ &-\lambda(A_{21}^T(\theta)P_2)\left(A_{22}^T(\theta)P_2(\theta)+P_2(\theta)A_{22}(\theta)+\frac{dP_2(\theta)}{dt}\right)^{-1}(P_2(\theta)A_{21}(\theta))<0\end{aligned} \tag{2.5}$$

The inequality (2.4) holds for any $\lambda>0$. For the rest of the proof, we find a positive real number $\lambda>0$, which satisfies (2.5). According to the assumption, there exist bounded positive definite matrices $P_1(\theta)$, $P_2(\theta)$ and positive real numbers r_1, r_2, respectively, which satisfy

$$A_{11}^T(\theta)P_1(\theta)+P_1(\theta)A_{11}(\theta)+\frac{dP_1(\theta)}{dt}\le -r_1 I, \tag{2.6}$$

$$-r_2 I\le A_{22}^T(\theta)P_2(\theta)+P_2(\theta)A_{22}(\theta)+\frac{dP_2(\theta)}{dt}<0. \tag{2.7}$$

Also, since $(P_2(\theta)A_{21}(\theta))^T(P_2(\theta)A_{21}(\theta))$ is a symmetric and bounded matrix, there exists a positive real number r_3, which satisfies

$$(P_2(\theta)A_{21}(\theta))^T(P_2(\theta)A_{21}(\theta))\le r_3^2 I \tag{2.8}$$

Substituting (2.6), (2.7) and (2.8) to (2.5) we obtain $\Theta\le\left(-r_1+\lambda r_2^{-1}r_3^2\right)I$. Finally, we see that inequality (2.5) holds for any $0<\lambda<r_1r_2r_3^{-2}$.

The upper triangle matrices can be deduced in the same way. Q.E.D.

Now we introduce a way of obtaining all parameter-dependent quadratically stabilizing controllers for a given LPV plant.

Theorem 1: *Let the LPV plant* $G(\theta)$ *be described in (2.1) and controllers* $K(\theta)$ *be connected by the general control architecture as shown in Figure 1. Then the set of all parameter-dependent quadratically stabilizing controllers can be parameterized as* $K(\theta) = F_l(J(\theta), Q(\theta))$ *where*

$$J(\theta) \triangleq \left[\begin{array}{c|cc} A(\theta) - B(\theta)F(\theta) - L(\theta)C(\theta) & L(\theta) & -B(\theta) \\ \hline F(\theta) & 0 & I \\ -C(\theta) & I & 0 \end{array}\right]. \tag{2.9}$$

This $J(\theta)$ *is an observer based stabilizing controller of the plant, where* $A(\theta) - B(\theta)F(\theta)$ *and* $A(\theta) - L(\theta)B(\theta)$ *should be parameter-dependent quadratically stable. The dynamics* $Q(\theta)$ *with any dimensions should be also parameter-dependent quadratically stable described as*

$$Q(\theta) \triangleq \left[\begin{array}{c|c} A_Q(\theta) & B_Q(\theta) \\ \hline C_Q(\theta) & D_Q(\theta) \end{array}\right] \tag{2.10}$$

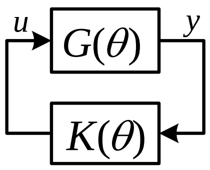

Figure 1. General LPV feedback control architecture.

The notation $F_l(\,)$ denotes lower linear fractional transformation and the structure of all parameter-dependent quadratically stabilizing controllers $K(\theta)$ can be parameterized as shown in Figure 2.

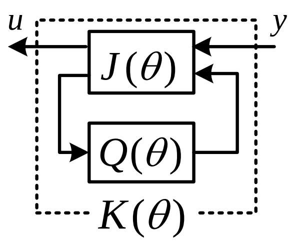

Figure 2. Parameterization of all parameter-dependent quadratically stabilizing controllers.

Proof:

Sufficiency: Using (2.9) and (2.10), the controller $K(\theta)$ is derived as

$$K(\theta) \triangleq \left[\begin{array}{cc|c} A(\theta)+B(\theta)F(\theta)+L(\theta)C(\theta)-B(\theta)D_Q(\theta)C(\theta) & B(\theta)C_Q(\theta) & B(\theta)D_Q(\theta)-L(\theta) \\ -B_Q(\theta)C(\theta) & A_Q(\theta) & B_Q(\theta) \\ \hline F(\theta)-D_Q(\theta)C(\theta) & C_Q(\theta) & D_Q(\theta) \end{array}\right] \quad (2.11)$$

Connecting (2.11) to (2.1), the closed-loop system can be expressed as

$$\begin{bmatrix} \dot{x} \\ \dot{e} \\ \dot{x}_Q \end{bmatrix} = \begin{bmatrix} A(\theta)+B(\theta)F(\theta) & B(\theta)F(\theta)-B(\theta)D_Q(\theta)C(\theta) & B(\theta)C_Q(\theta) \\ 0 & A(\theta)+L(\theta)C(\theta) & 0 \\ 0 & -B_Q(\theta)C(\theta) & A_Q(\theta) \end{bmatrix} \begin{bmatrix} x \\ e \\ x_Q \end{bmatrix} \quad (2.12)$$

where $e=\hat{x}-x$, $\hat{x}$ is the state of the observer $J(\theta)$ and x_Q is the state variable vector of the $Q(\theta)$.

Using lemma 1 twice in succession, first viewing (2.12) as upper triangle matrix and then viewing the second diagonal block of it as lower triangle matrix, we can find a parameter-dependent positive matrix $P_{cl}(\theta)$ with $\lambda_f>0$ and $\lambda_q>0$ formed as

$$P_{cl}(\theta)=\begin{bmatrix} \lambda_f P_f(\theta) & 0 & 0 \\ 0 & P_l(\theta) & 0 \\ 0 & 0 & \lambda_q P_q(\theta) \end{bmatrix} \quad (2.13)$$

where $P_f(\theta)>0$, $P_l(\theta)>0$ and $P_q(\theta)>0$ with compatible dimensions, satisfies (2.2) for system (2.12).

Necessity: We will show that the arbitrary stabilizing controller $K(\theta)$ can be expressed with an appropriate parameter-dependent quadratically stable $Q_0(\theta)$ as $K(\theta) = F_l(J(\theta), Q_0(\theta))$.

First, consider $Q_0(\theta) = F_l(\hat{J}(\theta), K(\theta))$, where $\hat{J}(\theta)$ is realized as

$$\hat{J}(\theta) \triangleq \left[\begin{array}{c|cc} A(\theta) & -L(\theta) & B(\theta) \\ \hline -F(\theta) & 0 & I \\ C(\theta) & I & 0 \end{array}\right]. \tag{2.14}$$

Since the (2,2) input-output relation of $\hat{J}(\theta)$ is the same as the original plant (2.1), then $K(\theta)$ stabilized not only the plant but also the $\hat{J}(\theta)$. Accordingly, $Q_0(\theta) = F_l(\hat{J}(\theta), K(\theta))$ is parameter-dependent quadratically stable.

The substitution of the $Q_0(\theta)$ into $F_l(J(\theta), Q_0(\theta))$ yields

$$\begin{aligned} F_l(J(\theta), Q_0(\theta)) &= F_l(J(\theta), F_l(\hat{J}(\theta), K(\theta))) \\ &= F_l(J_{tmp}(\theta), K(\theta)), \end{aligned}$$

where J_{tmp} is obtained using the star product formula as

$$J_{tmp} \triangleq \left[\begin{array}{cc|cc} A(\theta) + L(\theta)C(\theta) + B_u F(\theta) & -B(\theta)F(\theta) & -L(\theta) & B(\theta) \\ L(\theta)C(\theta) & A(\theta) & -L(\theta) & B(\theta) \\ \hline F(\theta) & -F(\theta) & 0 & I \\ -C(\theta) & C(\theta) & I & 0 \end{array}\right]. \tag{2.15}$$

A similar transform with

$$T = \begin{bmatrix} I & I \\ 0 & I \end{bmatrix},$$

and eliminating the stable uncontrollable and unobservable mode gives

$$J_{tmp} \triangleq \left[\begin{array}{c|c} 0 & I \\ \hline I & 0 \end{array}\right]$$

Consequently, the relation that we want to show is deduced

$$F_l(J(\theta), Q_0(\theta)) = F_l(J_{tmp}, K(\theta)) = K(\theta) \,. \qquad \text{Q.E.D.}$$

The result itself is a natural extension of the corresponding result for LTI systems. However, since eigenvalues of state matrices are not available for describing LPV, lemma 1 is needed to assure the stability of the closed loop LPV system.

3. TDOF controller design based on GIMC architecture

In this Section, we propose a TDOF architecture and its design strategy for LPV systems based on the Generalized Internal Model Control (GIMC) described directly with the Youla Parameterisation. First, we define doubly coprime factorisation description of LPV plants that resemble the case of LTI systems. Then, GIMC for LTI systems is extended to LPV systems.

Definition 3: The representation (3.1) is called a doubly coprime factorisation of LPV plants $G(\theta)$

$$G(\theta) = N(\theta)M(\theta)^{-1} = \tilde{M}(\theta)^{-1}\tilde{N}(\theta) \,, \tag{3.1}$$

where

$$\begin{bmatrix} M(\theta) & -U(\theta) \\ N(\theta) & V(\theta) \end{bmatrix} \triangleq \left[\begin{array}{c|cc} A(\theta)-B(\theta)F(\theta) & B(\theta) & L(\theta) \\ \hline -F(\theta) & I & 0 \\ C(\theta) & 0 & I \end{array}\right], \tag{3.2}$$

$$\begin{bmatrix} \tilde{V}(\theta) & \tilde{U}(\theta) \\ -\tilde{N}(\theta) & \tilde{M}(\theta) \end{bmatrix} \triangleq \left[\begin{array}{c|cc} A(\theta)-L(\theta)C(\theta) & B(\theta) & L(\theta) \\ \hline F(\theta) & I & 0 \\ -C(\theta) & 0 & I \end{array}\right], \tag{3.3}$$

where $F(\theta)$ and $L(\theta)$ are chosen such that $A(\theta)-B(\theta)F(\theta)$ and $A(\theta)-L(\theta)C(\theta)$ are both parameter-dependent quadratically stable. The product of (3.2) and (3.3) reduces a unit matrix, and this relationship supports the statement that (3.1) is doubly coprime factorisation.

According to doubly coprime factorisation and Youla Parameterisation for LPV plant, we can construct a GIMC architecture for LPV plants as shown in Figure 3, where u are control inputs, d are output disturbances, y are controlled outputs and r are reference inputs.

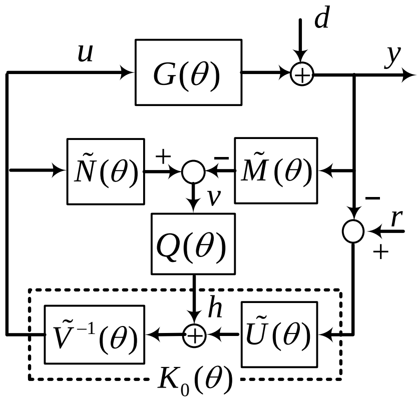

Figure 3. Generalized internal model LPV control architecture.

Just as introduced in (Zhou and Ren 2001), the GIMC architecture shown above is given by the modification of the Youla Parameterisation. In contrast to the Youla

Parameterisation, however, the adjustable dynamics $Q(\theta)$ will be active only when there exist output disturbances d or model uncertainties, thus this conditional controller can be used solely to deal with the external disturbances or model uncertainties. On the other hand, $K_0(\theta)$ is a nominal controller satisfying the adequate reference tracking performance.

The GIMC architecture leads to a two-step design procedure of TDOF-LPV controller. First, a feedback controller $K_0(\theta)$ that achieves good tracking performance is designed by a model matching strategy with L2-gain performance. Second, a conditional controller $Q(\theta)$ that rejects disturbances or model uncertainties, but does not affect tracking performance, is also designed with L2 gain performance. Each controller design problem can be reduced to the design problem of the gain-scheduled controller $K(\theta)$ in Figure 1 introduced by Apkarian and Adams (1998), by considering appropriate augmented plant. Also, each controller design problem can be formulated in terms of LMI expression.

3.1 *Construction of the nominal controller* $K_0(\theta)$

The approximation of transfer functions cannot be applied to solve the command tracking problem for LPV systems. Instead, we treat L2-gain performance between signals z and r by a controller $K_0(\theta)$ as shown in Figure 4 with reference model

$T_0(\theta)$ and the weighting function $W_r(\theta)$. The LTI reference model and weighting function are also available, although the latter can be omitted.

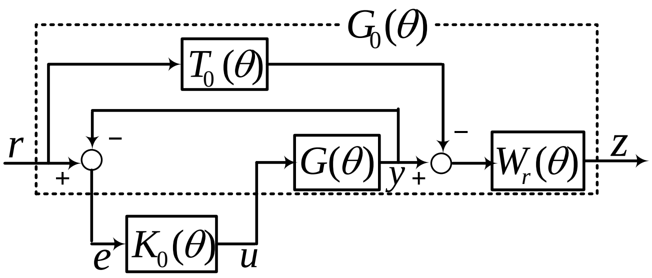

Figure 4. LFT configuration for the tracking problem.

According to Figure 4, the augmented plant $G_0(\theta)$ has the following relationship

$$\begin{bmatrix} z \\ e \end{bmatrix} = G_0(\theta)\begin{bmatrix} r \\ u \end{bmatrix} = \begin{bmatrix} -W_r(\theta)T_0(\theta) & W_r(\theta)G(\theta) \\ I & -G(\theta) \end{bmatrix}\begin{bmatrix} r \\ u \end{bmatrix}. \tag{3.4}$$

The state space realization of $G_0(\theta)$ can be derived as

$$G_0(\theta) \triangleq \left[\begin{array}{ccc|cc} A_{wr}(\theta) & -B_{wr}(\theta)C_{t0}(\theta) & B_{wr}(\theta)C(\theta) & -B_{wr}(\theta)D_{t0}(\theta) & 0 \\ 0 & A_{t0}(\theta) & 0 & B_{t0}(\theta) & 0 \\ 0 & 0 & A(\theta) & 0 & B(\theta) \\ \hline C_{wr}(\theta) & -D_{wr}(\theta)C_{t0}(\theta) & D_{wr}(\theta)C(\theta) & -D_{wr}(\theta)D_{t0}(\theta) & 0 \\ 0 & 0 & -C(\theta) & I & 0 \end{array}\right], \tag{3.5}$$

where, $W_r(\theta)$ and $T(\theta)$ have the realization $[A_{wr}(\theta),\ B_{wr}(\theta),\ C_{wr}(\theta),\ D_{wr}(\theta)]$ and $[A_{t0}(\theta),\ B_{t0}(\theta),\ C_{t0}(\theta),\ D_{t0}(\theta)]$, respectively.

Consequently, the design problem of the feedforward controller $K_0(\theta)$ obtaining L2 gain performance related to r and z is formulated in terms of the linear matrix inequality mentioned later in Subsection 3.3.

3.2 ***Construction of the conditional controller*** $Q(\theta)$

We also consider L2-gain performance between signals d and y by a conditional controller $Q(\theta)$. After the nominal controller $K_0(\theta)$ is fixed, Figure 3 can be rewritten by another LFT configuration as shown in Figure 5.

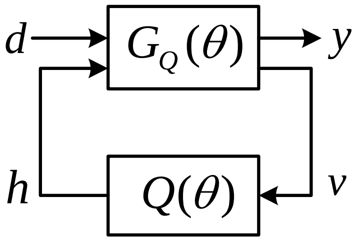

Figure 5. LFT configuration for the disturbance elimination.

The augmented plant $G_Q(\theta)$, consists of $G(\theta), K_0(\theta), \tilde{M}(\theta)$ and $\tilde{N}(\theta)$, has the following relationship

$$\begin{bmatrix} y \\ v \end{bmatrix} = G_Q(\theta)\begin{bmatrix} d \\ h \end{bmatrix} = \begin{bmatrix} (I+G(\theta)K_0(\theta))^{-1} & (I+G(\theta)K_0(\theta))^{-1}\tilde{M}^{-1}(\theta)\tilde{N}(\theta)\tilde{V}^{-1}(\theta) \\ -\tilde{M}(\theta) & 0 \end{bmatrix}\begin{bmatrix} d \\ h \end{bmatrix}. \tag{3.6}$$

Consequently, the state space realization of $G_Q(\theta)$ can be expressed as

$$G_Q(\theta) \triangleq \left[\begin{array}{ccc|cc} A(\theta)-B(\theta)D_{k0}(\theta)C(\theta) & B(\theta)C_{k0}(\theta) & 0 & B(\theta)D_{k0}(\theta) & -B(\theta) \\ -B_{k0}(\theta)C(\theta) & A_{k0}(\theta) & 0 & B_{k0}(\theta) & -L_{k0}(\theta) \\ 0 & 0 & A(\theta)-L(\theta)C(\theta) & -L(\theta) & 0 \\ \hline -C(\theta) & 0 & 0 & I & 0 \\ 0 & 0 & -C(\theta) & -I & 0 \end{array}\right] \tag{3.7}$$

where, $[A_{k0}(\theta),\ B_{k0}(\theta),\ C_{k0}(\theta),\ D_{k0}(\theta)]$ is a realization of $K_0(\theta)$. The details for designing of $Q(\theta)$ are given by the same manner as that of $K_0(\theta)$ and will also be introduced in the next Subsection.

Because $G_Q(\theta)$ becomes a high order system compared to the model plant, the resulting $Q(\theta)$ also tends to be of high order. Below we consider an alternative formulation to obtain a reduced order controller based on modified architecture shown in Figure 6.

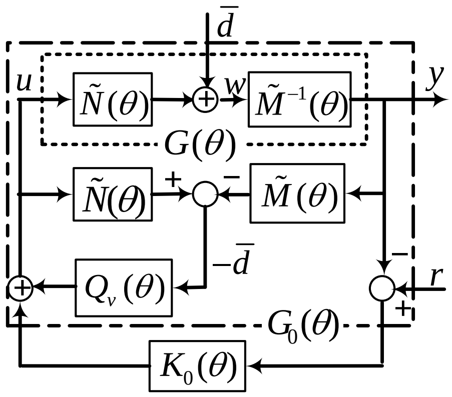

Figure 6. Modified architecture of Figure 3.

The disturbance $\bar{d}$ is called base-equivalent disturbances that represent real disturbances and model uncertainties. Robust control system design by means of rejection of the $\bar{d}$ has been proposed and refereed to as Robust Model Matching (RMM) in the References (Yali and Eisaka 2000, Eisaka and Xie 2004). We can apply the RMM to design of a conditional controller $Q_v(\theta)$. The design principle is to make the $G_0(\theta)$ as becoming close to the nominal $G(\theta)$ without using the feedback controller $K_0(\theta)$. A conditional controller $Q_v(\theta)$ should be obtained for this purpose. As for the configuration Figure 5, $G_0(\theta)$ can be rewritten in the form of LFT configuration as shown in Figure 7.

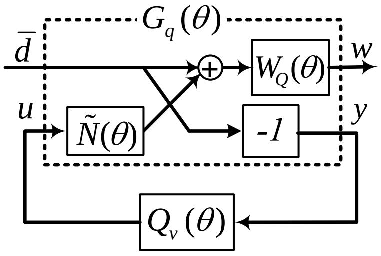

Figure 7. Simplified LFT configuration of $G_0(\theta)$.

The augmented plant $G_q(\theta)$ with the weighting function $W_Q(\theta)$ for this architecture can be derived as

$$G_q(\theta) = \begin{bmatrix} W_Q(\theta) & W_Q(\theta)\tilde{N}(\theta) \\ -I & 0 \end{bmatrix} \triangleq \left[\begin{array}{cc|cc} A_{wq}(\theta) & B_{wq}(\theta)C(\theta) & B_{wq}(\theta) & 0 \\ 0 & A(\theta)-L(\theta)C(\theta) & 0 & B(\theta) \\ \hline C_{wq}(\theta) & D_{wq}(\theta)C(\theta) & D_{wq}(\theta) & 0 \\ 0 & 0 & -I & 0 \end{array}\right], \tag{3.8}$$

where the realization of the $W_Q(\theta)$ is expressed as $\left[A_{wq}(\theta),\ B_{wq}(\theta),\ C_{wq}(\theta),\ D_{wq}(\theta)\right]$.

The controller $Q_v(\theta)$ is obtained without using information of nominal controller $K_0(\theta)$. The alternative architecture has advantages not only that it leads to lower order controller but also it is applicable to any existing control systems including nonlinear and/or non-closed form control scheme such as adaptive, model predictive, fuzzy or variable structure systems.

3.3 *Derivation of the controllers*

Design of each controller $K_0(\theta)$, $Q(\theta)$ and $Q_v(\theta)$ can be reduced to design of gain-scheduled controller $K(\theta)$ as in Figure 1, by considering an appropriate augmented plant $G_0(\theta)$, $G_Q(\theta)$ and $G_q(\theta)$, respectively.

Necessary and sufficient conditions of the existence of LPV controller $K(\theta)$ for the augmented plant $G_A(\theta)$ that assures both stability and L2-gain performance have been clarified. Here, we assume that the augmented plants have a unified formulation as

$$G_A \triangleq \left[\begin{array}{c|cc} A_p & B_{p1} & B_{p2} \\ \hline C_{p1} & D_{p11} & D_{p12} \\ C_{p2} & D_{p21} & 0 \end{array}\right]. \tag{3.9}$$

(Hereafter, dependency on the parameter θ will be omitted)

After the augmented plant (3.9) is given, corresponding controllers can be obtained by means of the well-known procedure given by Theorem 2.1 of Apkarian and Adams (1998). The two-step design of the controller is summarized below.

1. Solve for H, J, the factorisation problem $I - XY = HJ^T$.

2. Compute A_k, B_k, C_k with

$$A_k = H^{-1}(X\dot{Y} + H\dot{J}^T + \hat{A}_k - X(A_p - B_{p2}D_kC_{p2})Y - \hat{B}_kC_{p2}Y - XB_{p2}\hat{C}_k)J^{-T}$$

$$B_k = H^{-1}(\hat{B}_k - XB_{p2}D_k)$$

$$C_k = (\hat{C}_k - D_kC_{p2}Y)J^{-T}$$

where $X, Y, \hat{A}_k, \hat{B}_k, \hat{C}_k$ and D_k satisfies the following LMIs:

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \geq 0 \tag{3.10}$$

$$\begin{bmatrix} \dot{X} + XA_p + \hat{B}_k C_{p2} + (*) & * & * & * \\ \hat{A}_k^T + A_p + B_{p2} D_k C_{p2} & -\dot{Y} + A_p Y + B_{p2} \hat{C}_k + (*) & * & * \\ (XB_{p1} + \hat{B}_k D_{p21})^T & (B_{p1} + B_{p2} D_k D_{p21})^T & -\gamma I & * \\ C_{p1} + D_{p12} D_k C_{p2} & C_{p1} Y + D_{p21} \hat{C}_k & D_{p11} + D_{p12} D_k D_{p21} & -\gamma I \end{bmatrix} < 0 \quad (3.11)$$

Terms denoted * are induced by symmetry.

Finally, an admissible controller can be achieved as $[A_k,\ B_k,\ C_k,\ D_k]$.

It is noted that above design procedure contains an *NP-hard* computational problem. Several alternative design procedures with some restrictions but needing less computational power have been proposed in Packard (1994) and Scherer (2001). A simple idea for turning this infinite-dimensional problem into a finite set of LMIs is to grid the value set of θ.

4. Example

We briefly illustrate design procedure with a classical example of parameter-varying unstable plant that can be viewed as a mass-spring-damper system with time-varying spring stiffness. The state space equation of this unstable un-weighted LPV plant is expressed as follows (Xie and Eisaka 2004).

$$A(\theta) = \begin{bmatrix} 0 & 1 \\ -0.5 - 0.5\theta & -0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 0 \end{bmatrix}, \quad D = 0\,.$$

Here, for the simplicity, the scope of time-varying parameter $\theta(t)$ assumed to be varying in polytopic spaces $\Theta_1 := Co\{-1,1\}$ and the trajectory of this parameter is $\theta(t) = e^{-4t}\cos(100t)$.

From the above realization and Eq. (18), a right coprime factorisation of the plant is given as $G(\theta) = \tilde{M}^{-1}(\theta)\tilde{N}(\theta)$ with

$[\tilde{N} \quad \tilde{M}] \triangleq \left[\begin{array}{ccc|c} -29.4 & 1 & 0 & -29.4 \\ -381 & -0.2 & -1 & -380+0.5\theta \\ \hline 1 & 0 & 0 & 1 \end{array}\right]$ by setting $L(\theta) = \sum_{i=1}^{2}\alpha_i(t)L_i$, where $L_1 = [-29.4 \quad -381]^T$ and $L_2 = [-29.44 \quad -379.5]^T$. In this case, because the inverse of the $G(\theta)$ is stable, we can obtain an exact model matching controller $K_0(\theta)$ instead of the L2-gain approximation introduced in Subsection 3.1.

4.1 *Design of the nominal controller* $K_0(\theta)$

With a fast vertical convergence to the step reference, the LTI reference model was chosen as

$$T_0 = \frac{1}{s^2/\omega^2 + 2s\zeta/\omega + 1}, \text{ where } \omega = 22.4 rad/s \text{ and } \zeta = 0.8.$$

Since the inverse of $G(\theta)$ is stable, we obtain a simple controller that completes exact model matching derived as $K_0(\theta) = G(\theta)^{-1}(1-T_0)^{-1}T_0$.

The $K_0(\theta)$ is realized as $\sum_{i=1}^{2}\alpha_i(t)\begin{bmatrix} A_{ki} & B_{ki} \\ C_{ki} & D_{ki} \end{bmatrix}$ with $A_{k1} = A_{k2} = \begin{bmatrix} -35.8 & 2 \\ 0 & 0 \end{bmatrix}$, $B_{k1} = \begin{bmatrix} 139 \\ 0 \end{bmatrix}$, $B_{k2} = \begin{bmatrix} 139 \\ -1.95 \end{bmatrix}$, $C_{k1} = C_{k2} = [128 \quad 0]$ and $D_{k1} = D_{k2} = -500$.

where $\alpha_1(t) = (1-\theta(t))/2$ and $\alpha_2(t) = (1+\theta(t))/2$.

4.2 *Design of the conditional controller* $Q_v(\theta)$

Here we design a reduced order conditional controller $Q_v(\theta)$. Here, the LTI weighting function was set to be $W_Q = \dfrac{0.0001s^2 + 0.11s + 10}{1000s^2 + 110s + 1}$. According to the augmented plant formulation (3.8), using Matlab's LMI toolbox (Gahinet *et al*. 1995), we obtain a LTI $Q_v \triangleq [A_q,\ B_q,\ C_q,\ D_q]$ with optimal L2-gain performance $\gamma = 0.0023$ as

$$A_q = \begin{bmatrix} -2670 & 5.61e5 & 4750 & -1.06e4 \\ 12.6 & -2790 & -23.1 & 51.3 \\ 2900 & -6.14e5 & -5310 & 1.12e4 \\ -626 & 1.34e5 & 1140 & -2590 \end{bmatrix},\ B_q = \begin{bmatrix} -5.61e7 \\ 2.79e5 \\ 6.14e7 \\ -1.34e7 \end{bmatrix},\quad C_q = [67,\ \ -38,\ \ 81,\ \ 102],\quad D_q = 0.$$

4.3 *Simulation results*

Performance of the GIMC architecture for LPV plant is illustrated compared to the one-degree-of-freedom LPV control system with only nominal controller. Here, an indicial reference response with following step disturbance is compared.

$$d(t) = \begin{cases} 0 & 0 < t < 2.5 \\ 0.1 & 2.5 \le t \end{cases}$$

The conditional controller $Q_v(\theta)$ rejects the disturbance but does not affect the reference response given by the nominal controller, and good tracking performance and good disturbance rejection are compatibly achieved by a nominal controller and a conditional controller, respectively.

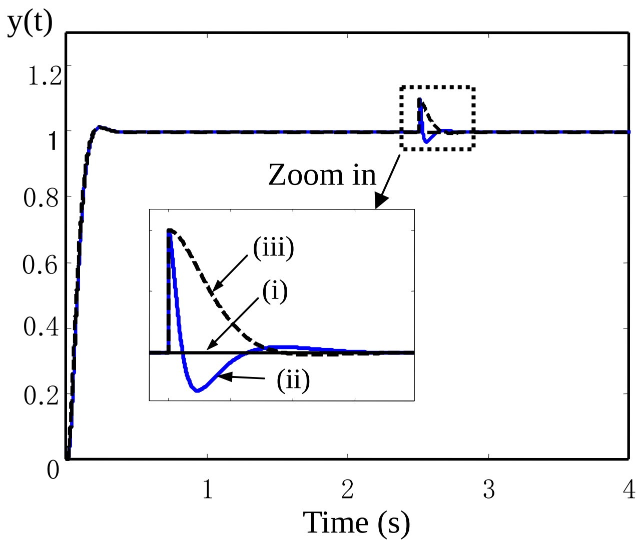

Figure 8. Indicial reference responses for;

(i) reference model $T_0(\theta)$

(ii) TDOF control system with $K_0(\theta) + Q_v(\theta)$

(iii) ODOF control system with: $K_0(\theta)$.

5. Conclusions

A Generalized Internal Model Control (GIMC) architecture and its design strategy for linear parameter varying (LPV) systems have been proposed. First, coprime factorisation and Youla Parameterisation described in state space formulas have been introduced for LPV systems based on parameter-dependent Lyapunov function. Second, according to these descriptions, GIMC for LTI systems has been extended to LPV systems with respect to parameter-dependent quadratic stability. And then, the standard design procedures of the nominal controller and the conditional controller have been proposed. In particular, a design of a lower order conditional controller has been considered. Because the lower controller is obtained without using information of the nominal controller, it is applicable to any existing control systems including nonlinear and/or non-closed form control scheme such as adaptive, model predictive, fuzzy or variable structure systems.

Consequently, the controller design problem is formulated in terms of LMIs. Based on the proposed architecture, we can design a two-degree-of-freedom controller systematically. Moreover, due to the separate structure of the conditional controller, the tuning of it to obtain expected control performance can be easily executed at an industrial site.

In the present paper, we have just focused on L2-gain performance to design controllers. Based on our results, however, the general Q-parameter approach can be applicable to LPV control systems. The Q-parameter approach will give us more practical validity to deduce the solution and covers more general control system designs including multi-objective and/or switching systems for LPV plants.

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