Title: Robust control system design for polytopic stable LPV systems

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Abstract

The present paper investigates robust control system design for polytopic stable linear parameter varying (LPV) plants using prior and non real-time knowledge of the parameter. Gain scheduled framework and robust model matching (RMM) strategy are combined to develop controllers. First, self-scheduled H-infinity method is applied to design a nominal controller using a known parameter. Then a robust compensator is added in order to reduce the influence of parameter perturbation due to the real parameter's deviation from the nominal parameter. Thus, a robust model matching design method, that is, a practical approach to the design of attachable robust compensators for the linear time invariant (LTI) plant, is extended for application to the LPV plant. Finally, robust stability of the overall system for possible parameter trajectories is confirmed. A design example and simulation results are presented in order to demonstrate the proposed method.

Text:

1. Introduction

In practical, most dynamical systems have nonlinear and/or time-variant properties, and a certain class of these systems can be represented as linear time variant (LTV) systems. Basic analysis and synthesis of control systems for LTV systems has been examined in previous studies (D'Angelo (1970); Stubberud (1964); Zadeh & Desoer (1964)). LTV system design, including tracking, stabilization, optimization and robust control, has been investigated comprehensively in several studies recently (see for example, Arvanitis (1992); Barmish (1985); Boyd (1994); Chen (1998); Feintuch (2002); Ichikawa (2001); Limebeer (1992)). However, unlike the linear time invariant (LTI) systems, few powerful tool or algebraical frequency-domain description exists to study LTV systems. As such, a systematic control system design method for the general LTV system has not yet been developed.

On the other hand, Shamma & Athans (1990, 1991) formalized a certain type of nonlinear system as a linear parameter varying (LPV) system, and succeeded in developing a control strategy for this system based on classical gain scheduled methodology. Basically, this LPV control system design method, known as the frozen parameter method, deals with only parameters that vary slowly with time. Recently, significant progress has been made in this area, and a unified H-infinity approach is being developed that is reducible to a linear matrix inequality (LMI) optimization problem (see Apkarian *et al.* (1995); Apkarian & Gahinet (1995); Becker *et al.* (1993); Boyd *et al.* (1994); Gahinet *et al.* (1995); Packard (1994); Wang & Balakrishnan (2002)). Compared to the classical gain scheduled method, these approaches take into consideration the time-varying nature of plants and grow out of ad-hoc interpolation. During the last couple of years, tutorial paper and special publications concerning this problem have appeared in Int. J. of Robust and Nonlinear Control (2002); Leith.& Leithead(2000); Rugh & Shamma (2000); Wu (2001). The recent gain scheduled method assures a quadratic H-infinity property and robust stability for all possible parameter trajectories.

Such approaches are applicable under assumption that the dependent parameters can be measured on-line. In practical control, this requirement is often difficult to satisfy. In contrast, nominal information concerning the dependent parameters is available in several applications. In this case, the systematic gain-scheduled control design technique is also applied to design of the nominal controller using the nominal trajectory. However, because the real trajectory differs from the nominal trajectory, a robust control technique is needed to compensate for this error.

Turning now to robust control design method, a practical approach to the design of attachable robust compensators has been developed by Eisaka *et al.* (1989); Kimura *et al.* (1985); Tagawa (1985); Yali & Eisaka (2000); Zhong (1996, 2002), for the LTI plant. The principle behind this method is robust model matching (RMM), which adjust 'a real plant with a robust compensator' to 'a nominal plant' by equivalent-disturbance attenuation without changing desirable response to reference in two-degree-of-freedom control scheme.

In the present paper, RMM has been developed for application to LPV plants in combination with gain-scheduled strategy. Namely, the present paper investigates robust control system design for stable polytopic LPV plants using prior and non real-time knowledge of the dependent parameter. Since the additional robust compensator is designed without information of previously designed controllers, moreover, the robust compensator is constructed separately with the previous controllers; novel RMM is applicable for any existing control systems. Among them, first, a standard design procedure of a controller for a nominal LPV plant is proposed based on a self-scheduled H-infinity method by Apkarian *et al.* (1995). Then, a robust compensator is added to reduce the influence of parameter perturbation due to the real parameter's deviation from the nominal parameter. Finally, the robust stability of the overall system for feasible trajectories is confirmed. A design example and simulation results are presented in order to illustrate the proposed method.

2. Plant description

The notation used in this paper is as follows:

$w \in \Re^p$: exogenous inputs (reference, disturbance, etc.),

$x \in \Re^l$: state vector,

$u \in \Re^q$: control inputs,

$z \in \Re^m$: controlled outputs,

$y \in \Re^g$: measurable outputs,

$\theta(t) = [\theta_1(t),\ \theta_2(t), \cdots, \theta_r(t)]^T \in \Re^r$: time-varying parametric uncertainty,

$d \in \Re^g$: equivalent disturbances representing influence on the controlled outputs due to trajectory error between the real dependent parameters and the nominal ones,

I_k : $k \times k$ unit matrix,

$0_k, 0_{a \times b}$: respectively, $k \times k$ and $a \times b$ zero matrix,

Co : convex hull.

Consider an LPV plant: $P(\theta(t))$ described by state space equations as:

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \left[\begin{array}{c|cc} A(\theta(t)) & B_w(\theta(t)) & B_u \\ \hline C_z(\theta(t)) & D_{wz}(\theta(t)) & D_{uz} \\ C_y & D_{wy} & 0_{g \times q} \end{array}\right] \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix}. \tag{2.1}$$

Here state-space matrices have compatible dimensions. Moreover we have the following assumptions. Notations follow the reference (Apkarian(1995)).

(1) The state-space matrices $A(\theta), B_w(\theta), C_z(\theta), D_{wz}(\theta)$ depend affinely on $\theta(t)$.

(2) The real parameter $\theta(t)$ is not real-time measurable but nominal one $\theta_0(t)$ can be known in advance. Both θ and θ_0 vary in the same polytope Θ of vertices $\omega_1, \omega_2, \cdots, \omega_N$, $N = 2^r$; they can be expressed respectively as:

$$\theta(t) \in \Theta := Co\{\omega_1, \omega_2, \cdots, \omega_N\} = \left\{\sum_{i=1}^{N} \alpha_i(t)\omega_i : \alpha_i(t) \geq 0, \sum_{i=1}^{N} \alpha_i(t) = 1\right\}, \tag{2.2}$$

$$\theta_0(t) \in \Theta := Co\{\omega_1, \omega_2, \cdots, \omega_N\} = \left\{\sum_{i=1}^{N} \alpha_{0i}(t)\omega_i : \alpha_{0i}(t) \geq 0, \sum_{i=1}^{N} \alpha_{0i}(t) = 1\right\}. \tag{2.3}$$

(3) The pair $(A(\theta), C_y)$ is quadratically detectable over Θ.

(4) The nominal LPV plant is stable.

With above assumptions, the LPV plant is called polytopic when it ranges in a matrix polytope. Namely, rewriting (2.1) with (2.3), the nominal LPV polytopic plant $P(\theta_0)$ can be expressed as:

$$\begin{pmatrix} A(\theta_0) & B(\theta_0) \\ C(\theta_0) & D(\theta_0) \end{pmatrix} = \sum_{i=1}^{N} \alpha_{0i}(t) \begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix} \quad \text{with} \quad \alpha_{0i} \geq 0, \Sigma_{i=1}^{N} \alpha_{0i} = 1. \tag{2.4}$$

Here, $\begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix} := \begin{pmatrix} A(\omega_i) & B(\omega_i) \\ C(\omega_i) & D(\omega_i) \end{pmatrix}$

Also, the real plant: $P(\theta)$ can be expressed as:

$$\begin{pmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{pmatrix} = \sum_{i=1}^{N} \alpha_i(t) \begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix} \quad \text{with} \quad \alpha_i \geq 0, \textstyle\sum_{i=1}^{N} \alpha_i = 1. \tag{2.5}$$

3. Controllers design

In practice, parameter $\theta(t)$ is not always available in real-time. Instead, there is a case, the nominal trajectory $\theta_0(t)$ can be settled a priori. We present a method for designing controllers for such occasions. As we mentioned, we can use any existing controllers as nominal controllers. However, here, we introduce standard design method of a nominal controller according to the nominal LPV plant, based on self-scheduled H-infinity control method by Apkarian (1995), which have nice properties discuss later. Then, we propose a design method of a robust compensator that reduce the influence of parameter perturbation due to the real parameter's deviation from the nominal one based on the method shown in Yali & Eisaka(2000).

3.1 Nominal controller design

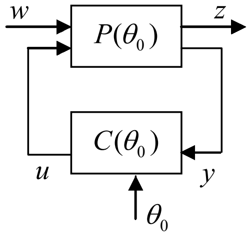

Fig.1. Gain-scheduled control scheme

Consider control scheme of Fig.1. Here, $P(\theta_0)$ is weighted nominal LPV plant A nominal controller: $C(\theta_0)$ also depends affinely on θ_0 and it is designed to satisfy the following control objectives:

(i) Desirable response to reference for $P(\theta_0)$,

(ii) Disturbance rejection for $P(\theta_0)$,

(iii) Robust stability for all feasible $P(\theta)$,

The systematic gain-scheduling control design technique is applied to design the nominal controller. The resulting control system has the quadratic H-infinity performance that guarantees L_2 gain of the map from w to z less than γ and global asymptotic stability for all feasible plant $P(\theta)$.

Design of the controller is reduced to solve LMI optimization problem similarly formulated by the method proposed in section five of reference Apkarian(1995) for nominal plant $P(\theta_0)$. We can obtain vertex state space matrices of the controller, and then the resulting c-th order continuous controller is led as:

$$\begin{pmatrix} A_c(\theta_0) & B_c(\theta_0) \\ C_c(\theta_0) & D_c(\theta_0) \end{pmatrix} = \sum_{i=1}^{N} \alpha_{0i}(t) \begin{pmatrix} A_c(\omega_i) & B_c(\omega_i) \\ C_c(\omega_i) & D_c(\omega_i) \end{pmatrix}, \quad (3.1)$$

concerning input y, outputs u and state $x_c(t) \in \Re^c$.

3.2 Robust compensator design

Because the real trajectory $\theta(t)$ will differ from nominal ones $\theta_0(t)$, the LPV control system that consists of real plant of $\theta(t)$ and nominal controller of $\theta_0(t)$ may not satisfy the desired specifications one and two mentioned above. A robust compensator should be added into the control system to recover the specifications.

In this subsection, we introduce the principle of robust model matching (RMM) method briefly, and develop this method to apply for LPV systems.

3.2.1 Principle of RMM

We see the robust compensator have structures separate from nominal control system compared Fig.1 with Fig.2. Here, the augmented plant is composed of a real plant : P and a robust compensator: R .

The philosophy of RMM is to make input-output property of the augmented plant approaches to the nominal model. This objective is achieved by means of rejecting the equivalent disturbance that represents the modeling errors. It must be noted that unlike Fig.1, P , C and R are LTI system.

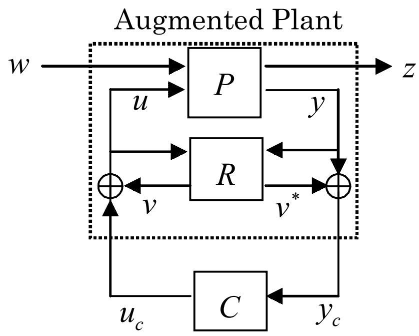

Fig.2 RMM control scheme

The robust compensator : $R(s)$ consists of following elements.

(i) Observer of equivalent disturbances: $R_o(s)$, and

(ii) Zeroing element: $R_z(s)$,

(iii) Robust filter: $R_f(s)$.

The observer calculates equivalent disturbances from measurable variables, y and u . The zeroing element cancels the effect of plant's changes by minimizing transfer matrix of over all system from equivalent disturbances to measurable outputs. Because the R_0 multiplied by the R_z is not always proper matrix, differentiators in it should be eliminated by a low-pass filter R_f called a robust filter. Another purpose of the robust filter is to consist disturbance rejection with robust stability.

Now we develop the RMM strategy to apply for LPV systems, and explain design procedure.

3.2.2 Robust compensator design for polytopic LPV systems

Because there is no algebraic transfer function like LTI system, unlike conventional RMM, we propose a robust compensator based on state-space expression.

(i) Observer: $R_o(\theta_0)$

The real signals around the plant can be expressed with the nominal plant and disturbances as:

$$\begin{aligned} y &= P(\theta)u \\ &= P(\theta_0)u + d \end{aligned} . \tag{3.2}$$

The vector $d \in \Re^g$ represents the influence of trajectory error on the measurable outputs, and called equivalent disturbance of LPV plants.

The state space equation of the observer $R_o(\theta_0)$ can be derived from substituting (2.1) into (3.2) as the following:

$$\begin{bmatrix} \dot{x}_o(t) \\ d(t) \end{bmatrix} = \begin{bmatrix} A(\theta_0) & \begin{pmatrix} B_u, & 0_{l\times g} \end{pmatrix} \\ -C_y & \begin{pmatrix} 0_{g\times q}, & I_g \end{pmatrix} \end{bmatrix} \begin{bmatrix} x_o(t) \\ \begin{pmatrix} u(t) \\ y(t) \end{pmatrix} \end{bmatrix}, \tag{3.3}$$

here, $x_o(t) \in \Re^l$ stands for the states of the observer.

(ii) Zeroing element: $R_z(\theta_0)$

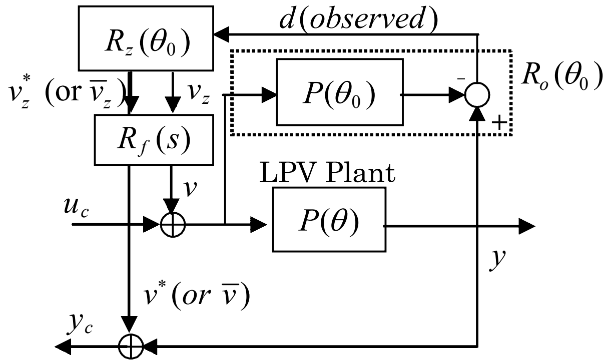

Fig.3 LPV plant with robust compensator

In RMM control scheme (Fig.2), robust compensator can be constructed as Fig.3 .

The role of the zeroing element $R_z(\theta_0)$ is to realize quadratic H-infinity performance for the LPV plant, applying gain scheduling methodology (Apkarian(1995)). We can obtain bounded input/output map of the augmented LPV plant for all possible trajectories as:

$$\|y_c\|_2 \le \gamma \|d\|_2 . \tag{3.4}$$

If v^* is restricted to $\overline{v}$ that adds just to $\overline{z}$ (y except z), Fig.3 can be rewritten simply to LFT (Linear Fractional Transformation) structure as in Fig.4

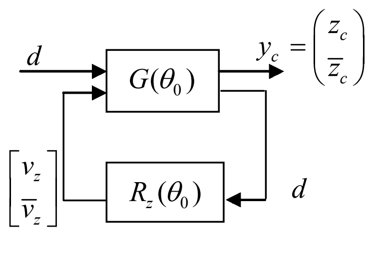

Fig.4 Rearranged figure of Fig.3 except the observer

Here, state-space expression of $G(\theta_0)$ is expressed as:

$$\begin{bmatrix} \dot{x}(t) \\ z_c(t) \\ \bar{z}_c(t) \\ d(t) \end{bmatrix} = \left[\begin{array}{c|cc} A(\theta_0(t)) & 0_{l\times g} & (B_u, 0_{l\times(g-m)}) \\ \hline \begin{pmatrix} C_{zc} \\ C_{\bar{z}c} \end{pmatrix} & I_g & \begin{pmatrix} 0_{m\times q} & 0_{m\times(g-m)} \\ I_{(g-m)\times q} & 0_{g-m} \end{pmatrix} \\ 0_{g\times l} & I_g & (0_{g\times q}, 0_{g\times(g-m)}) \end{array}\right] \begin{bmatrix} x(t) \\ d(t) \\ v_z(t) \\ \bar{v}_z(t) \end{bmatrix}, \tag{3.5}$$

and, C_y in (2.1) is rewritten as $\begin{pmatrix} C_{zc} \\ C_{\bar{z}c} \end{pmatrix}$ corresponding with $\begin{pmatrix} z_c \\ \bar{z}_c \end{pmatrix}$. Above $G(\theta_0)$ is produced from unweighted nominal plant, but if necessary we can use weighting function that can be selected based on a frozen time analysis and follows the same way as conventional H-infinity synthesis.

Similarly to nominal controller design mentioned in subsection 3.1, we obtain vertex state space matrices of the compensator $R_z(\theta_0)$, and then the resulting continuous compensator is led as:

$$\begin{pmatrix} A_z(\theta_0) & B_z(\theta_0) \\ C_z(\theta_0) & D_z(\theta_0) \end{pmatrix} = \sum_{i=1}^{N} \alpha_{0i}(t) \begin{pmatrix} A_z(\omega_i) & B_z(\omega_i) \\ C_z(\omega_i) & D_z(\omega_i) \end{pmatrix}, \tag{3.6}$$

concerning input d, outputs $\begin{pmatrix} v_z \\ \bar{v}_z \end{pmatrix}$ and state $x_z(t) \in \Re^z$.

(iii) Robust filter: $R_f(\theta_0)$

In order to consist disturbance rejection with robust stability and keep the closed-loop state-space matrix affine dependent on $\theta(t)$ or $\theta_0(t)$, a transfer function matrix called robust filter is used. To satisfy these requirements, the robust filter should have adequate band-width and be strictly proper as the form of:

$$
\begin{bmatrix} \dot{x}_f \\ v(t) \\ \overline{v}(t) \end{bmatrix} = \left[\begin{array}{c|c} A_f & \begin{pmatrix} B_{fv}, & B_{f\overline{v}} \end{pmatrix} \\ \hline \begin{pmatrix} C_{fv} \\ C_{f\overline{v}} \end{pmatrix} & 0_{q+g-m} \end{array}\right] \begin{bmatrix} x_f(t) \\ \begin{pmatrix} v_z(t) \\ \overline{v}_z(t) \end{pmatrix} \end{bmatrix}, \tag{3.7}
$$

besides, $x_f(t) \in \Re^f$ stands for the states of the robust filter.

4. Robust stability analysis of whole closed-loop system

If we design nominal controller $C(\theta_0)$ based on self-scheduled H-infinity method, we can assure robust stability of the control system consist of $P(\theta)$ with $C(\theta_0)$ for all possible θ. On the other hand, robust compensator has a capability to make augmented $P(\theta)$ approaches to $P(\theta_0)$. Therefore, it is reasonable to suppose that over-all system including the robust compensator is also stable for all $\theta(t)$.

In this section, first we derive the closed loop state-space matrix with real plant. Then we show two ways to test the stability of the closed system following the way described in Gahinet *et al.*(1995).

The closed loop autonomous state space expression is given by combination of (2.1), (3.1), (3.3), (3.6) and (3.7) as:

$$
\dot{x}_{cl} = A_{cl}(\tilde{\theta})x_{cl} \tag{4.1}
$$

Here, $x_{cl}(t)^T = \begin{bmatrix} x(t)^T & x_o(t)^T & x_z(t)^T & x_f(t)^T & x_c(t)^T \end{bmatrix}$, $\tilde{\theta} = [\theta^T, \theta_0{}^T]^T$ and

$$A_{cl}(\tilde{\theta}) = \begin{bmatrix} A(\theta)+B_u D_c(\theta_0)C_y & 0_{l\times l} & 0_{l\times z} & B_u(C_{fv}+D_c(\theta_0)\begin{bmatrix}0_{m\times f}\\ C_{f\bar{v}}\end{bmatrix}) & B_u C_c(\theta_0) \\ B_u D_c(\theta_0)C_y & A(\theta_0) & 0_{l\times z} & B_u(C_{fv}+D_c(\theta_0)\begin{bmatrix}0_{m\times f}\\ C_{f\bar{v}}\end{bmatrix}) & B_u C_c(\theta_0) \\ B_z(\theta_0)C_y & -B_z(\theta_0)C_y & A_z(\theta_0) & 0_{z\times f} & 0_{z\times c} \\ (B_{fv}D_{zv}(\theta_0)+B_{f\bar{v}}D_{z\bar{v}}(\theta_0))C_y & -(B_{fv}D_{zv}(\theta_0)+B_{f\bar{v}}D_{z\bar{v}}(\theta_0))C_y & B_{fv}C_{zv}(\theta_0)+B_{f\bar{v}}C_{z\bar{v}}(\theta_0) & A_f & 0_{f\times c} \\ B_c(\theta_0)C_y & 0_{c\times l} & 0_{c\times z} & B_c(\theta_0)\begin{bmatrix}0_{m\times f}\\ C_{f\bar{v}}\end{bmatrix} & A_c(\theta_0) \end{bmatrix},$$

here, C_z and D_z in (3.6) is rewritten as $\begin{pmatrix} C_{zv} \\ C_{z\bar{v}} \end{pmatrix}$ and $\begin{pmatrix} D_{zv} \\ D_{z\bar{v}} \end{pmatrix}$ corresponding with output $\begin{pmatrix} v_z \\ \bar{v}_z \end{pmatrix}$. Note that $A_{cl}(\widetilde{\theta}(t))$ is affinely dependent on both $\theta(t)$ and $\theta_0(t)$.

4.1 Quadratic stability

The system is said to be quadratically stabilizable via a dependent parameter if there exists a $(2l+z+f+c)\times(2l+z+f+c)$ positive definite matrix P such that:

$$\mathrm{P}A_{cl}(\widetilde{\theta}) + A_{cl}^T(\widetilde{\theta})\mathrm{P} < 0 . \tag{4.2}$$

Inequality (4.2) is reduced to be similar problem as after-mentioned (4.5) with common matrix P.

4.2 Parameter-dependent Lyapunov functions

Less conservative sufficient conditions for stability over the entire polytope are as follows:

Since $A_{cl}(\widetilde{\theta})$ varies in the convex envelope of a set of LTI models as:

$$A_{cl}(\tilde{\theta}) \in Co(A_1,\cdots,A_{2N}) = \left\{ \sum_{i=1}^{2N} \beta_i A_i : \beta_i \ge 0, \sum_{i=1}^{2N} \beta_i = 1 \right\}, \tag{4.3}$$

we seek a parameter-dependent Lyapunov function of the form $V(x,\beta) = x^T \mathrm{P}(\beta)^{-1} x$, where, $$\mathrm{P}(\beta) = \beta_1 \mathrm{P}_1 + \cdots + \beta_{2N} \mathrm{P}_{2N} . \tag{4.4}$$

If there exists $(2l+z+f+c)\times(2l+z+f+c)$ symmetric matrices $\mathrm{P}_1,\cdots,\mathrm{P}_{2N}$, and scalars $t_{ij} = t_{ji}$ such that,

$$\begin{aligned} & A_i \mathrm{P}_j + \mathrm{P}_j A_i^T + A_j \mathrm{P}_i + \mathrm{P}_i A_j^T < 2t_{ij} I_{(2l+z+f+c)} , \\ & \mathrm{P}_j > I_{(2l+z+f+c)} , \\ & \begin{bmatrix} t_{11} & \cdots & t_{1(2N)} \\ \vdots & \ddots & \vdots \\ t_{(2N)1} & \cdots & t_{(2N)(2N)} \end{bmatrix} < 0 , \end{aligned} \tag{4.5}$$

for all $i, j \in \{1, 2, \cdots, 2N\}$, then the Lyapunov function $V(x,\beta)$ establishes stability of the whole closed-loop system.

5. Example

The state space equation of an unweighted LPV plant is assumed as:

$$A(\theta(t)) = \begin{bmatrix} -2\theta(t) & -1 \\ 1 & 0 \end{bmatrix}, B_w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{5.1}$$

$$C_z = \begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 1 & -2 \end{bmatrix},\ C_y = \begin{bmatrix} -1 & 2 \end{bmatrix},\ D_{wz} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ D_{uz} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ D_{wy} = 0.$$

Here the scope of nominal time-varying parameter $\theta(t)$ is in polytopic spaces $\Theta := Co\{1.5, 3.5\}$. Also, the nominal trajectory of dependent parameter $\theta_0(t)$, $\alpha_{01}(t)$ and $\alpha_{02}(t)$ are assumed as:

$$\theta_0(t) = 2.5 + \sin(0.01t),$$

$$\alpha_{01}(t) = (3.5 - \theta_0)/2, \tag{5.2}$$

$$\alpha_{02}(t) = 1 - \alpha_{01}(t).$$

5.1 The design of nominal controller

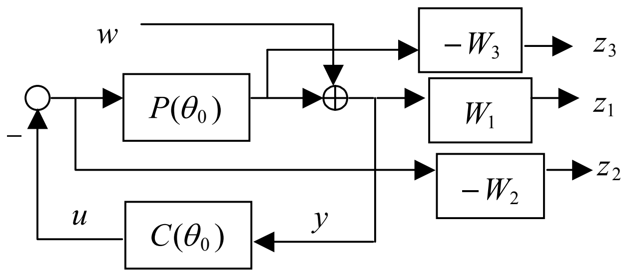

Fig.5 Block diagram of the nominal control system

To enforce the performance and robustness requirements, we treat L_2 gain of the map from w to z_1, z_2 and z_3 less than γ as the following inequality (5.3) and global asymptotic stability for all feasible parameter trajectories θ_0 in the polytopic space Θ;

$$\left\| \begin{matrix} W_1 S \\ W_2 SC \\ W_3 T \end{matrix} \right\| \le \gamma. \tag{5.3}$$

Here S, SC and T denotes maps from w to z_1, z_2 and z_3, respectively.

The weighting functions were chosen as follows:

$$W_1 = \frac{6s+1}{25s+0.04}, \; W_2 = 0.01, \; W_3 = \frac{s+1.5}{5s+100}.$$

The gain diagram of the weighs is shown in Fig. 6.

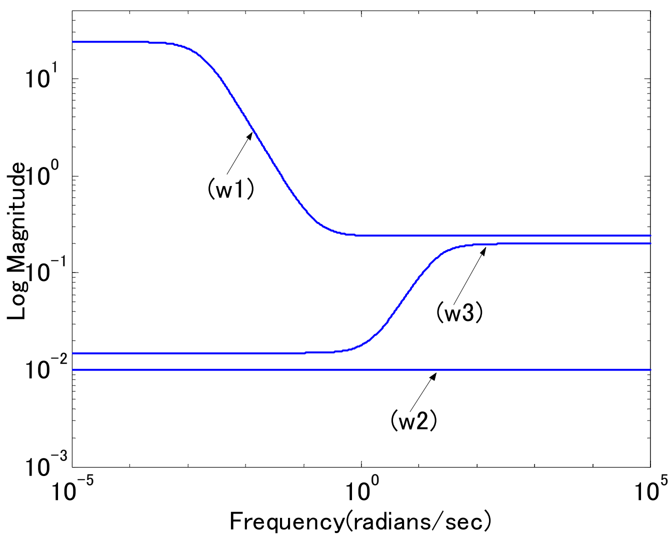

Fig.6 Gain diagram of weighting functions

Using standard software from the Matlab LMI toolbox (see Gahinet(1995)), we got controller vertex matrices as:

$$A_{c1} = \begin{bmatrix} -5.61e-3 & 0.53 & -6.47e-2 & 1.64e-2 \\ -0.42 & -4.62 & 13.6 & 8.02 \\ -1.63e-2 & -0.88 & -5.29 & 16.7 \\ 9.12e-2 & -0.30 & 1.91 & -13.3 \end{bmatrix},$$

$$B_{c1}^T = \begin{bmatrix} 0.10 & 2.66e-2 & 3.17e-2 & 1.06e-1 \end{bmatrix},$$

$$C_{c1} = \begin{bmatrix} 5.43e-2 & -1.64 & 13.1 & 8.48 \end{bmatrix}, \qquad D_{c1} = 0,$$

$$A_{c2} = \begin{bmatrix} -5.64e-3 & 0.55 & -6.45e-2 & 1.10e-2 \\ -0.41 & -8.60 & 13.5 & 8.27 \\ -1.60e-2 & -0.96 & -5.29 & 16.7 \\ 8.87e-2 & -0.32 & 1.92 & -13.4 \end{bmatrix},$$

$$B_{c2}^T \approx B_{c1}^T, C_{c2} \approx C_{c1}, \ \ D_{c2} = 0. \tag{5.4}$$

Then the nominal controller can be constructed as:

$$\begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} = \alpha_{01}(t)\begin{bmatrix} A_{c1} & B_{c1} \\ C_{c1} & D_{c1} \end{bmatrix} + \alpha_{02}(t)\begin{bmatrix} A_{c2} & B_{c2} \\ C_{c2} & D_{c2} \end{bmatrix}. \tag{5.5}$$

The H-infinity norm of the above optimal problem γ is 0.50 after 11 iterations of the algorithm.

5.2 design of robust compensator

(i) Observer

Observer of the base-equivalent disturbance R_o is derived from substituting (5.1) into (3.3) as:

$$\begin{bmatrix} \dot{x}_o(t) \\ d(t) \end{bmatrix} = \left[\begin{array}{cc|cc} -2\theta_0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ \hline 1 & -2 & 0 & 1 \end{array}\right] \begin{bmatrix} x_o(t) \\ \begin{pmatrix} u(t) \\ y(t) \end{pmatrix} \end{bmatrix}. \tag{5.6}$$

(ii) Zeroing element

According to the subsection 3.2.2 we consider the minimization problem as (3.5). The following weighting function: $k(s)$ was used in both d to d and d to y_c relation of $G(\theta_0)$ in Fig.4.

$$k(s) = \frac{1}{s+0.1} \tag{5.7}$$

Using Matlab LMI Toolbox, solving for the zeroing element yielded a performance level of $\gamma = 1.03$ after 13 iterations of the algorithm, we got the optimization result as:

$$A_{Z1} = \begin{bmatrix} -34.6 & 4.10 & 7.88 & 226 \\ 1001 & -126 & -217 & -6580 \\ -449 & 57.6 & 96.9 & 2951 \\ 0.28 & 0.71 & 1.36 & -6.84 \end{bmatrix}, \quad B_{Z1} = \begin{bmatrix} -0.82 \\ -0.19 \\ -0.36 \\ 0.79 \end{bmatrix},$$

$$C_{Z1} = \begin{bmatrix} 1096.9 & -135.7 & -237.7 & -7215.4 \end{bmatrix}, \; D_{Z1} = 0,$$

and $A_{Z2} \approx A_{Z1}, B_{Z2} \approx B_{Z1}$, $C_{Z2} = \begin{bmatrix} 1096.6 & -131.9 & -239.0 & -7215.4 \end{bmatrix}$, $D_{Z2} = 0$.

Consequently, R_z whose inputs and outputs are respectively d and v_z is given as:

$$\begin{bmatrix} A_z & B_z \\ C_z & D_z \end{bmatrix} = \alpha_{01}(t) \cdot \begin{bmatrix} A_{Z1} & B_{Z1} \\ C_{Z1} & D_{Z1} \end{bmatrix} + \alpha_{02}(t) \cdot \begin{bmatrix} A_{Z2} & B_{Z2} \\ C_{Z2} & D_{Z2} \end{bmatrix}. \tag{5.8}$$

(iii) Robust filter

In this case, as the tuning function of robust stability of whole system robust filter is chosen as:

$$R_f = \frac{1}{0.67s + 1}. \tag{5.9}$$

5.3 Stability test of whole system

According to the section 4, $A_{cl}(\widetilde{\theta})$ is described as:

$$A_{cl} = \begin{bmatrix} -2\theta & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.05 & -1.6 & 13.8 & 8.4 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2\theta_0 & -1 & 0 & 0 & 0 & 0 & 1 & & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05 & -1.6 & 13.8 & 8.4 \\ 0.82 & -1.62 & -0.82 & 1.62 & -34.6 & 4.09 & 7.88 & 226.4 & 0 & 0 & 0 & 0 & 0 \\ 0.19 & -0.38 & 0.19 & 0.38 & 1000.9 & -126.4 & -216.7 & -6580.2 & 0 & 0 & 0 & 0 & 0 \\ 0.36 & -0.72 & 0.36 & 0.72 & -449 & 57.6 & 96.9 & 29513 & 0 & 0 & 0 & 0 & 0 \\ 0.79 & 1.58 & 0.79 & -1.58 & 0.28 & 0.71 & 1.36 & -6.84 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1644 & -207 + 2.8\theta_0 & -355 - \theta_0 & -10823 & -1.5 & 0 & 0 & 0 & 0 \\ -0.1 & 0.2 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & -5.5e-3 & 0.52 & -6.4e-2 & 0.02 \\ -0.02 & 0.04 & 0 & 0 & 0 & 0 & 0.02 & 0 & 0 & -0.43 & -1.63 - 1.98\theta_0 & 13.6 & 7.83 + 0.12\theta_0 \\ -0.03 & 0.06 & 0 & 0 & 0 & 0 & 0.03 & 0 & 0 & -1.6e-2 & -0.82 & -5.3 & 16.7 \\ -0.11 & 0.22 & 0 & 0 & 0 & 0 & 0.11 & 0 & 0 & 0.09 - 1.3e-3\theta_0 & -0.77 + 0.31\theta_0 & 1.9 & -13.3 \end{bmatrix}$$

We found the existence of positive-definite quadratic Lyapunov function as：

$$V(x) = x^T \mathrm{P} x$$

such that $\dot{V}(x) < 0$ where P was given as:

$$P = \begin{bmatrix}
1.12e-6 & -1.49e-6 & 0 & 0 & 0 & 0 & -1.08e-6 & 1.57e-6 & 0 & 8.2e-6 & 5.8e-4 & 0.01 & 1.08e-3 \\
-1.49e-6 & 4.28e-6 & 0 & 0 & 0 & 0 & 1.4e-6 & -4.1e-6 & 0 & -1.9e-5 & -2e-3 & -0.02 & -2e-3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.6e-5 & -4e-4 & -4.7e-5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2e-4 & 1.1e-4 & 1.35e-5 \\
0 & 0 & 0 & 0 & 1.23e-6 & 1.69e-6 & 0 & 0 & 0 & 2.33e-6 & 1.1e-4 & 5.8e-3 & 4.5e-4 \\
0 & 0 & 0 & 0 & 1.69e-6 & 2.48e-6 & 0 & 0 & 0 & 2.56e-6 & 1.6e-4 & 5.8e-3 & 5.8e-3 \\
-1.08e-6 & 1.4e-6 & 0 & 0 & 0 & 0 & 0 & -1.43e-6 & 0 & -8.07e-6 & -4.3e-4 & -0.01 & -1e-3 \\
1.57e-6 & -4.1e-6 & 0 & 0 & 0 & 0 & -1.43e-6 & 4.3e-6 & 0 & 2e-5 & 2.5e-3 & 0.028 & 2.8e-3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.6e-4 & 7.8e-4 & 7.6e-5 \\
8.2e-6 & -1.9e-5 & 0 & 0 & 2.33e-6 & 2.56e-6 & -8.07e-6 & 2e-5 & 0 & 7e-4 & 0.016 & 0135 & 1.34e-5 \\
5.8e-4 & -2e-3 & -1.6e-5 & -2e-4 & 1.1e-4 & 1.6e-4 & -4.3e-4 & 2.5e-3 & 2e-4 & 0.016 & 3.1 & 16.8 & 1.66 \\
0.01 & -0.02 & -4e-4 & 1.1e-4 & 5.8e-3 & 5.8e-3 & -0.01 & 0.028 & 7.8e-4 & 0.135 & 16.86 & 191.6 & 19 \\
1.08e-3 & -2e-3 & -4.7e-5 & 1.35e-5 & 4.5e-4 & 5.8e-3 & -0.001 & 2.8e-3 & 7.57e-5 & 0.0134 & 1.66 & 19 & 1.88
\end{bmatrix}.$$

Here absolute values of elements less than $1e-7$ are expressed as zeros. Eigenvalues of P are (195, 1.6, 5.8e-4, 3.4e-6, 6.8e-7, 2.8e-7, 6.2e-8, 1.2e-8, 7.7e-8, 4.9e-8, 2.8e-8, 3.0e-8, 3.6e-8).

5.4 Simulation results

The proposed method is illustrated by indicial responses. Proposed control systems are compared with nominal control system designed based on reference Apkarian(1995) under the following two cases about real trajectories in the plant.

Case one: $\theta(t) = 2.5 + \cos(0.05t)$

Case two: $\theta(t) = \begin{cases} 2.5 + \sin(0.05t) & t \le 50 \\ 3.1 & t > 50 \end{cases}$

Trajectories and indicial responses are shown in Fig.8 and Fig.9, respectively each case.

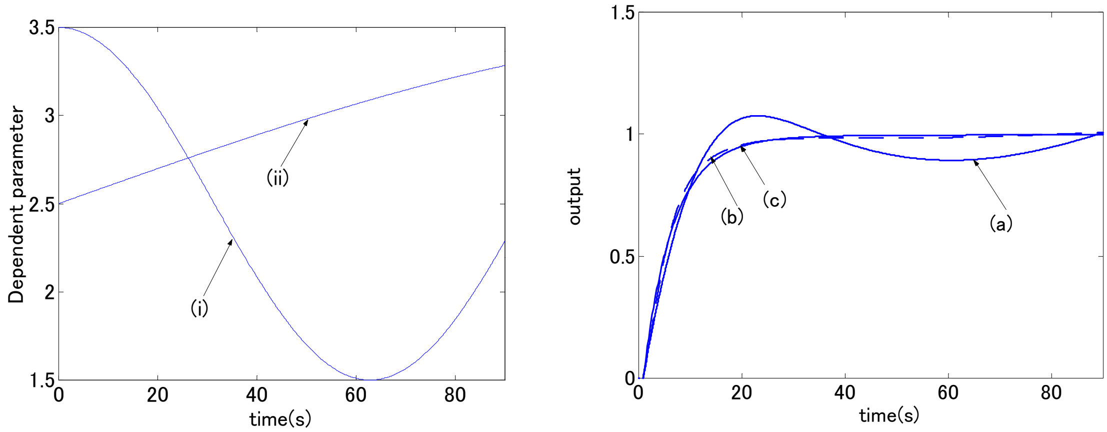

Fig.7 Parameters trajectories and indicial responses of Case one.

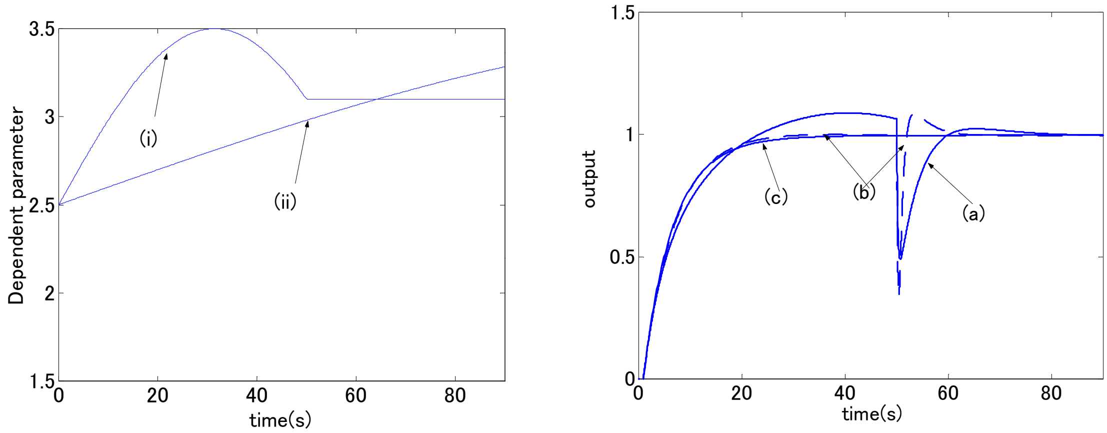

Fig.8 Parameters trajectories and indicial responses of Case two

(i) Trajectories of real dependent parameters

(ii) Trajectories of nominal dependent parameters

(a) Control system without robust compensator (b) control system with robust compensator design(c) nominal system

In both simulations, even if parameter changes quickly and far deviates from nominal parameter, the proposed method shows near response with desirable ones. From Fig.7 although real parameter shows opposite phase to nominal one, the response of proposed method has almost the same as nominal response. In Fig.8, real parameter has a nasty variation at 50 seconds later, the response of proposed method converges to nominal response more steeply than the case without robust compensator.

6. Conclusions

In the present paper, an approach by which to design a robust compensator for an LPV polytopic control system using prior and non real-time knowledge of the dependent parameter has been proposed. First, a nominal polytopic LPV plant has been selected based on the trajectory of the nominal dependent parameters. Second, a nominal controller has been developed to design a robustly stable control system for the LPV plant with polytopic time-varying uncertainty. In order to address parameter perturbation that occurs due to the real dependent parameter's deviation from nominal dependent parameter, a design method involving adding an attaching robust compensator to the nominal control system has been proposed. The role of the robust compensator is to eliminate the influence of parameter perturbation on the controlled output. Therefore, a robust model matching design method that is applicable to the LPV plant has been developed. The problem is reduced to an optimization problem having LMI constraints for the vertex matrix derived from the LPV plant. The robust compensator is designed using only information from the plant, so the robust compensator can be attached to any type of existing control system. Finally, robust stability of the overall system for possible parameter trajectories has been confirmed. The design procedure has been demonstrated in an example design, and the performance of the proposed method has been examined.

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