Thesis

Quantification of uncertainty of material properties and its application

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Doctoral Thesis

Quantification of uncertainty of material properties and its application

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Abstract

The material(s) used to produce a component or product often determine(s) its economic, functional, and sustainable characteristics. On the other hand, there are different types of materials (metals and alloys, composites, natural materials, plastics, ceramics, and foams), and each type consists of a large number of members. Therefore, selecting an optimal material for a given component or product is not only an important problem but also a difficult one to solve. Usually, the entities called material indices (*MI*) are used to solve a material selection problem. *MI* is a function that consists of some of the material properties (mechanical properties, electrical properties, magnetic properties, and sustainability properties) and applicable only to a machine element (e.g., beam). In real-life, a component cannot be characterized by a single machine element; it is a combination of several machine elements (e.g., a combination of beam, plate, and column). Therefore, there is uncertainty regarding the material indices themselves while selecting an optimal material in a real-life setting, i.e., a material index-free procedure makes a material selection process a more pragmatic one. In addition, the data regarding a material property (e.g., data regarding the tensile strength) exhibits a great deal of variability. Therefore, uncertainty associated with the data of material properties needs to be quantified before starting the process of selecting an optimal material either by a material index-based procedure or by any other means. Based on the abovementioned contemplation, one of the objectives of this thesis is to shed some light on the uncertainty quantification of material properties. The other objective is to develop a material selection model that does not depend on any material indices.

Accordingly, the following chapters have been incorporated in this thesis. Chapter 1 describes the background of the thesis and reports a literature review on the uncertainty, material selection methods. It also describes the experimental works for determining the properties of the relevant materials. Chapter 2 describes the mathematical entities needed to define the uncertainty in statistical,

probabilistic, and possibilistic means. In addition, the mathematical entities needed for the formal computation while selecting a material is also described. Chapter 3 shows the experimental results regarding the mechanical properties, namely, tensile strength, modulus of elasticity, and stain to failure of a natural material called Jute. The uncertainty associated with the properties mentioned above has been quantified by using the statistical, probabilistic, and possibilistic approaches. It has been found that out of the three approaches, the possibilistic approach quantifies the uncertainty more reliably. Therefore, when one uses a material property for making a decision, its uncertainty can be put into the formal computation using a possibility distribution (e.g., a triangular fuzzy number) rather than using a probability distribution (e.g., Weibull distribution) or statistical approach. Based on this conclusion, the uncertainties associated with the tensile strength, modulus of elasticity, density, CO_2 footprint, recycle fractions, and water usages of 197 types of Aluminum alloys, 45 types of Titanium alloys, and 30 types of Magnesium alloys are represented by possibility distributions, as reported in Chapter 4. In addition, a decision model is also developed in Chapter 4 to select an optimal material out of the three alternatives namely, Aluminum, Titanium, and Magnesium alloys. In this decision model, the objective functions (e.g., maximize tensile strength, minimize CO_2 footprint, and so on) are set by the possibility distributions, too. Three of the possibility distributions (i.e., objective functions) are for maximizing the tensile strength, modulus of elasticity, and recycle fractions, respectively, and the other three are for minimizing the density, CO_2 footprint, and water usages. The compliance between the possibility of distribution of a material property of a type of alloys (e.g., possibility distribution of the tensile strength of Aluminum alloys) and the possibility distribution of the corresponding objective function (e.g., possibility distribution of maximizing the tensile strength) are used to make a decision. It is found that the decision model selects an optimal material even though the material properties are uncertain and the underlying material indices are not known. In Chapter 5 discusses the implications of this study in eco-product development. It also describes how the stakeholders (research organizations and researchers, designers, producers) should interact centering the material related decision making processes. Finally, Chapter 6 provides the concluding remarks of this thesis.

Chapter 1: Introduction

The notion of uncertainty has earned a great deal of attention from the researchers belonging to various academic disciplines. In this study, the uncertainty in the material properties is considered and its application, particularly, the material selection for developing a product has been investigated. The remainder of the chapter is structured as follows: General Background, Core Idea, Scope, Objectives, Literature Review, and the Thesis Structure.

1.1 General Background

Engineering materials and their properties, the concept of sustainability, uncertainty and their categories, uncertainties in product development particularly material selection are described in this subsection.

1.1.1 Materials and their Properties

In this artificial world, there are many products. Each product has components and each component consists of materials, has a specific shape, performs a specific function, and is a part of a system in the product. The materials can be metals and alloys, ceramics, polymer, rubber, natural material, composite or any other combination. Engineering materials are evolving day by day, that means the number of engineering materials is uncertain. Therefore, material selection is one of the important issues in product development. For example, consider a product called a bag as schematically illustrated in Figure 1.1. It has a chamber, handle, zipper, and lock. When we develop a bag, we need to define the materials for the chamber, handle, zipper, and lock. This also means that we know how to select the optimal materials for each component mentioned in the above, from a large number of alternatives.

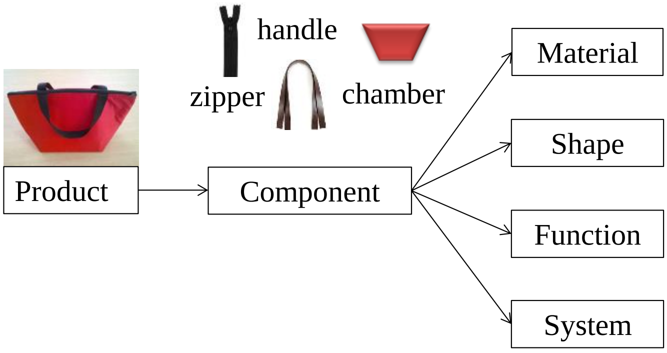


Figure 1.1: Dependency of a product on different elements.

In 1945, there were only four categories of engineering materials, namely, Metals and Alloys, Polymers and Elastomers, Ceramics and Glasses, and Natural Materials. Now, there are two more categories, namely, Foams and Composites, resulting more than 80,000 members in the Universe of Materials. Refer to (Mousavi-Nasab and Anvari, 2017) and (Ashby, 2007) for more details regarding the types of materials. Therefore, material selection is a difficult problem to solve. Usually, the material is selected using a function called Material Index, *MI* (Ashby, 2007). The material selection procedure is briefly described as follows.

Consider an object schematically shown in Figure 1.2. Depending on the shape, support, and loading condition, it can be considered a Column, Plate, Tie, Beam, or Panel or any combinations. It is made from a material, which has different properties such as mechanical, chemical, optical, thermal, sustainable, electrical, magnetic, atomic, and manufacturing properties.

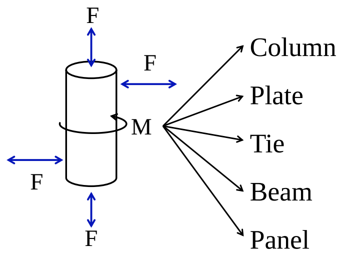


Figure 1.2: Schematic diagram of a mechanical object under load and torque.

From the sustainable manufacturing point of view for a mechanical object along with other material properties, sustainable properties are important. In this study, mechanical and sustainable properties are emphasized. The mechanical properties include tensile strength (*TS*), modulus of elasticity (*E*), hardness, density (ρ), endurance limit (σ_e), fracture toughness, and compressive strength. On the other hand, the sustainable properties (Ashby, 2007) include Carbon dioxide (CO_2) Footprint, Water usage, Recycle fraction, Cost, Reservation, Safety, and Environmental damage. Consider a case, to select materials for a mechanical object 'tie'. Nowadays, while selecting materials, the sustainability is taken as one of the key considerations by engineers and researchers. The idea of sustainability is described in the following section.

1.1.2 Sustainability

In this section, the concept of sustainability is enhanced. Sustainability means fulfilling the present-day needs without jeopardizing the potential of fulfilling the future needs (N.N, 1987; Ullah, et al., 2014). The salient point of sustainability is schematically illustrated in Figure 1.3. As seen in Figure 1.3, two worlds, artificial (marked as 1) and natural worlds (marked as 2), simultaneously exist around us, and they must be synergistic to each other. The concept of 'sustainability' deals with issues related to the coexistence of natural and artificial worlds (Umeda, et al., 2009; Ullah, et al., 2017). In particular, the natural world consists of natural resources (water, air, land, ore, biomass, and hydrocarbon) whereas the artificial world consists of products (car, road, building, plane, train, pen, computer, paper, and many more). Using the natural resources, primary energy and materials are produced. Afterward, the primary energy and materials are used to produce products and support their lifecycles (marked as 4). The artificial world is full of man-made products; each product has a life cycle. A lifecycle means the chronological stages of a product, namely, conceptualization, design, manufacturing, use, recycle, and landfill. To obtain primary materials and energy, resources (marked as 3) are required which are obtained from the natural world. If a product (or its lifecycle) needs a large amount of energy and materials, it puts burden on the natural resources, and, thereby, on the natural world. This means the natural world is exhausted to enrich the artificial world by producing and maintaining products.

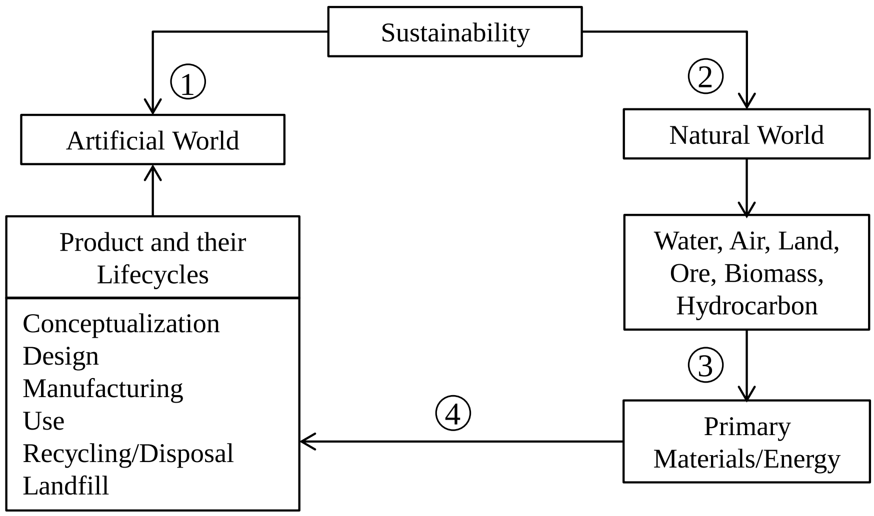


Figure 1.3: The concept of the life cycle from the viewpoint of the primary energy.

This means that the sustainability is jeopardized if the demands of primary energy and materials are not kept within the stipulated limits. Numerous studies have shown that the types and usages of materials in the products (i.e., the constituents of the artificial world) heavily affect the sustainability (Ullah, et al., 2014; Ullah, et al., 2013; Shahinur and Ullah, 2017). However, for sustainability, both these worlds must coexist, and we must not overburden the natural world as well as fulfill the needs of the artificial world.

Many strategic goals have been set to achieve sustainability of the environment, one important goal is to reduce the global *CO_2 Footprint* by half, by the year 2050 (Allwood, et al., 2010) schematically shown in Figure 1.4.

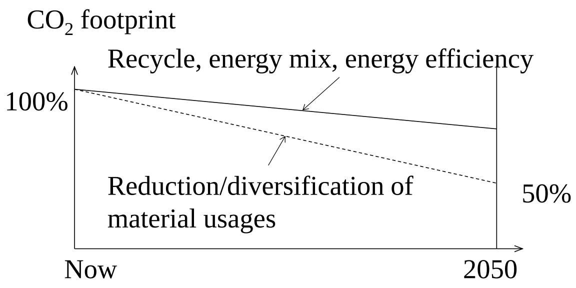


Figure 1.4: Important strategy to maintain the sustainability.

Effective CO_2 reduction requires the simultaneous attainment of efficiency in terms of materials, energy, and components of products (Allwood, et al., 2010; Milford, et al., 2013; Allwood, et al., 2011; Ullah, et al., 2013; Ullah, et al., 2014). Improving material efficiency means increasing the use of environmental friendly materials, increasing material yields, and making lightweight products (Allwood, et al., 2010; Milford, et al., 2013; Allwood, et al., 2011; Ullah, et al., 2013; Ullah, et al., 2014). Energy efficiency denotes deploying renewable energy sources, and decreasing energy usage (Ullah, et al., 2013; Ullah, et al., 2014). It has been found that material efficiency is more effective than energy efficiency in achieving the strategic goals of sustainability (Allwood, et al., 2010; Milford, et al., 2013; Allwood, et al., 2011; Ullah, et al., 2013; Ullah, et al., 2014). One of the options for achieving material efficiency is to increase the amount of usage of natural materials in various products as much as possible (Alves, et al., 2010). The natural materials have low density as schematically shown in Figure 1.5 and require low energy for the primary material productions. Thus, natural materials are better option for sustainability of the product compared to other materials.

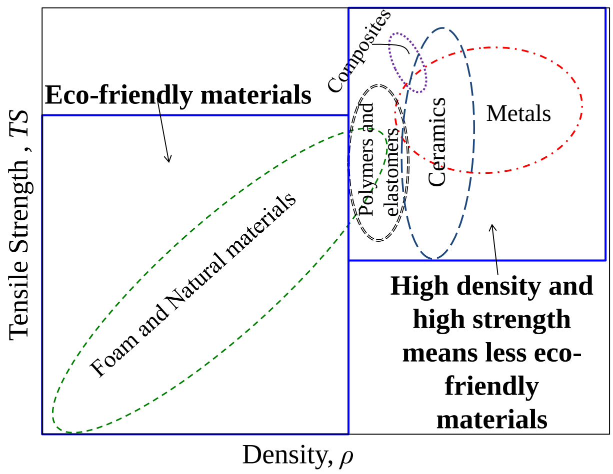


Figure 1.5: Importance of natural materials on the view point of eco-product (Ashby, 2007).

In this study, a widely used natural material, 'jute' has been selected, which grows mainly in Bangladesh, India, and China (Anon., 2017). The motivation behind selecting the jute fiber is due to their abundant availability in the nature followed by cotton and bamboo (Barth and Carus, 2015). It is a natural fibrous material having good mechanical (Xia, et al., 2009), thermal (Pandey, et al., 1993), and sustainable properties. For instance, jute exhibits excellent weight per strength ratio compared to the metal. Many researchers have gained interest in jute for the sustainable product due to its biodegradable and nontoxic nature. However, the knowledge of the natural material is uncertain to take a decision on the eco-product manufacturing. The concept of uncertainty is explained in the following section.

1.1.3 Uncertainty

Uncertainty is often understood (semantics) by classifying it into different categories, (Booker and Ross 2011; Ross et al., 2013) as follows:

a) *Aleatory uncertainty,*
b) *Epistemic uncertainty,*

c) *Reducible uncertainty,*
d) *Irreducible uncertainty, and*
e) *Inference uncertainty*

In general, aleatory uncertainty refers to uncertainty due to random variability or stochastic processes. In case of aleatory uncertainty, distribution is known at the beginning and the data vary due to that distribution as schematically illustrated in Figure 1.6(*a*).

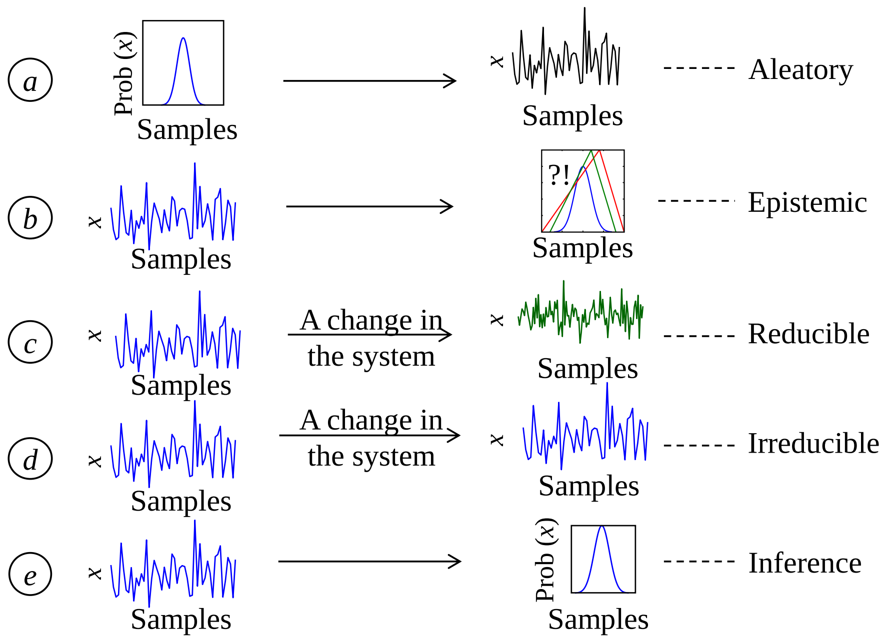


Figure 1.6: The variability of data due to different uncertainties.

Epistemic uncertainty refers to uncertainty due to lack of knowledge or imprecision associated with the data and information. In case of epistemic uncertainty, it is not known which distribution data will follow as shown in Figure 1.6(*b*). The possibility distribution is generally used to quantify the epistemic uncertainty and it is the neutral representation of the uncertainty (Ullah and Shamsuzzaman, 2013; Ullah, 2016). The mathematical procedure related to this distribution is described in Appendix-A.

Reducible uncertainty refers to the uncertainty that can be reduced by applying different conditions as shown in Figure 1.6 (*c*). Irreducible uncertainty refers to uncertainty due to natural variability that can be quantified but cannot be reduced. In such case, the variability or uncertainty in the data cannot be reduced or controlled by physical means (for example chemical modification, x-ray, and radiation) as shown in Figure 1.6 (*d*).

Inference uncertainty refers to predicting the future from the past, inferring the population behavior from a sample, and inferring the system behavior from its subsystems. In such case, infinite estimation can be made from finite world as shown in Figure 1.6 (*e*).

To compute uncertainty in a formal manner (syntax), theories have been developed, for example, probability theory, imprecise probability theory, evidence theory, possibility theory, and random interval theory (Dempster, 1968; Walley, 1991; Walley 2000; Shafer, 1976; Klir, 1990; Zadeh, 1978; Dubois and Prade, 1988; Joslyn and Booker, 2004). In certain cases, the theories are based on different categories of uncertainty. For example, the probability theory deals mainly with the aleatory uncertainty, whereas the possibility theory deals with the epistemic uncertainty. Certain theories can deal with multiple categories of uncertainty, e.g., imprecise probability theory can deal with aleatory uncertainty and the epistemic uncertainty associated with the probabilities of events. Nevertheless, the uncertainty of a category can be interpreted in terms of the uncertainty of a different category as schematically shown in Figure 1.7 (Klir, 1999; Dubois et al., 2004; Ullah and Shamsuzzaman 2013).

Uncertainty in the data can be reduced by physical means (treated by chemical, thermal and etc.) as schematically shown in Figure 1.7 and marked as 1. Reducible or irreducible uncertainty can be transformed into inference uncertainty after application of some theory such as probability, possibility and evidence as schematically illustrated in Figure 1.7.

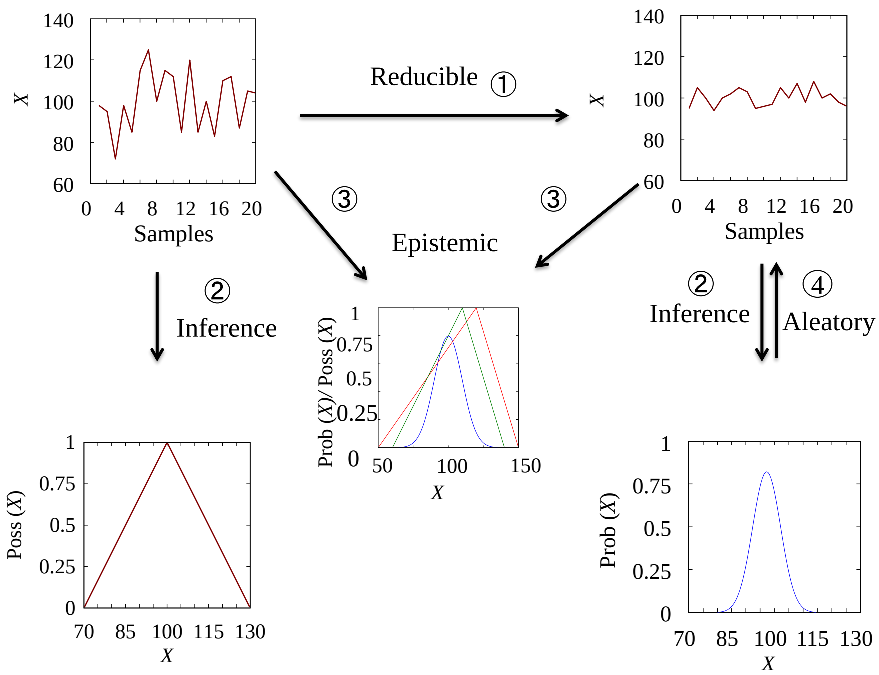


Figure 1.7: One types of uncertainty can be transformed into other uncertainty.

Again, when the distribution of reducible and irreducible uncertainty is not known, they become as epistemic uncertainty marked as 3 as schematically shown in Figure 1.7 As it is discussed before the epistemic uncertainty can be quantified by using previously mentioned theory. Moreover, if the distribution is known (probability distribution) they are named as aleatory uncertainty and can be quantified using probability theory as shown in Figure 1.7 marked as 4.

That means the aleatory uncertainty, epistemic uncertainty, and any other uncertainty are different from each other in the sense of semantics, but all these uncertainties are somewhat same in the computational science, and, thereby, can be integrated while developing systems for making decisions under uncertainty irrespective of its category. There is uncertainty regarding the material selection method, *MI*, is discussed below.

1.1.4 Uncertainty in Material Selection

Material is an important issue for a component of the product and needs to be considered in selecting material. Thus, to select a material for an object from the Material Universe, it is required to differentiate one material from other materials. Usually, to select a material for a component of the product, a function termed as, *MI*, (Ashby, 2007) is used. The general formula for *MI* (particularly for stiffness and strength limited design for an object) is given by equation (1.1).

$$MI = TS^{a} E^{b} \rho^{c} \tag{1.1}$$

The *MI* is related to three mechanical properties, namely *TS*, *E* and ρ which are important in selecting a material. In case of optimal material selection, the maximum value of the *MI* should be considered (Ashby, 2007). For example, if a tie is considered, it will bear only tensile load and the values of the exponentials of the equation (1.1) are a = 1, b = 0, and c = -1. Therefore, the *MI* (one types of knowledge) for tie can be defined using the equation (1.2) (Ashby, 2007).

$$MI = \frac{TS}{\rho} \tag{1.2}$$

There are uncertainties in selection of materials using *MI*. These issues are discussed in next section.

1.1.4.1 Uncertain Material Properties

Knowledge of the material properties comes from experimental investigations (e.g., tensile tests, flexural tests, hardness tests, etc.). Let Table 1-a shows the mechanical properties (*TS* and ρ) and *MI* of a tie for two different materials, A and B. It shows that B has higher *MI* compared to A material. Thus, B material is selected for a tie. To select a material based on *MI*, the material properties are required. The mechanical properties of the tie are determined through the tensile test, by applying the tensile load on a sample. Thereafter calculation, the results of TS and ρ are found. The procedure is repeated for different samples. The experimental results vary from sample to sample. Therefore, it can be inferred that there is uncertainty in the data of material properties. As the data of material properties are uncertain, it is tough to judge a material according to *MI***.**

Table 1-a. Data of the material properties of A and B for tie regarding *MI*.

Material	**Properties** $\boldsymbol{TS}$	$\boldsymbol{\rho}$	$\boldsymbol{MI = {}^{TS}/_{\rho}}$
A	300	100	3
B	200	50	4

1.1.4.2 Uncertainty in *MI* Calculation

There is uncertainty in the *MI* itself. Suppose, there is a 'Table' (as shown in Figure 1.8) where the top of the 'Table' is considered as a plate, two sides of the 'Table' are considered as column and base of the 'Table' is considered as a beam. That means a product is a combination of multiple components. Moreover, the *MI* of each component is different. Though the *MI* of the column ($\frac{E^{\frac{1}{2}}}{\rho}$), beam ($\frac{E}{\rho}$) and plate ($\frac{E^{\frac{1}{3}}}{\rho}$) (Ashby, 2007) are known, however, it is difficult to know what will be the *MI* of the 'Table'.

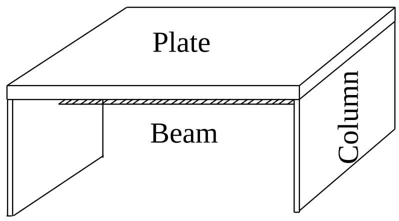


Figure 1.8: A mechanical object ('Table') is a combination of multiple components.

Generally, the shapes of the mechanical components are more complex. Therefore, it is difficult to calculate the *MI* of the complex shape. Therefore, *MI* itself is uncertain or cannot be derived. Even though *MI* can be calculated for the different machine elements, *MI* cannot be calculated for a given product. As *MI* is uncertain, an alternative option for selecting material is required.

1.1.4.3 Uncertainty in *MI* Method

There is uncertainty in the material selection using *MI* method. Consider the material selection procedure using *MI*, from the Material Universe, for a mechanical object 'tie'. As discussed in the previous section, *MI* of a tie is the ratio of *TS* and ρ. Now if the straight line is drawn in the engineering Material Universe plot as schematically shown in Figure 1.9, the value of the slope of the straight line will represent the *MI* of the tie.

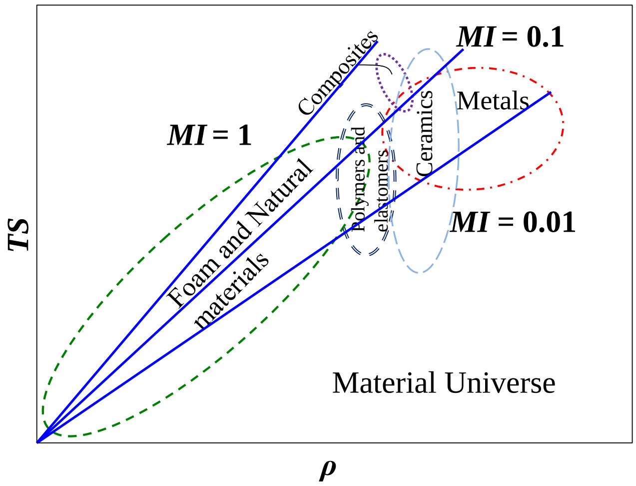


Figure 1.9: Various types of materials under same slope line (Ashby, 2007).

The equation of straight line is $y = mx + c$, where $x = \rho$, $y = TS$, $m = MI$, and $c = 0$. To select an optimal material, *MI* should always be maximized. From Figure 1.9, it can be informed that the materials which lie on *MI* = 1 are better material than those having *MI* = 0.1 and *MI* = 0.01. All materials correspond to a single slope line, say M_1, M_2, and M_3 of *MI* are equivalent. That means there are many good materials which can be selected for a tie under *MI* =1 alone. Now, which good material can be selected for the tie? It is difficult to choose a material for the tie even though the *MI* is known. Therefore, it can be inferred that *MI* does not guarantee the selection of a single material.

From the above description, it is clear that there is uncertainty, (epistemic), in the data of material properties, uncertainty in the *MI*. When *MI* is uncertain it cannot give a guarantee of a single material selection. Thus, it is hard to compare one material with another and to make the list of preference of materials. Furthermore, sustainable properties cannot be incorporated in the material selection procedure under *MI.* Therefore, it is essential to deal and manage these types of uncertainty to ensure the selection of material for the sustainable product. Furthermore, what will be the tool to deal these types of uncertainties? It is an important issue.

1.1.5 Objectives or Requirement

At the early stage of the product development, objectives are partially known. These objectives are the customer requirements and it is referred as 'zero stage' of a solution. For example, if a strong material is required then objective is to have a material with high *TS.* For structural integrity, *TS* and ρ are needed to be maximized and minimized, respectively. For sustainability, *CO_2 Foot print*, *Water usages*, *Environmental damage* are needed to be minimized whereas the *Recycle fraction* and *Safety* are needed to be maximized. Therefore, there are some conflicting objectives regarding the material selection. Furthermore, if the structural integrity is maximized, then it may create a conflict with the environmental impact. Furthermore, the shape of the objective function (e.g. linear and nonlinear) for a specific requirement is also unknown. As such, how to define and manage these conflicting objectives is an important issue.

1.2 Core Idea/ How to Deal with Different Uncertainties

This section describes the core idea to deal with uncertainty of the material properties, uncertainty in the *MI* regarding the selection of material.

1.2.1 Uncertainty Quantification

As described in Section 1.1.4.1 the raw data of material property (for example *TS*) is not a single value and it varies a lot. In this study, this type of variation in material property means a kind of uncertainty, so material cannot be selected based on this uncertainty. Therefore, quantification and maintenance of the uncertainty for selection of material is a vital issue. Quantification of the uncertainty can be done by different approaches. Through literature review, it is found that three quantification approaches such as statistical, probabilistic and possibilistic are most widely used. The quantified data of the material properties can be represented by statistical range or probabilistic form or possibilistic form as schematically shown in Figure 1.10. From these three methods which one will be used for uncertainty quantification and which is reliable method to quantify the uncertainty, is an important issue. In this study, a reliable approach to quantify the uncertainty is investigated.

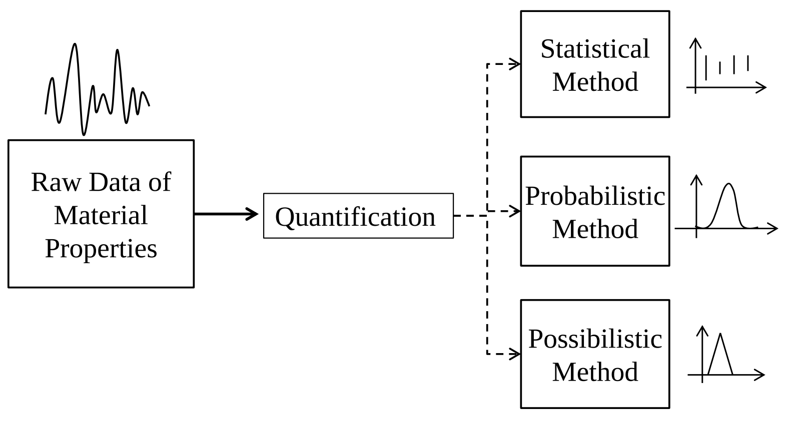
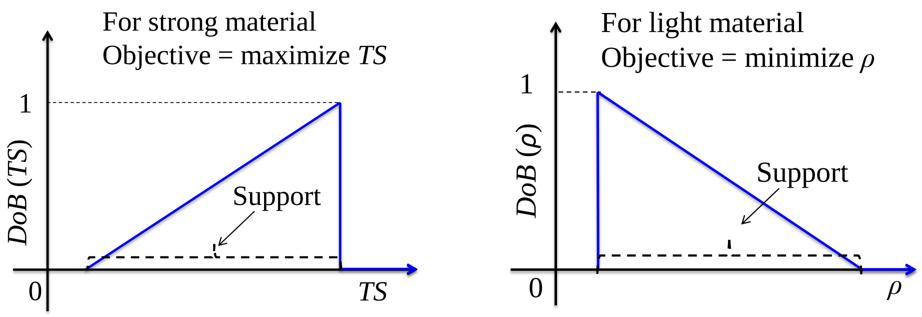


Figure 1.10: Quantification of the uncertainty using three approaches.

1.2.2 Tool for Material Selection

At the initial stage of product development, *MI* is unknown and objectives are partially known. As the information is not known or partially known, they can be represented by a fuzzy function. In this study, two types of possibility objective functions have been proposed, namely, maximization and minimization as shown in Figure 1.11. Possibility Objective function is selected based on requirement and common sense. The maximization function will be selected, when a high value of a criterion (say *TS*) is required as schematically depicted in Figure 1.11(*a*) and vice versa.



(*a*) (*b*)

Figure 1.11: Conflicting objectives representation by (*a*) Maximization and (*b*) Minimization function

The membership value of the possibility objective function lies between 0 and 1. Therefore, to build the objective function one must consider the range of *x*-axis, named as support. The support as shown in Figure 1.11 is derived from the Material Universe. It may be local, semi-local, deterministic or global (detail in Chapter 2). If minimum and maximum values of a property are plotted, support will be the range of the values of the *x*-axis.

Suppose designer's objective is to have a strong as well as light material for a product (like a vehicle body). There are two objectives in designer requirements, such as strong material and light material. High *TS* is required for strong material and low ρ is required for light material. Therefore, for strong material, *TS* needs to be maximized whereas, for light material, ρ needs to be minimized. Thus, the objective function for *TS* will be maximization function as shown in Figure 1.11(*a*). However, the objective function for ρ will be minimization function as shown in Figure 1.11(*b*). In Figure 1.11 *x*-axis represents the *TS* and ρ, *y*-axis represents the degree of belief (*DoB*).

1.2.3 Compliance

MI (related with shape and size) of a product is customer dependent (Sharif Ullah et al., 2016) or unknown, *MI* is itself uncertain, and it does not guarantee the selection of a single material. Thus, an alternative method to select a material is essential. The objectives are known and they are conflicting in nature, which can be represented by proposed possibility objective functions. Based on the conflicting objectives (general requirements) this section describes the alternative way to rank the materials. Consider, the previous example of Section 1.2.2 where, the two requirements are represented by two objectives functions, minimization, and maximization, as shown in Figure 1.11. The value of the material property (*TS*) is assumed as a single value. Now if anyone wants to know the material A will fit with the required objective function or not. If fit how much fit? Furthermore, how will it be possible to know? To take a decision or select a strong material interaction is required between the material properties (*TS*) and objective function. This type of interaction is termed as compliance (discuss in Chapter 2). The value of the compliance lies between 0 and 1. The physical meaning of the compliance is how much far or close the material properties from the ideal condition or objective function.

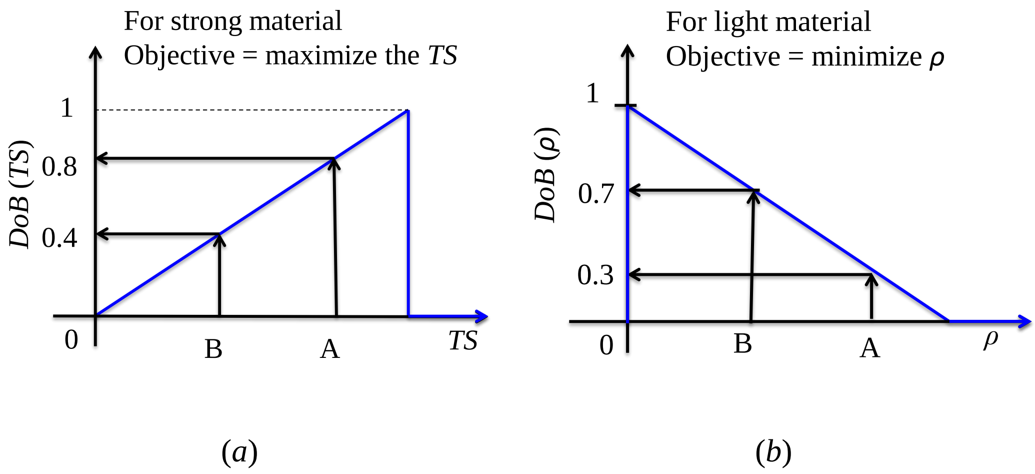
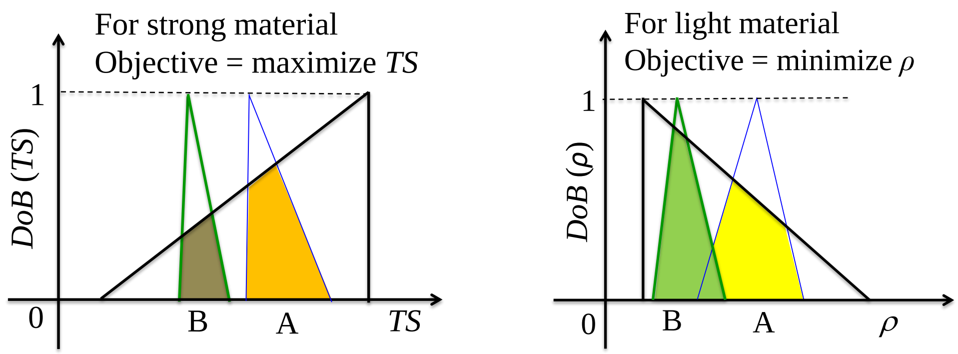


Figure 1.12: Compliance between material properties and (*a*) maximization and (*b*) minimization function.

It is assumed that there are two materials A and B and their corresponding degree of believe (*DoB*) for *TS* are 0.8 and 0.4, respectively as shown in Figure 1.12(*a*). It can be inferred that material A is a better option than B for strong material. Similarly, for light materials selection, the objective is to minimize the ρ which can be represented in Figure 1.12(*b*). From Figure 1.12(*b*), it can be inferred that B material (*DoB* = 0.7) is better than A for a lighter material. Therefore, a strong, as well as light material can be selected using possibility distribution function without *MI*. Thus, using the possibility objective function one can represent ones feeling. Furthermore, one can also represent one's requirement through these types of functions. Using compliance one can make a ranking of the materials and finally, can select a single material for product development. Therefore, *MI* can be replaced by Compliance calculation between the possibility objective function and the data of material property (or criteria). It can be said that it is an alternative representation of the *MI*.

However, when the data of the material properties is uncertain, the compliance calculations are different. According to literature, the uncertainty can be represented by statistical range, probabilistic form, and possibilistic form. In the previous example of Section 1.2.2, the two requirements are represented by two objectives functions, minimization, and maximization, as shown in Figure 1.11. Let, the uncertainties of material properties (*TS* and ρ) are represented by possibility distributions as shown in Figure 1.13. To obtain a strong material between A and B, interaction is required between the maximization objective functions (Figure 1.13(*a*)) and the possibility distributions of *TS* for corresponding materials.

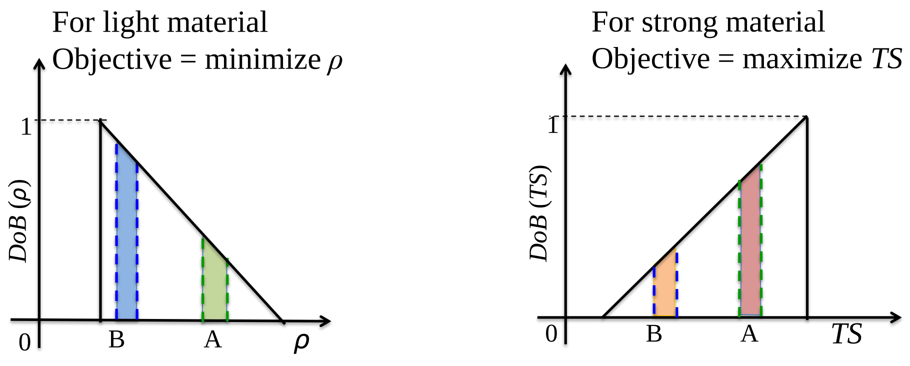


(*a*) (*b*)

Figure 1.13: Compliance between (*a*) maximization and (*b*) minimization function uncertain (possibility distribution) material properties.

Now, if the values of compliances of two materials A and B for *TS* are 0.75 and 0.39, respectively, it can be inferred that material A is a better option than B for strong material. Similarly, for light materials selection, the objective is to minimize the ρ which can be represented in Figure 1.13(*b*). Let the values of compliance of A and B are 0.4 and 0.85, respectively. Thus, it can be inferred that material B is better than A for a lighter material based on the calculated compliances values.

Similarly, when the uncertainty of the data is represented by statistical range the interaction between the objective function and the uncertain data for ρ and *TS* are shown in Figure 1.14. For this type of uncertainty, the compliance means the ratio of objective function area and the data range. Material ranking is made based on the compliance calculation (detail in Chapter 2) and finally, a material is selected base on the importance of designer on the individual criteria.



(a) (b)

Figure 1.14: Compliance between (a) minimization and (b) minimization function uncertain (range) material properties.

Now a new approach has been developed for material selection, conflicting objectives are represented by two types of possibilistic objective functions, and the uncertainty in the data of material properties are quantified and investigate a reliable method for quantification. Using compliance analyses between the objective functions and the decision-relevant information (quantified data), one can compare one material with other materials (using our proposed method). Finally, a preference list of materials can be made and a decision can be taken based on the preference of criteria as schematically shown in Figure 1.15. In this study, this whole method is proposed as a decision model (discuss in Chapter 4).

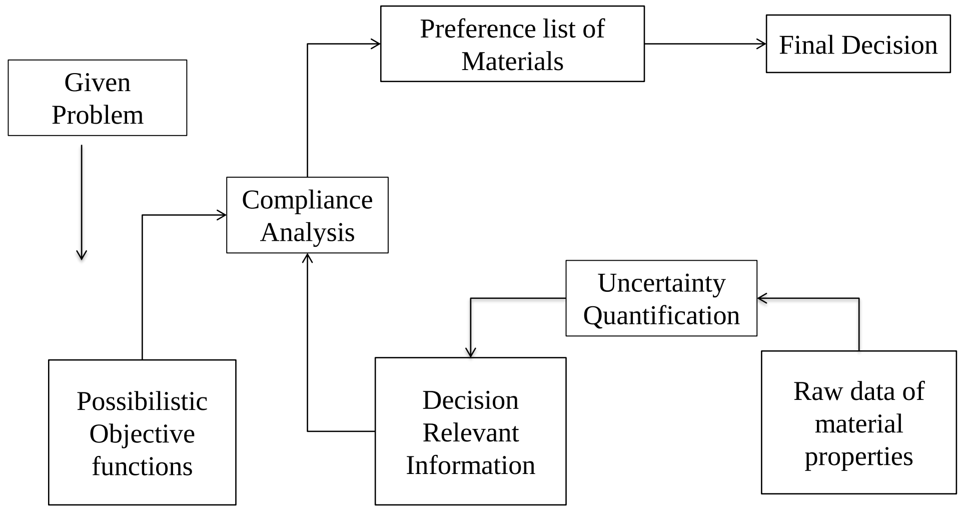


Figure 1.15: Dealing with epistemic uncertainty.

To produce a component of the product, material selection is required and to select the optimal material, specific *MI* is needed. However, a degree of uncertainty is associated with the *MI*, data of material properties as well as material selection method. Therefore, uncertainty quantification is an important aspect of product manufacturing. Without uncertainty quantification, product development processes may not provide the reliability, durability, quality, and functionality of the product. As quantification of uncertainty is necessary, one of the objectives of this study is to quantify the uncertainty in the data of material properties, particularly mechanical and sustainable properties, and to identify a reliable method to quantify the uncertainty. After that, using the quantified data, an optimal material will be selected. Basically, now the designers are giving importance to the function and shape of the product not emphasizing the material. For example, for a long time, there has not been any change in the selection of material for the car body. However, the same car is being used in cold region as well as in the warm region. If the designer considered the materials which would be suitable for both cold and warm weather, a car may become more user-friendly and more efficient. Nevertheless, 80% cost of the product depends on the materials. Furthermore, researchers are now giving emphasize on the material in terms of sustainability. GRANTA with other two companies tries to incorporate the material selection at the beginning of the sustainable product designing. Nevertheless, they are not considering the uncertainty of the *MI*, material selection, and the data of material properties. In that sense, this study has a good impact on the sustainable product development sector.

1.3 Scope of the Work

Quantification is necessary to select an optimal material for a component of the product. Material selection is required for a product to maintain the complexity of the production system as well as reliability, durability, sustainability, energy, and cost of the product**.**

1.4 Objectives of the Work

Objectives of this study are laid below:

1. To quantify the uncertainty of material properties
2. To develop a decision model to select a material under uncertainty.

1.5 Literature Review

In the life cycle of a product, most of the energy consumption (Stoffelsa, et al., 2017) is due to the material. The materials have been considered as a key factor for managing the complexity while designing engineering products (McDowell et al., 2010). To achieve a sustainable future, the reduction and diversification of material usages (i.e., materials efficiency) are considered as more effective than other measures (e.g., energy efficiency) (Allwood et al., 2011; Ullah et al., 2013; Ullah et al., 2014). Mayyas et al. (2012a-b) and Poulikidou et al. (2015) have shown that the environmental impact of a product depends heavily on the materials used in different elements. In addition, materials are limited and some of the materials will not be available after a certain time. Therefore, it is important to select proper material for a product, otherwise, sustainability may be hampered, and this may be a critical issue for next generation. Therefore, if a designer has a clear idea about the appropriateness of a set of materials for making the parts of a product at the early stage of the design process, then it would be easy for the designer to control the complexity of the subsequent design activities (McDowell et al., 2010; Omar, 2011).

There are different types of tools, used by different researchers to select material. Some of them are conventional method (AL-Oqla and Salit, 2017), the famous Ashby chart (Ashby, 2007) are procedure based tools. Some of the researchers select material using advanced material selection tool and technique using based on artificial intelligent (AL-Oqla and Salit, 2017; Gul, et al., 2017) system like fuzzy MCDM (Fuzzy VIKOR, fuzzy TOPSIS, and fuzzy ELECTRE) PROMETHEE (Gul, et al., 2017), and so. The researchers have selected the materials from small component to large and sophisticated product for example automotive instrumental panel (Lorenzo, et al., 1995), for high entropy alloy (Fu, et al., 2017), sustainable products (Stoffelsa, et al., 2017), nuclear machinery (Hosemanna, et al., 2017), for products development. Some of the researchers have developed the model for a specific field, while some researchers have developed a system which is suitable for material selection and not for calculation. However, the developed method or models are complicated, mathematical operation, and not efficient, it is necessary to develop an efficient method to select the optimal material for uncertain data.

For environmental performance, materials are highly important (Prendeville, et al., 2014). To maintain the sustainability aimed at reduction of the energy consumption, material selection is required. For example, to produce light-weight product and the optimal selection of the material (for sustainability) low dense material i.e. Natural materials (according to engineering material) are the better option compared to other materials. There are different types of researches on the natural material characterization (Biswas, et al., 2013; Biswas, et al., 2011; Hossain, et al., 2014), modification (Shahinur, et al., 2017; Jafrin, et al., 2009), and eco-product development.

Development of product using materials requires an understanding of their material properties (Alves, et al., 2010). To understand the properties of natural fiber based products, two types of experiments have been performed. The first type deals with the characterization of composites, where one natural fiber and other natural/artificial fiber are used as reinforcing materials (Jawaid, et al., 2011; Li, et al., 2015; Matějka, et al., 2013; Prachayawarakorn, et al., 2013; Shanmugam and Thiruchitrambalam, 2013; Vijaya Ramnath, et al., 2013). The main concern of these studies is to elucidate the efficacy of methods for improving the material properties of composites. The following issues have been studied: improving the dynamic mechanical properties of a jute composite (Katogi, et al., 2016) improving the adhesion between a matrix and fibers using chemically treated fibers (Ahmed, et al., 2007; Rawal and Sayeed, 2014) improving the performance of a composite by changing the weight percentages of fibers (Rawal and Sayeed, 2014; Aggarwal, et al., 2013), improving the performance of a composite by changing the fiber length (Hu, et al., 2010; Zhou, et al., 2013) and orientation (Abdellaoui, et al., 2015; Vijaya Ramnath, et al., 2014), improving the performance of a composite through lamination (Ahmed, et al., 2007; Abdellaoui, et al., 2015; Vijaya Ramnath, et al., 2014; Sabeel Ahmed, et al., 2012; Santulli, et al., 2013), improving the performance of a composite by mixing natural fibers with other natural or artificial fibers (Vijaya Ramnath, et al., 2014; Jawaid, et al., 2011; Li, et al., 2015; Matějka, et al., 2013; Prachayawarakorn, et al., 2013; Shanmugam andThiruchitrambalam, 2013; Vijaya Ramnath, et al., 2014), etc. Here, "improving performance" means improving the mechanical, thermal, environmental degradability, and durability properties of a composite.

The other type of experiments performed on natural materials (including jute) have aimed to determine the properties of raw or chemically treated fibers collected from various segments of the respective plants. Some studies have determined the different properties such as mechanical (Biswas, et al., 2013; Biswas, et al., 2011; Shahinur, et al., 2015), thermal (Ray, et al., 2002; Ouajai and Shanks, 2005; Tomczak, et al., 2007; Nechwatal, et al., 2003; D'Almeida, et al., 2006) of raw fiber. There are other studies that have determined the properties of chemically treated fibers (Jafrin, et al., 2014; Jafrin, et al., 2009), as well as the fibers collected from various segments of plants (Shahinur, et al., 2015). Under the mechanical properties some of them have worked on the method or process development (Biswas, et al., 2013; Biswas, et al., 2011; Hossain, et al., 2014) and some of them are on theoretical model development or product development (Shahinur, et al., 2017). There are many types of researches on the thermal properties including other properties (Shahinur and Ullah, 2017; Anon., 2017; Ahmed, et al., 2007; Rawal and Sayeed, 2014; Aggarwal, et al., 2013) of the natural fibers. The thermal studies emphasize on DTG, DSC (Ray, et al., 2002; Shahinur, et al., 2015) gas chromatography (Ranganathan, et al., 2008), fire resistance (Pandey, et al., 1993), thermal stability (Shahinur, et al., 2017; Ray, et al., 2002), activation energy (Ouajai and Shanks, 2005) of natural and treated natural materials like jute (Pandey, et al., 1993; Ray, et al., 2002; Shahinur and Ullah, 2017; Shahinur, et al., 2013) hemp, bamboo (Biswas, et al., 2015), coir (Biswas, et al., 2013), banana, sisal (Oliveira, et al., 2017; Mariano, et al., 2016), okra (Hossain, et al., 2013), silk and other natural materials.

The goal of these types of studies has been to gain scientific knowledge of the natural fiber itself, which can then be applied in designing (natural fiber based composite products) eco product. For example, see (Alves, et al., 2010) to understand how the material properties of natural fibers have been used to develop an engineering component used in automobiles. In such engineering practices, it may not be wise to rely solely on the material properties of a natural fiber only. Because most of the time jute based products are produced from jute yarns. However, the natural materials, as well as other materials, have uncertainty in their material properties.

This uncertainty is more noticeable in natural material. The properties of natural materials along with other materials vary significantly as the microscopic structures of a naturally growing material cannot be tightly controlled (Fidelis, et al., 2013). This causes variability in the underlying properties of a natural material. Therefore, understanding natural materials require a clear understanding of the uncertainty associated with each relevant material property (Shahinur and Ullah, 2017) that means quantification is necessary. Otherwise, it would not be possible to make design and manufacturing decisions that might ensure the functionality, quality, reliability, and durability of natural material-based products. For this reason, the reliability, durability, quality, and sustainability of the design decision may be uncertain.

To compute uncertainty in a formal manner (syntax), numerous theories have been developed, e.g., (to name a few) probability theory (Dempster, 1968), imprecise probability theory (Walley, 1991; Walley 2000), evidence theory (Shafer, 1976; Klir, 1990), possibility theory (Zadeh, 1978; Dubois and Prade, 1988), and random interval theory (Joslyn and Booker, 2004).

Information regarding quantification of natural material can be extracted from the works of numerous authors. There are many studies have been conducted to quantify the uncertainty exhibited by the material properties of natural materials using a probability distribution called a Weibull distribution. For example, Silva et al. (2008) quantified the variability in material properties of sisal fibers using a Weibull distribution and correlated sisal microstructures with tensile strength. Defoirdt et al. (2010) used a Weibull distribution to explain variability in the tensile properties of jute, bamboo, and coconut fibers. Fidelis et al. (2013) examined the morphology of the natural fibers and correlated their mechanical properties with their morphology using a Weibull distribution. Hossain et al. (2014) created a histogram for the morphological structures of natural fiber cross-sectional areas and provided ranges for their material properties. In these studies (Fidelis, et al., 2013; Silva, et al., 2008; Defoirdt, et al., 2010; Hossain, et al., 2014), significant variability was observed for the respective material properties studied. Most of the researchers quantify the data using statistical analysis.

Information regarding quantification of data can be extracted from the works of several authors. The Statistical approach is widely used to quantify the data. Though this approach has problem and limitation, this approach is familiar due to quick and easy access. Some of the common application of the statistical approach is robotics (Birglen and Schlicht, 2018), clinical application (Andy, et al., 2017), environmental (Jorge, et al., 2017), mechanical, physical (Lewis, et al., 2017), electrical engineering (Zhao, et al., 2017) and metallurgical engineering sector (Mir, et al., 2013) to quantify the data. Therefore, it is an important issue to check the through this method.

As there are different approaches to quantify the uncertainties, estimated values, and the assumption are different from each other. Thus, it is an important issue to select a reliable approach to quantify the uncertainty for a product development. Now fuzzy method is widely used to quantify the uncertainty to take the decision in every sector. This approach is linguistically representable; any problem can be transformed into linguistically and solved by fuzzy number. The use of a fuzzy number to deal the uncertainty is increasing day by day. Hence, it is important to quantify the uncertainty of natural material using possibilistic approach

Numerous academic communities, engineering design community has also recognized the theorization (syntax) and categorization (semantics) of uncertainty, and developed numerous models and tools for making design decisions under uncertainty (Antonsson and Otto 1995; Huang and Jiang, 2002; Nikolaidis et al., 2003; Nikolaidis et al., 2004; Youn and Choi, 2004; Gurnani and Lewis, 2005; Ullah, 2005a-b; Ullah and Harib, 2008; Sharif Ullah and Tamaki, 2011; Achiche and Ahmed-Kristensen, 2011; Matsumura and Haftka, 2013; Ullah and Shamsuzzaman 2013; Jiang et al., 2015; Rezaee et al., 2015). The methods and tools for dealing with uncertainty bring benefits for solution-based design and problem-based design. In particular, the aleatory uncertainty-based measures (e.g., probability distributions and Bayesian inferences) are useful for the solution-based design, where the robustness or reliability of a given design solution is enhanced, without making any drastic changes in the geometric and material specifications of the given design solution. On the other hand, in the case of problem-based design, the geometric and material specifications are not clearly defined or known; rather numerous problems are introduced and solved (determining customer needs, concept selection, materials selection) by using the epistemic uncertainty-based measures (e.g., possibility measures and fuzzy numbers). The goal here is to transform a problem-based design to a solution-based design. Some authors have integrated both aleatory uncertainty and epistemic uncertainty based measures to make the design decision-making process an even more robust and user-friendly process (e.g., see the works of Nikolaidis et al., 2003; Sharif Ullah and Tamaki, 2011; Ullah and Shamsuzzaman, 2013).

At the initial case of the product development, there is epistemic uncertainty. The optimum material selection is required from this epistemic uncertainty, due to limited information at the early stage of the designing. Therefore, the goal of this study is to transform a problem-based design to a solution-based design.

There are researches on product development and uncertainty quantification as well as decision tools. However, there is limited research on the combination of these three. In this study, the under uncertainty, a material is selected using a compliance.

1.6 Thesis Structure

Thesis structure is organized as follows. Chapter 2 describes the mathematical entities needed to define the uncertainty in statistical, probabilistic, and possibilistic means. In addition, the mathematical entities needed for the formal computation while selecting a material is also described. Chapter 3 and 4 describe the findings and outcome of this study.

Chapter 3 shows the experimental results regarding the mechanical properties, namely, tensile strength, modulus of elasticity, and stain to failure of a natural material called Jute. The uncertainty associated with the properties mentioned above has been quantified by using the statistical, probabilistic, and possibilistic approaches. It has been found that out of the three approaches, the possibilistic approach quantifies the uncertainty more reliably. Therefore, when one uses a material property for making a decision, its uncertainty can be put into the formal computation using a possibility distribution (e.g., a triangular fuzzy number) rather than using a probability distribution (e.g., Weibull distribution) or statistical approach.

Based on this conclusion, the uncertainties associated with the *Tensile Strength*, *modulus of elasticity, density*, *CO_2 Footprint*, *Recycle fractions, and Water usages* of 197 types of Aluminum alloys, 45 types of Titanium alloys, and 30 types of Magnesium alloys are represented by possibility distributions, as reported in Chapter 4. In addition, a decision model is also developed in Chapter 4 to select an optimal material out of the three alternatives namely, Aluminum, Titanium, and Magnesium alloys. In this decision model, the objective functions (e.g., maximize *tensile strength*, minimize *CO_2 Footprint*, and so on) are set by the possibility distributions, too. Three of the possibility distributions (i.e., objective functions) are for maximizing the *tensile strength*, *modulus of elasticity*, and *Recycle fractions*, respectively, and the other three are for minimizing the density, *CO_2 Footprint*, and *Water usages*. The compliance between the possibility of distribution of a material property of a type of alloys (e.g., possibility distribution of the tensile strength of Aluminum alloys) and the possibility distribution of the corresponding objective function (e.g., possibility distribution of maximizing the tensile strength) are used to make a decision. It is found that the decision model selects an optimal material even though the material properties are uncertain and the underlying material indices are not known.

Chapter 5 discusses the implications of this study in eco-product development. It also describes how the stakeholders (research organizations and researchers, designers, producers) should interact centering the material related decision making processes.

Finally, Chapter 6 provides the concluding remarks of this thesis.

Chapter 2: Mathematical Settings

This section deals with the mathematical settings used in this study. In particular, the following mathematical concepts are defined: Probability Distribution, Mean, Variance, Weibull Distribution, Average, Standard Deviation, Ranges of Mean and Variance, Possibility Distribution or Fuzzy Number, Trapezoidal Fuzzy Number, Triangular Fuzzy Number, Ramp Up Fuzzy Number, Ramp Down Fuzzy Number, Degree of Compliance, Degree of Compliance of Crisp Value, Degree of Compliance of Range, and Degree of Compliance of Possibility Distribution. Besides providing the definitions, the relevant usages of the concepts regarding to this thesis have also been described.

2.1 Probability Distribution

Let x be a random variable associated with a physical parameter and it takes values in the interval $x \in [\pi 1, \pi 2] \in \Re$. If Pr($x$) be the probability of x, then following conditions hold: Pr(x) $\geq$ 0, Pr([π1,π2]) = $\int_{\pi 1}^{\pi 2} Pr(x)\, dx$ = 1. The cumulative distribution of x denoted as $F(x)$ is given as follows:

$$F(x) = \int_{\pi 1}^{x} Pr(x)\, dx \tag{2.1}$$

The expected value or mean of the x denoted as μ is given as follows:

$$\mu = \frac{\int_{\pi 1}^{\pi 2} Pr(x) x dx}{\int_{\pi 1}^{\pi 2} Pr(x) dx} = \int_{\pi 1}^{\pi 2} Pr(x)\, x dx \tag{2.2}$$

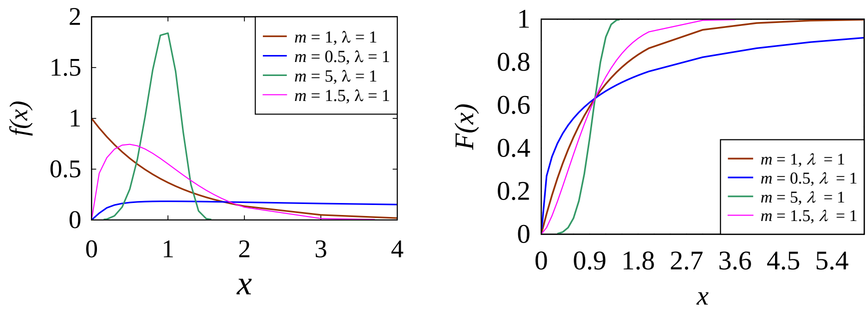
The variance of x denoted as σ^2 is given as follows:

$$\sigma^2 = \mu\big(x - \mu(x)\big)^2 \tag{2.3}$$

There are different types of probability distributions, e.g., Normal distribution, *t*-distribution, *chi*-square distribution, Poisons distribution, Binomial distribution, Degenerate distribution, Skellam distribution, Gamma distribution, and Weibull distribution (Figliola and Beasley, 2000). In this study, Weibull distribution is used to represent the uncertainty in the material properties. Weibull distribution has many forms. However, in this study the Weibull distribution refers to a probability distribution $f(x)$, which is given by the following equation (Defoirdt et al., 2010; Trujillo E. , et al., 2014).

$$f_w(x) = \frac{m}{\lambda}\left(\frac{x}{\lambda}\right)^{m-1} e^{-\left(\frac{x}{\lambda}\right)^m} \tag{2.4}$$

Here, $x \in [0,\propto]$, m is called the form or shape parameter and λ is called the scale parameter. The effects of m and λ are shown in Figure 2.1(a).



(a) (b)

Figure 2.1:Weibull (a) density function and (b) cummulative distribution for different shape and scale factor.

The cumulative Weibull distribution takes the following form:

$$F_w(x) = \int_0^x f_w(x)dx = 1 - e^{-\left(\frac{x}{\lambda}\right)^m} \tag{2.5}$$

The shape of $F_w(x)$ is shown in Figure 2.1(b) for different values of shape and scale parameters. The expected value and the variance of Weibull distribution are given as follows:

$$\mu = \lambda\Gamma\left(1+\frac{1}{m}\right) \tag{2.6}$$

$$\sigma^2 = \lambda^2\Gamma\left(1+\frac{2}{m}\right) - \mu^2 \tag{2.7}$$

Here, Γ is the gamma function. Confidence interval for μ and σ^2 at 95% confidence is calculated using Mean Time to Failure (MTTF) (Santiago; Lane, 2017; Sullivan, 2017) method.

When $f_w(x)$ is unknown but a set of data points regarding x is known, i.e., $\{x_i \mid i = 1,\ldots,m\}$ is known, in such case, one can apply a procedure to determine $F_w(x)$ first and estimate the shape and scale parameters. This way, one can determine the underlying $f_w(x)$ associated with the data points of x. There are different types of procedures to determine $F_w(x)$ from a set of data points (Kececioglu, 1994; Weibull, 1992-2018; Wikipidia, 2017). One of the widely used procedures, median rank approximations, is described below.

This cumulative distribution function is a step function that steps up by 1/n at each of the n data points. There are different approaches to obtaining the empirical distribution function from data. In statistics, an empirical distribution function is the distribution function associated with the empirical measure of a sample. An empirical measure is a random measure arising from a particular realization of a (usually finite) sequence of random variables. In this study, the approximation method represented by equation (2.8) is used to calculate the empirical distribution function:

$$F(x_i) = \frac{x_i - 0.3}{n + 0.4} \tag{2. 8}$$

For the step by step calculation of mean order number or empirical distribution function $F\ (x_i)$, (Kececioglu, 1994) x_i is the rank of the data point and n is the number of data points (Wikipidia, 2017). The approximation method is used to find the Weibull cumulative function for different properties and shown in Section 3.6.2.

2.2 Average, Standard Deviation and Ranges of Mean and Variance

Let $x(i) \in \Re$, $i = 1,...,m$, be the given data points regarding a parameter x where m is the number of data points. The average denoted as $\bar{x}$ of these data points is given by equation (2.9)

$$\bar{x} = \frac{\sum_{i=1}^{m} x(i)}{m} \tag{2. 9}$$

The average provides the central tendency of the data points and is used for statistical analysis of uncertainty, as it is shown in Section 3.6.1.

The standard deviation denoted as sd of the data points, $x(i) \in \Re$, $i = 1,...,m$, is given as follows:

$$sd = \sqrt{\frac{\sum_{i=1}^{m}\left(x(i) - \bar{x}\right)^2}{m-1}} \tag{2. 10}$$

The standard deviation represents the dispersion in the data points and is used to quantify the uncertainty, as it is shown in Section 3.6.1.

The ranges of the expected value and variance denoted as μ and σ^2, respectively, of a given set of data points, $x(i) \in \Re$, $i = 1,...,m$, are given as follows:

$$\mu \in \left[\bar{x} \pm t_{1-\alpha/2,\upsilon} \frac{sd}{\sqrt{m}}\right] \tag{2. 11}$$

$$\sigma^2 \in \left[\frac{\upsilon \times sd^2}{\chi^2_{1-\alpha/2,\upsilon}}, \frac{\upsilon \times sd^2}{\chi^2_{\alpha/2,\upsilon}}\right] \tag{2. 12}$$

In equations (2.11)–(2.12), ν is the degree of freedom equal to $m-1$, α is the significance level, $t_{1-\alpha/2,\nu}$ is the critical value of a Student's *t*-distribution for a two-sided test, $\chi^2_{\alpha/2,\nu}$, and $\chi^2_{1-\alpha/2,\nu}$ are the lower and upper critical values of a χ^2 distribution for a two-sided test. Refer to (Hiller, Lieberman, Nag, and Basu, 2005; Montgomery, 2001; Tables for Probability Distributions) for the details of the relevant distributions. Note that for a two-sided test, $(1-\alpha/2)\times 100\%$ is the confidence interval. Usually, α = 0.05% or 97.5% confidence interval is used to calculate the ranges of μ and σ^2. The ranges of expected value and variance as defined above are widely used in quantifying the uncertainty associated with a given set of data points, as it is shown in Section 3.6.1.

2.3 Possibility Distribution or Fuzzy Number

A fuzzy number, *DoB* (degree of belief) is a function *DoB*: $\Re \rightarrow [0, 1]$, and it must follow the following four conditions (Zadeh, 1975; Dubois and Prade, 1978; Dijkman, et al., 1983)

a) *Normal*,
b) *Compactly supported*,
c) *Convex*, and
d) Upper semi-continuous

The function is normal means that there is, at least, one real number f_0 for which $DoB\ (f_0) = 1$. It is compactly supported means that the set $\{f \in \Re \mid DoB(f) > 0\}$ is bounded. It is convex means that if $f_1 \leq f_2 \leq f_3$, then $\min\ (DoB(f_1), DoB(f_3)) \leq DoB(f_2)$ for all $f_1, f_2, f_3 \in \Re$. It is upper semi-continuous means that the set $\{f \mid DoB(f) \geq \alpha\}$ is closed for each $\alpha \in [0, 1]$. The points corresponding to $DoB(.) = 1$ constitutes an interval called core. The closed interval $S = [a, b] \in \Re$ beyond which the fuzzy number $DoB(.) = 0$ is called support. As such, $DoB(a) = 0 \wedge DoB(a + \varepsilon) > 0$ and $DoB(b-\varepsilon) > 0 \wedge DoB(b) = 0$ where ε is very small positive number. The support is the largest alpha-cut. The concept of alpha-cut is useful when one needs an interval or a set of intervals for a given triangular fuzzy number. In this sense, all alpha-cuts belong to the support [*a*, *b*].

To infer the most plausible value and logically consistent ranges, the truth-value of the proposition is assumed to equal to its degree of membership which gives possibility distribution. It is possible to infer the most plausible value and logically consistent ranges of *TS* using possibility distribution as shown in Figure 2.2.

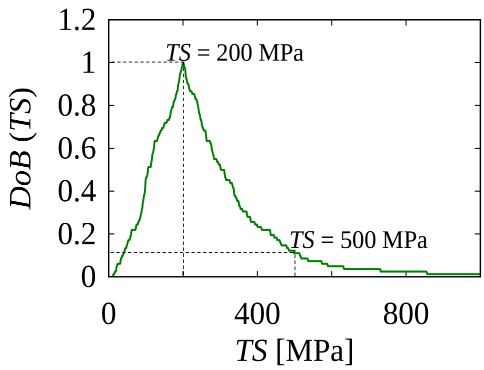
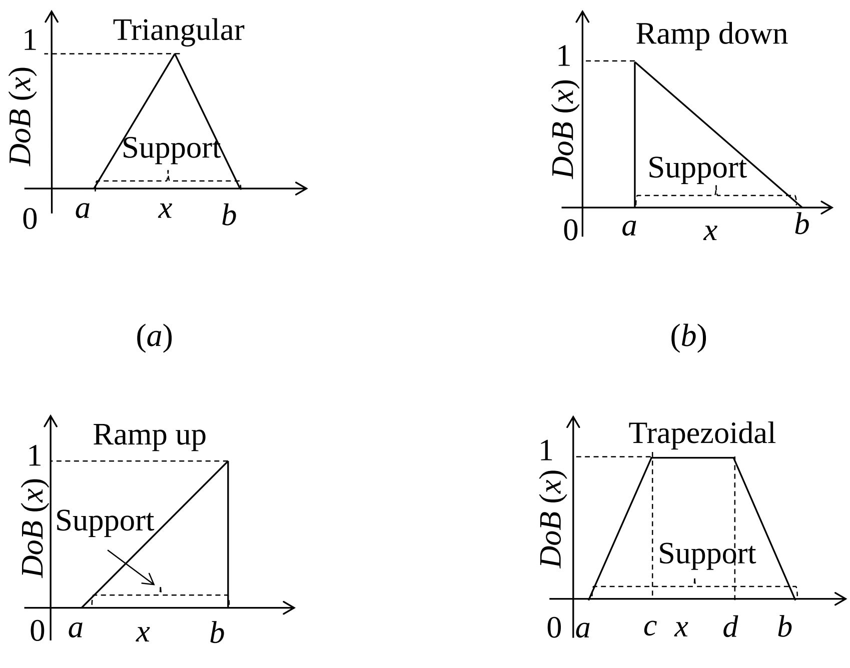


Figure 2.2: A typical nature of fuzzy number.

In order to infer the most plausible value and logically consistent range, for example, the propositions of following form: p (Z, Y, X, Q) = Z of Y jute yarn is XQ. Here, $Z \in$ {*TS*}, $Y \in$ {raw}, $X \in \Re$, and Q = {MPa}. The truth-value of the proposition T (p) is equal to the fuzzy number *DoB* (.), i.e., T (p) = *DoB*(X). For example, if Z = *TS*, Y = raw, X = 500, and Q = MPa, then the proposition is as follows: *TS* of raw jute yarn is 500 MPa. The truth-value of this proposition is equal to its degree of belief as given by the possibility distribution in Figure 2.2 i.e., 0.109756098. This means that "*TS* of raw jute yarn is 500 MPa" is more false than true. In addition, if Z = *TS*, Y = jute yarn, X = 200, and Q = MPa, then the proposition is as follows: *TS* of raw jute yarn is 200 MPa. The truth-value of this proposition is equal to its degree of belief as given by the possibility distribution in Figure 2.2 i.e., 1. This means that "*TS* of raw jute yarn is 200 MPa" is true and there is no doubt about it. The above explanation implies that the value of Z corresponding to $DoB(.) = 1$ is the most plausible value. The range of values of Z corresponding to $DoB(Z) \geq 0.5$ is the logically consistent range of values of Z because $DoB(.) \geq 0.5$ corresponds to the truth-values that is more true than false. One can also determine the expected value of Z using the centroid method (Ullah and Harib, 2006).

There are different categories of fuzzy numbers according to the shape of function as shown in Figure 2.3. In this study triangular Figure 2.3 (*a*)), ramp up (Figure 2.3 (*b*)), ramp down (Figure 2.3 (*c*)), and trapezoidal (Figure 2.3(*d*)) fuzzy numbers are considered. In Figure 2.3, the support of fuzzy numbers is [a, b].



(*a*) (*b*)

(*c*) *Ramp up* (*d*) *Trapezoidal*

Figure 2.3: Different shapes of the fuzzy numbers(*a*) triangular (*b*) ramp up (*c*) Ramp down and (*d*)trapezoidal.

2.3.1 Trapezoidal Fuzzy Number

For example, Figure 2.4 shows the pictorial representation of membership function of a trapezoidal fuzzy number, *DoB(x),* where [*b*, *c*] is the core, [*a*, *d*] is the support, alpha cut at 50% is partially true and partially false.

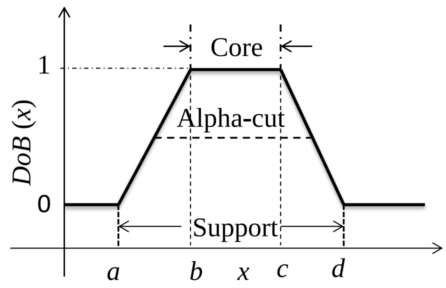


Figure 2.4: Trapezoidal fuzzy number.

The following equation (2.13) represents the function of *DoB (x)*

$$DoB(x) = \begin{cases} \dfrac{x-a}{b-a} & a < x \leq b \\ 1 & b \leq x \leq c \\ \dfrac{c-x}{d-c} & c \leq x \leq d \\ 0 & otherwise \end{cases} \tag{2. 13}$$

In this study trapezoidal fuzzy number is used to quantify the uncertainty of material properties, *TS*, as detail is shown in Chapter 3.

2.3.2 Triangular Fuzzy Number

Membership function of a triangular fuzzy number *T* is a fuzzy number that has a triangularly shaped membership function (*DoB*) expressed by equation (2.14)

$$T(x) = \begin{cases} \dfrac{x-a}{c-a} & a < x \leq c \\ \\ \dfrac{b-x}{b-c} & c \leq x \leq b \end{cases} \tag{2. 14}$$

In equation (2.14), $x \in \Re$, and $a < c < b \in \Re$. As such, the support of the triangular fuzzy number T is $[a, b]$. The core of T is c because $T(x = c) = 1$ as shown in Figure 2.5

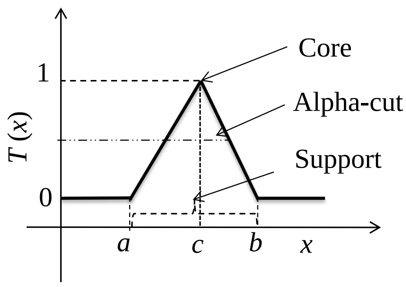


Figure 2.5: A triangular fuzzy number.

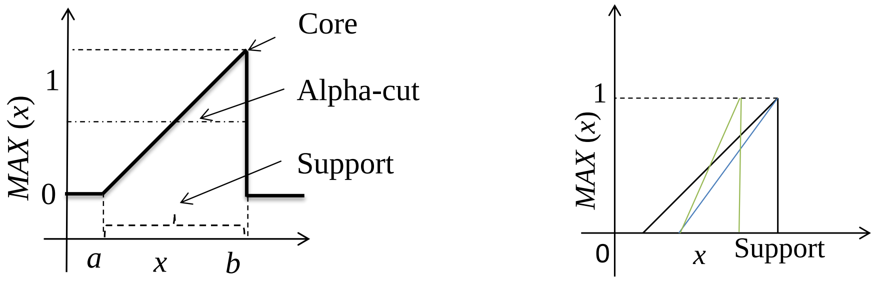
The function $(x-a)/(c-a)$ is called the left function and the function $(b-x)/(b-c)$ is called the right function. The alpha-cuts of a triangular fuzzy number are the intervals $[a + (c-a)\alpha, b- (b-c)\alpha]$, $\forall \alpha \in (0, 1)$. Triangular fuzzy number is used to quantify the uncertainty of the physical parameter such as modulus of elasticity (E) and strain to failure (s) as shown in Chapter 3.

2.3.3 Ramp Up Fuzzy Number

A ramp up fuzzy number denoted as *MAX* is also a fuzzy number. It defines a possibilistic objective function for maximizing a quantity. The expression of *MAX* is given by equation (2.15):

$$MAX(x) = \begin{cases} \max\left\{0, \dfrac{x-a}{b-a}\right. & x \le b \\ 0 & otherwise \end{cases} \tag{2. 15}$$

The core of *MAX* is equal to b and the support is equal to $[a, b]$ as shown in Figure 2.8(a). *MAX* linearly increases with the x in the interval of its support. Since *MAX* is for maximizing a quantity, it can be used as maximization function and setting its support $[a, b]$ is a critical issue.



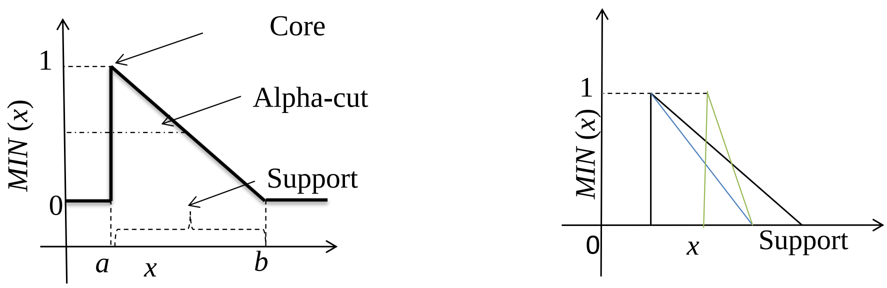
(*a*) (*b*)

Figure 2.6: (*a*) Ramp up and (*b*) typical pattern of ramp up function.

The ramp up function is used to define the maximization function as shown in Chapter 4. In this study maximization function is used to represent the objective function of a criterion which is needed to maximize. Depending on the support the area of the *MAX* is changed as shown in Figure 2.6**(***b*). This issue of support is described in Chapter 4 and 5.

2.3.4 Ramp Down Fuzzy Number

A ramp down fuzzy number denoted as *MIN* is also a fuzzy number as shown in Figure 2.7(*a*). It is defined as possibilistic objective function for minimizing a quantity. The expression of *MIN* is given by equation (2.16):



(*a*) (*b*)

Figure 2.7: (*a*) Ramp down and (*b*) typical pattern of ramp down function.

$$MIN(x) = \begin{cases} \max\left\{0, \dfrac{b-x}{b-a}\right. & x \geq a \\ 0 & otherwise \end{cases} \tag{2. 16}$$

The ramp down function is used to define the minimization function is shown in Chapter 4. The core of *MIN* is equal to *a*, and the support is equal to [*a*, *b*]. *MIN* linearly decreases with the increase in *x* in the interval of its support. Since *MIN* is for minimizing a quantity, it can be used as minimization function and setting its support [*a*, *b*] is a critical issue (see Figure 2.7(*b*)), similar to *MAX*. This issue is also described in Chapter 4. In this study minimization function is used to represent the objective function of a criterion which is needed to minimize. Application of the *MIN* is shown in Chapter 4 and the critical issue regarding the support selection is discussed in Chapter 5.

2.4 Degree of Compliance

The interaction between the information and objective function is termed as compliance. The degree of compliance means how well the information complies with the objective functions. Its value lies between 0 and 1. When the information fully complies with the objective, the value of the compliance is 1 otherwise it is less than 1. The degree of compliance calculation is different depending on the categories of information or data. There are two broad categories of information as listed below:

a) *Crisp information and*
b) *Granular information*

A piece of crisp information is referred to a sharp numerical value (e.g., density is 10 Mg/m^3). The other category of information, granular information (Zadeh, 2005; Khozaimy et al., 2011), is referred to a set of numerical values and has numerous forms. The simplest form of granular information is called crisp granular information that refers to a numerical range (e.g., density is [10, 15]

Mg/m^3). Probability granular information refers to a piece of information given (say) by a probability distribution (e.g., density is normally distributed with mean 12 Mg/m^3 and standard deviation 1 Mg/m^3). Fuzzy granular information refers to linguistically define pieces of information that are often modeled by the fuzzy sets or numbers (e.g., density is "low" where low is defined by a triangular fuzzy number with core 12 Mg/m^3 and support [8, 20] Mg/m^3). The terms called triangular fuzzy number, core, and support will be discussed in a moment. There are other complex forms of granular information, e.g., fuzzy-probability granular information (density is most-likely normally distributed with mean 10 Mg/m^3 and standard deviation 1 Mg/m^3). If the probability distribution is unknown, one can model a piece of information using a fuzzy number or possibility distribution (Dubois, 2004; Ullah and Shamsuzzaman, 2013). This means that a fuzzy number is a general form of granular information that subsumes other forms of information.

In this study, both crisp information and various forms of granular information are used. To formally compute the crisp information and various forms of granular information in an integrated manner, certain mathematical entities are needed. The compliance calculation systems for three different categories information, namely, crisp value, granular crisp value, and probability granular value are described in the following section.

2.4.1 Degree of Compliance of Crisp Value

Let d be a point in the support of *MAX* or *MIN*, i.e., $d \in [a, b]$ as shown in Figure 2.8. Figure 2.8(a) shows the degree of compliance of *MAX* and Figure 2.8 (b) shows the degree of compliance of *MIN*. The degree of compliance of *MAX* and *MIN* are denoted as CC_{MAX} and CC_{MIN}, respectively, is its membership value or *DoB*.

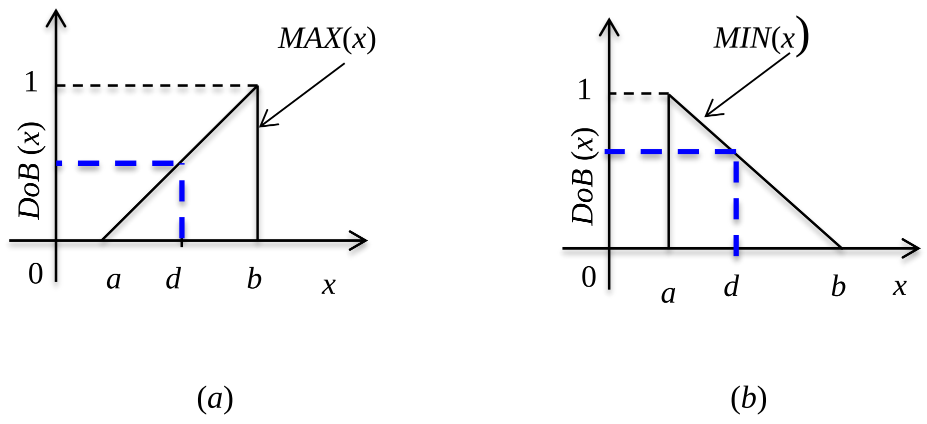


Figure 2.8: Interaction between crisp value and (*a*) *MAX* and (*b*) *MIN* fuzzy numbers.

The degree of compliance of *MAX* is expressed in equation (2.17) and *MIN* is expressed in equation (2.18).

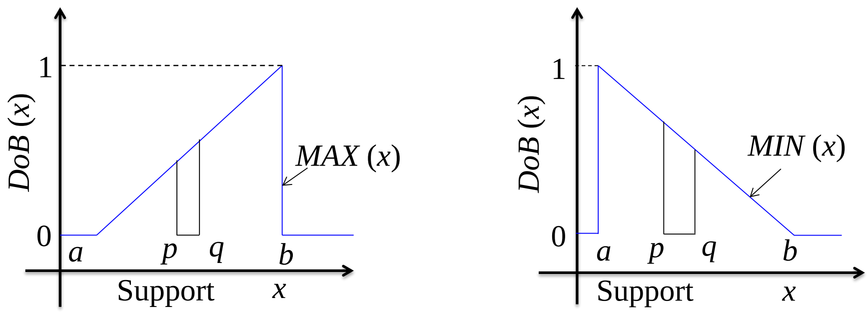
$$CC_{MAX} = \frac{d-a}{b-a} \tag{2. 17}$$

$$CC_{MIN} = \frac{b-d}{b-a} \tag{2. 18}$$

For example, assume $[a, b] = [10, 30]$ for both *MAX* and *MIN*. As such, if $d = 15$, then $CC_{MAX} = 0.25$ and $CC_{MIN} = 0.75$. Needless to say, the nature of CC_{MAX} or CC_{MIN} resembles the nature of *MAX* or *MIN*, respectively. Higher the value of CC_{MAX} or CC_{MIN}, better the d from the viewpoint of maximization or minimization, respectively.

2.4.2 Degree of Compliance of Crisp Granular Value or Range

Let $P = [p, q]$ be an interval in the support $[a, b]$ of *MAX* or *MIN*, i.e., $p \geq a \wedge q \leq b$, as schematically illustrated in Figure 2.9.



(*a*) (*b*)

Figure 2.9: Interaction between crisp granular value and (*a*) maximization and (*b*) minimization fuzzy number.

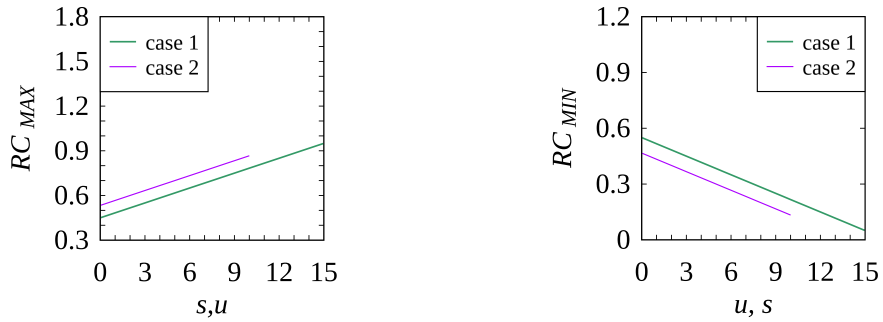
The compliance of *P* with respect to *MAX* or *MIN* denoted as RC_{MAX} or RC_{MIN}, respectively, is the average membership value of *P* with respect to *MAX* or *MIN* (Ullah, 2008; Rashid, et al., 2011; Shamasuzzaman, et al., 2013). The degree of compliance of *MAX* and *MIN* are expressed by equation (2.19) and equation (2.20) respectively.

$$RC_{MAX} = \frac{\int_P MAX(x)dx}{|q-p|} = \frac{MAX(p)+MAX(q)}{2} = \frac{p+q-2a}{2(b-a)} \quad (2.\ 19)$$

$$RC_{MIN} = \frac{\int_P MIN(x)dx}{|q-p|} = \frac{MIN(p)+MIN(q)}{2} = \frac{2b-(p+q)}{2(b-a)} \quad (2.20)$$

As such, RC_{MAX} and RC_{MIN} take a value in the interval [0, 1]. The plot shown in Figure 2.10(*a*) for two arbitrary cases shows the typical nature of RC_{MAX}. Case 1 corresponds to $p = 12 + s$, $q = 15 + s$, $s = 0,...,15$. The other case corresponds to $p = 12 + u$, $q = 20 + u$, $u = 0,...,10$. The range corresponding to the first case is relatively slim whereas the other is relatively fat. In both cases, RC_{MAX} linearly increases when it approaches the upper limit of the maximization (i.e., $b = 30$). RC_{MAX} becomes unit if it is a point equal to the core of *MAX* (i.e., $p = q = b$). RC_{MAX} becomes zero if it is a point equal to *a*, (i.e., $p = q = a$), otherwise,

$RC_{MAX} < 1$ as shown in Figure 2.10 (a). Higher the value of RC_{MAX}, better the P from the viewpoint of maximization.



(a) (b)

Figure 2.10: A typical nature of (a) RC_{MAX} and (b) RC_{MIN}.

On the other hand, Figure 2.10(b) shows two arbitrary cases of the typical nature of RC_{MIN}. Case 1 corresponds to $p = 12 + s$, $q = 15 + s$, $s = 0,...,15$. The other case corresponds to $p = 12 + u$, $q = 20 + u$, $u = 0,...,10$. The range corresponding to the first case is relatively slim whereas the other is relatively fat, similar to that in RC_{MAX}. In the both cases, RC_{MIN} linearly decreases when it approaches the lower limit of minimization (i.e., $a = 10$). RC_{MIN} is unit if it is a point equal to the core of MIN (i.e., $p = q = a$). RC_{MIN} is zero if it is a point equal to b, (i.e., $p = q = b$), otherwise, $RC_{MIN} < 1$ as shown in Figure 2.10(b)). Higher the value of RC_{MIN}, better the P from the viewpoint of minimization. This kind of degree of compliance is used to calculate the interaction between objective function and the material properties (in the form of upper value and lower value) as shown in Chapter 4.

2.4.3 Degree of Compliance of Triangular Fuzzy Number/ Possibility Distribution

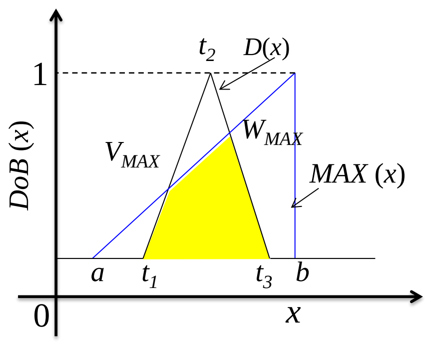
This sub-section employs the notion of triangular fuzzy number, as defined in equation (2.14). Let $t_1 = a$, $t_2 = b$, and $t_3 = c$ be three points in the ascending order on the real-line, i.e., $t_1 \le t_2 \le t_3 \in \Re$. Let the interval $[t_1, t_3]$ and the point t_2 be the support and core, respectively, of a triangular fuzzy number denoted as D. The triangular fuzzy number is expressed by equation (2.21).

$$D(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1} & t_1 < x \le t_2 \\ 0 & otherwise \\ \dfrac{t_3 - x}{t_2 - t_1} & t_1 < x \le t_2 \end{cases} \tag{2.21}$$

This kind of degree of compliance is helpful to see the closeness or farness of the criteria (data was as a form of uncertainty) from the objective function. The application of the compliance is shown in Chapter 4. The compliance of this triangular fuzzy number can be calculated as follows:

2.4.3.1 Interaction of D with *MAX*

The maximization fuzzy number *MAX* defined in equation (2.15) and its support $[a, b]$. Assume that the support of D belongs to the support of *MAX*, i.e., $a \le t_1$ and $b \ge t_3$. This assumption is illustrated in Figure 2.11(a), where the points of intersections of D and *MAX* are V_{MAX} (V_{MAXx}, V_{MAXy}) and W_{MAX} (W_{MAXx}, W_{MAXy}), and are given by equation (2.22) and (2.23).



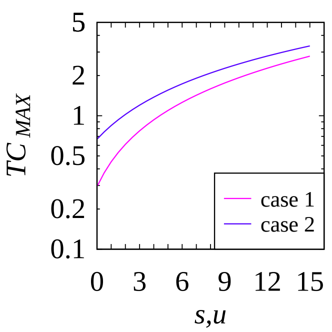


Figure 2.11: (a) Interaction between the triangular and maximization fuzzy numbers and (b) A typical nature of TC_{MAX}.

$$V_{MAXx} = \frac{bt_1 - at_2}{(b-a)-(t_2 - t_1)} \quad V_{MAXy} = \frac{t_1 - a}{(b-a)-(t_2 - t_1)} \tag{2. 22}$$

$$W_{MAXx} = \frac{bt_3 - at_2}{(b-a)+(t_3 - t_2)} \quad W_{MAXy} = \frac{t_3 - a}{(b-a)+(t_3 - t_2)} \tag{2.23}$$

Let, the area under the function min ($D(x)$, $MAX(x)$) is A_{MAX} and simplified expression is given by (2.24).

$$\begin{aligned} A_{MAX} &= \int_{t_1}^{t_2} \min(D(x), MAX(x))dx \\ &= \frac{V_{MAXy}(W_{MAXx} - t_1) + W_{MAXy}(t_3 - V_{MAXx})}{2} \end{aligned} \tag{2. 24}$$

The maximum possible A_{MAX} is $\frac{1}{2}(t_3 - t_1)$, which occurs if $t_1 = a$ and $t_2 = t_3 = b$, i.e., if D takes the shape of MAX. Therefore, if A_{MAX} is normalized by the above mentioned maximum possible area, then the resulting quantity denoted as TC_{MAX} measures the degree of compliance of D with respect to MAX in the interval [0, 1]. The equation (2.25) is used to express this relationship.

$$TC_{MAX} = \frac{A_{MAX}}{\frac{1}{2}(t_3 - t_1)} = \frac{V_{MAXy}(W_{MAXx} - t_1) + W_{MAXy}(t_3 - V_{MAXx})}{t_3 - t_1} \tag{2. 25}$$

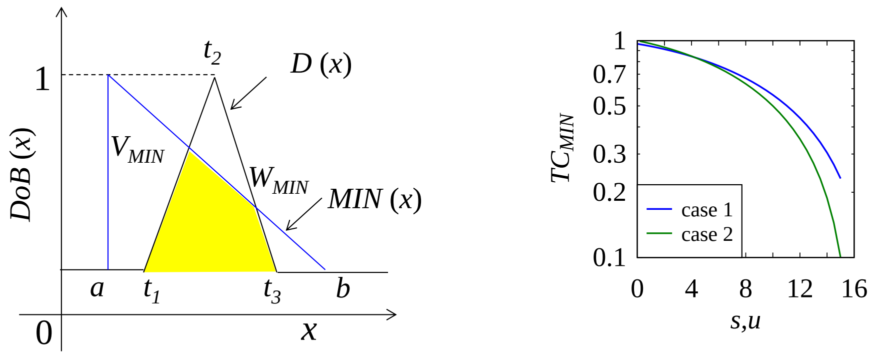
A typical nature of TC_{MAX} is shown in Figure 2.11(*b*) for two arbitrary cases. Case 1 corresponds to $t_1 = 10 + s$, $t_2 = 12 + s$, $t_3 = 15 + s$, $s = 0, \ldots 15$. The other case corresponds to $t_1 = 10 + u$, $t_2 = 15 + u$, $t_3 = 20 + u$, $u = 0 \ldots, 10$. The triangular fuzzy number corresponding to the first case is relatively slim whereas the other one is relatively fat. In both cases, an exponential increase in the value of TC_{MAX} is observed, if the triangular fuzzy numbers approach the upper limit of the maximization (i.e., $b = 30$). TC_{MAX} becomes unit if D takes the shape of MAX (i.e., $t_1 = a$, $t_2 = t_3 = b$), otherwise, $TC_{MAX} < 1$ (see Figure 2.11(*b*)). The more the D resembles MAX, the higher is the value of TC_{MAX}. In other words, the higher is the value of TC_{MAX}, the better is the D from the viewpoint of maximization. In Chapter 4 using this type of compliance calculation, the ranking of material is made when the requirement was to maximize the criteria. The criteria of the material are at first represented by possibility distribution.

2.4.3.2 Interaction of D with *MIN*

The minimization fuzzy number *MIN* is defined by the equation (2.9) and its support [a, b]. It is assumed that the support of D belongs to the support of *MIN*, i.e., $a \le t_1$ and $b \ge t_3$. This assumption is illustrated in Figure 2.12(a) where the points of intersections between D and *MIN* are V_{MIN} (V_{MINx}, V_{MINy}) and W_{MIN} (W_{MINx}, W_{MINy}), and given by equation (2.26) and (2.27).

$$V_{MINx} = \frac{bt_2 - at_1}{(b-a)+(t_2-t_1)} \quad V_{MINy} = \frac{b-t_1}{(b-a)+(t_2-t_1)} \tag{2.26}$$

$$W_{MINx} = \frac{bt_2 - at_3}{(b-a)-(t_3-t_2)} \quad W_{MINy} = \frac{b-t_3}{(b-a)-(t_3-t_2)} \tag{2.27}$$



(a) (b)

Figure 2.12: (a) Interaction between a triangular and a minimization fuzzy number and (b) Typical nature of TC_{MIN}.

Let, the area under the function min ($D(x)$, $MIN(x)$) is A_{MIN} and simplified expression is given by (2.28).

$$A_{MIN} = \int_{t_1}^{t_3} \min(D(x), MIN(x))dx$$
$$= \frac{V_{MINy}(W_{MINx} - t_1) + W_{MINy}(t_3 - V_{MINx})}{2} \tag{2. 28}$$

The maximum possible A_{MIN} is $\frac{1}{2}(t_3 - t_1)$, which occurs when $t_1 = t_2 = a$ and $t_3 = b$, i.e., when D takes the shape of MIN. Therefore, if A_{MIN} is normalized by the above mentioned maximum possible area, then the resulting quantity denoted as TC_{MIN} measures the degree of compliance of D with respect to MIN in the interval [0, 1]. The expression of TC_{MIN} can be expressed as equation (2.29).

$$TC_{MIN} = \frac{A_{MIN}}{\frac{1}{2}(t_3 - t_1)} = \frac{V_{MINy}(W_{MINx} - t_1) + W_{MINy}(t_3 - V_{MINx})}{t_3 - t_1} \tag{2. 29}$$

The typical nature of TC_{MIN} is shown in Figure 2.12(b) for two different cases. Case 1 corresponds to $t_1 = 10 + s$, $t_2 = 12 + s$, $t_3 = 15 + s$, $s = 0,...,15$. The other case corresponds to $t_1 = 10 + u$, $t_2 = 15 + u$, $t_3 = 20 + u$, $u = 0,...,10$. The triangular fuzzy number corresponding to the first case is relatively slim compared to that of the other case. In both cases, TC_{MIN} linearly increases if the triangular fuzzy number approaches the upper limit of the maximization (i.e., $b = 30$). It is worth mentioning that TC_{MIN} becomes unit if D takes the shape of MIN (i.e., $t_1 = a$, $t_2 = t_3 = b$), otherwise, $TC_{MIN} < 1$ as shown Figure 2.12(b)). Higher the value of TC_{MIN}, better the D from the viewpoint of minimization. In Chapter 4 using this type of compliance calculation, the ranking of material is made when the requirement is to minimize the criteria.

2.5 Induction of Fuzzy Number

This section describes the mathematical entities used to quantify the epistemic uncertainty. A possibility distribution is a probability-distribution-neutral representation of the uncertainty associated with a physical quantity. In certain cases, the uncertainty associated with a set of numerical data that can be represented by a possibility distribution of triangular form i.e., a triangular fuzzy number, particularly when the set of numerical data shows a central tendency. It can be also represented by other fuzzy numbers (e.g., trapezoidal fuzzy number). To create a triangular fuzzy number it is important to develop a transformation mechanism based on the probability-possibility consistency principle, which states that lessening of the possibility of an event tends to lessen its probability - but not vice-versa (Zadeh, 1978 Figure 2.13 illustrates a triangular fuzzy number induction process using an arbitrary set of numerical data $X = \{(i, x(i)) \in \Re \mid i = 0,...,100\}$ (say) and the mathematical procedure is described as follows.

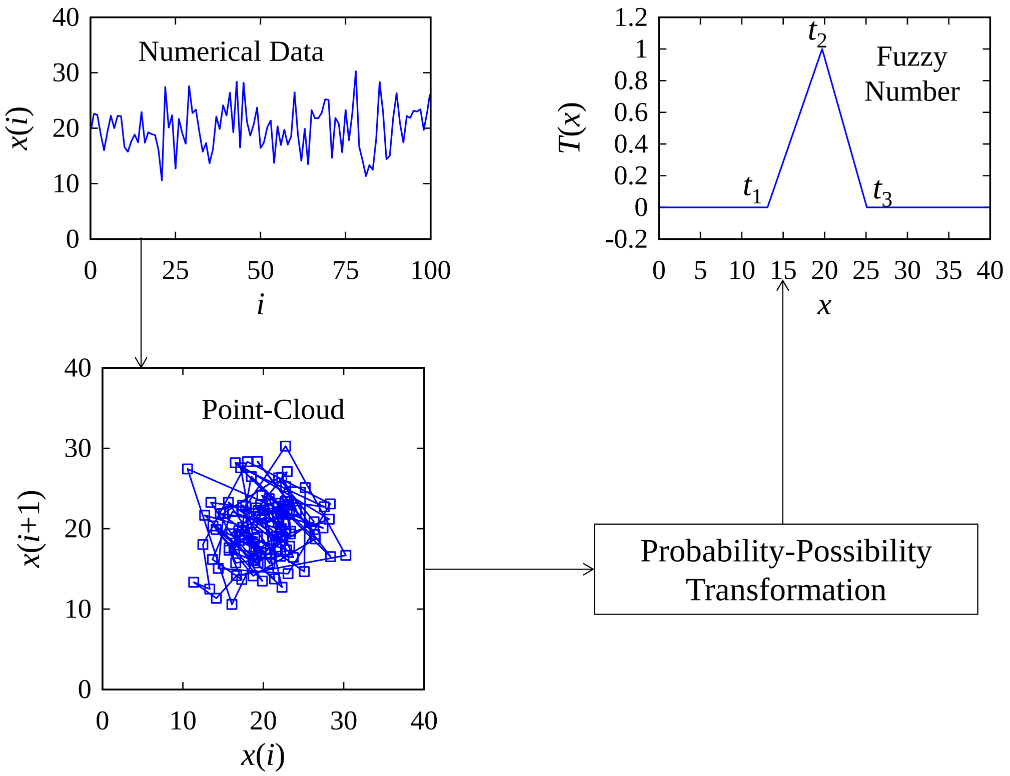


Figure 2.13: Representing the uncertainty of numerical data using a triangular fuzzy number.

The variability associated with a variable X is first represented by a point-cloud that is the plot in ordered-pairs $\{(x(i), x(i+1)) \mid i = 0,...,99\}$ as shown in Figure

2.13. Using a probability-possibility transformation, the point-cloud is transformed into a triangular fuzzy number. See Ullah and Shamsuzzaman (2013) for detail procedure that transforms a point-cloud to a triangular fuzzy number. The induced triangular fuzzy number is used to calculate the degree of compliance of the supplied set of data on X with respect to *maximization or minimization*. These issues are described in Section 2.3 and Section 2.4. It is worth mentioning that the set of numerical data must lie in the support of *MAX* or *MIN*, i.e., $x(i) \in [a, b]$, $i = 0,...,n$. Otherwise, the calculation of the degree of compliance cannot be performed. In addition, if a variable X takes values from a unimodal probability distribution (e.g., from uniform, normal, or triangular distribution), then its equivalent possibility distribution (a triangular fuzzy number) is used while calculating the degree of compliance in accordance with the procedure described in Section 2.4. Ullah and Shamsuzzaman (2013) show the equivalent triangular fuzzy numbers for the uniform and normal distributions.

Let $x(t) \in \Re$, $t = 0,...,n$-1 be n data points, as shown in Figure 2.14

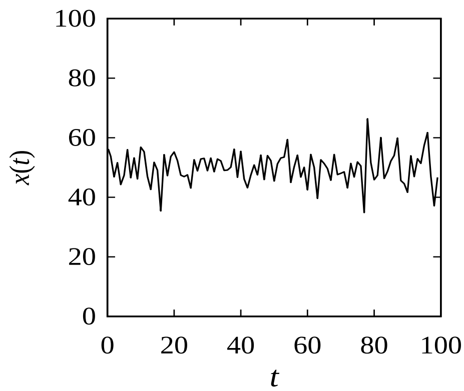


Figure 2.14: A given set of numerical data.

Let $(x(t), x(t+1))$, $t = 0,...,n$-1, be a point-cloud in the universe of discourse $X = [x_{min}, x_{max}]$ so that $x_{min} < \min(x(t)|\forall t \in \{0,...,n\})$ and $x_{max} > \max(x(t)|\forall t \in \{0,...,n\})$. Let A and B are two square boundaries so that the vectors of the vertices of A and B (in the anti-clockwise direction) are $((x_{min}, x_{min}), (x, x_{min}), (x, x), (x_{min}, x))$

and ((x_{max}, x_{max}), (x, x_{max}), (x, x), (x_{max}, x)), respectively, $\forall x \in X$. As such, (x, x) is their common vertex of A and B. For example, consider the arbitrary point-could show in Figure 2.15. As from Figure 2.15, the universe of discourse X = [20, 80]. Notice the relative positions of boxes denoted by *A* and *B* in Figure 2.15. The boxes are connected at their common vertices.

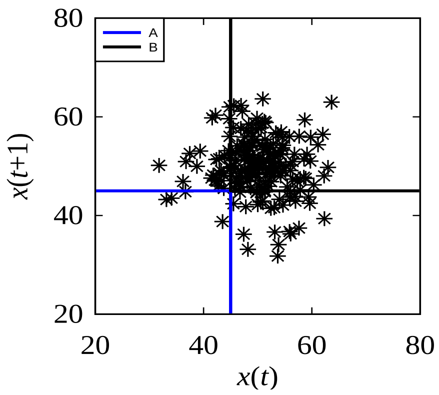


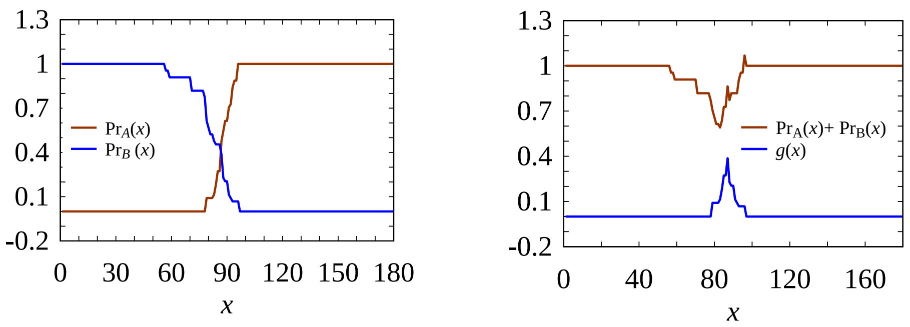
Figure 2.15: Relative position of *A* and *B* in the point-cloud ($x(t)$, $x(t+1)$).

Let $\Pr_A(x)$ and $\Pr_B(x)$ are two subjective probability wherein $\Pr_A(x)$ and $\Pr_B(x)$ represent the degrees of chances that the points in the point-cloud be in *A* and *B*, respectively. As such, these functions are defined by the following mappings:

$$
\begin{aligned}
&X \rightarrow [0,1] \\
&x \;\alpha\;\; \Pr_A(x) = \frac{\sum_{i=0}^{n-1} \Theta(t)}{n-1} \\
&\Theta(t) = \begin{cases} 1 & ((x(t) \le x) \wedge (x(t+1) \le x)) \\ 0 & otherwise \end{cases}
\end{aligned}
\tag{2. 30}
$$

$$
\begin{aligned}
&X \rightarrow [0,1] \\
&x \;\alpha\;\; \Pr_B(x) = \frac{\sum_{i=0}^{n-1} \Omega(t)}{n-1} \\
&\Omega(t) = \begin{cases} 1 & ((x(t) \ge x) \wedge (x(t+1) \ge x)) \\ 0 & otherwise \end{cases}
\end{aligned}
\tag{2.31}
$$

The typical nature of the functions defined in equations (2.30) and (2.31) are illustrated in Figure 2.16 using the information of the point-cloud shown in Figure 2.15. Note that $\Pr_A(x)$ increases with the increase in x and the opposite is true for $\Pr_B(x)$. It is worth mentioning that $\Pr_A(x) + \Pr_B(x) \leq 1$ for the point-cloud, though for some cases $\Pr_A(x) + \Pr_B(x) = 1$ (see Figure 2.16(*a*)). This means that $\Pr_A(x) + \Pr_B(x)$ does not serve the role of "cumulative probability distribution." A cumulative probability distribution can be formulated by using the information of $\Pr_A(x)$ and $\Pr_B(x)$, as follows:



(*a*) (*b*)

Figure 2.16: (*a*)The typical nature of $\Pr_A(x)$ and $\Pr_B(x)$ for unimodal quantity and (*b*) Nature of $\Pr_A(x)+\Pr_B(x)$ and $\min(\Pr_A(x),\Pr_B(x))$ for unimodal data

Consider a mapping that maps x into the minimum of $\Pr_A(x)$ and $\Pr_B(x)$, as follows:

$$\begin{aligned} &X \to [0,a] \\ &x \mapsto g(x) = \min\left(\Pr_A(x), \Pr_B(x)\right) \end{aligned} \tag{2.32}$$

In equation (2.32), $a = 1$, if the point-cloud is a point; otherwise, $a < 1$. Figure 2.16(*b*) shows the nature of $g(x)$ for $\Pr_A(x)$ and $\Pr_B(x)$. The area under $g(x)$ is given by equation (2.33).

$$Q = \int_X g(x)dx \tag{2. 33}$$

There is no guarantee that Q = 1. Otherwise $g(x)$ could have been considered a probability distribution of the underlying point-cloud. However, a function $F(x)$ can be defined as follows:

$$[0, a] \to [0,1]$$
$$x \alpha \quad F(x) = \frac{\int_{x_{\min}}^{x} g(x)dx}{Q} \tag{2. 34}$$

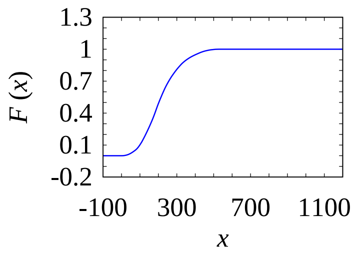


Figure 2.17: Nature of cumulative probability distribution of a point-cloud.

$F(x)$ can be considered a cumulative probability distribution because max($F(x)$)=1, $F(x) \geq F(z)$ for $x \geq z$, $F(x) \in [0,1]$, $\forall x, z \in X$. Figure 2.17 shows the nature of $F(x)$ derived from $g(x)$ shown in Figure 2.16(b). The cumulative probability distribution defined in equation (2.34) produces a probability distribution Pr(x). , the following formulation holds:

$$\Pr(x) = \frac{dF(x)}{dx} \tag{2.35}$$

Figure 2.18(a) shows the probability distribution Pr(x) underlying $F(x)$ shown in equation (2.34). It is needless to say that, the area under the probability distribution Pr(x) is unit and Pr(x) remains in the bound of [0, 1].

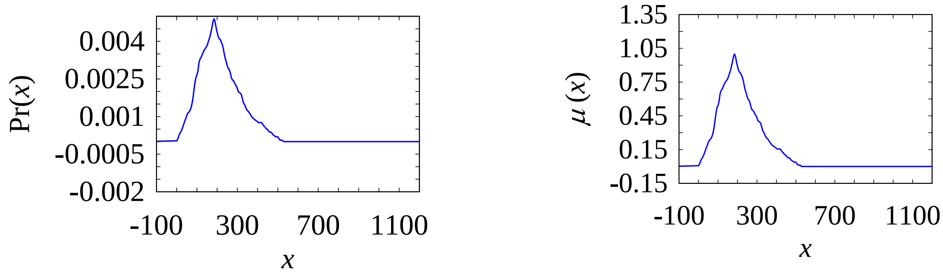


Figure 2.18: The nature of (*a*) probability (*b*) possibility distribution of a unimodal point-cloud.

From the induced probability distribution Pr(x), a possibility distribution given by the membership function $\mu_I(x)$) can be defined based on the heuristic rule of probability-possibility transformation that the degree of possibility is greater than or equal to the degree of probability. The easiest formulation is to normalize Pr(x) by its maximum value, max(Pr(x)| $\forall x \in X$). Therefore,

$$[0,1] \to [0,1]$$
$$\Pr(x)\,\alpha\quad \mu_I(x) = \frac{\Pr(x)}{\max(\Pr(x) \mid \forall x \in X)} \tag{2.36}$$

Figure 2.18 (*b*) shows the possibility distribution $\mu_I(x)$ derived from the probability distribution Pr(x) shown in Figure 2.18(*a*). The shape of the induced probability distribution and the shape of the induced possibility distribution are identical, as evident from Figure 2.18(*a*) and Figure 2.18 (*b*). Other formulations can be used instead of the formation (2.36) as suggested by others.

However, it is observed that when the point-cloud resembles the point-cloud of a bimodal quantity, the induced possibility distribution resembles a trapezoidal fuzzy number. In addition, when the point-cloud is a point, the induced possibility distribution becomes fuzzy singleton. Moreover, when the point-cloud resembles the point-cloud of a unimodal data, the induced probability/possibility distribution resembles a triangular fuzzy number. To define the membership function of an induced fuzzy number in the form of a triangular fuzzy number, the following formulation can be sued.

Let u, v, and w be three points in the ascending order in the universe of discourse X, $u \leq v \leq w \in X$. Let the interval $[u, w]$ be the *support* of a triangular fuzzy number and the point v be the *core*. The following procedure can be used to determine the values of u, v, and w from the induced fuzzy number $\mu_I(x)$ (equation (2.36)):

$$\begin{aligned}
&u \leq v \leq w \in X \\
&u = x \quad (\mu_I(x) = 0 \wedge \mu_I(x + dx) > 0) \\
&v = x \quad (\mu_I(x - dx) < 1 \wedge \mu_I(x) = 1) \\
&w = x \quad (\mu_I(x - dx) > 0 \wedge \mu_I(x) = 0)
\end{aligned} \tag{2. 37}$$

As defined in equation (2.37), u is the point after which the membership value $\mu_I(x)$ is greater than zero, v is the point corresponding to the maximum membership value $\max(\mu_I(x))$, and w is the point beyond which the membership value $\mu_I(x)$ again becomes zero. Therefore, the membership function $\mu_T(x)$ of the induced triangular fuzzy number is as follows:

$$\begin{aligned}
&X \to [0,1] \\
&x \ \alpha \ \ \mu_T(x) = \max\left(0, \min\left(\frac{x-u}{v-u}, \frac{w-x}{w-v}\right)\right)
\end{aligned} \tag{2. 38}$$

It is needless to say that this formation is valid only for the point-cloud exhibiting the nature of a unimodal quantity. The general detail flow diagram of the possibility distribution formulation from numerical data to triangular fuzzy number is shown in Figure 2.19.

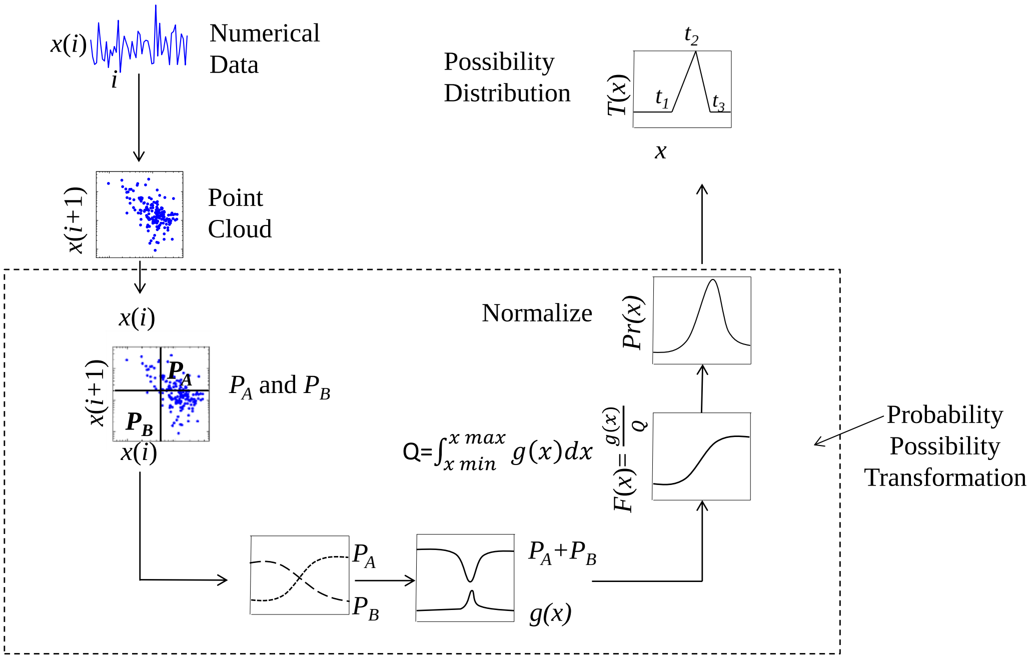


Figure 2.19: Numerical data to Possibility distribution transformation.

Using this Probability possibility transformation the uncertainty of the material properties are quantified as shown in Chapter 3 and Chapter 4.

Chapter 3: Uncertainty Quantification of Mechanical Properties of Jute Yarn

Uncertainty quantification in the data of material properties is an important issue to select a material for a product development. The objective of this chapter is to quantify the uncertainty in the data of mechanical properties and identify a reliable approach to quantify the related uncertainty. This chapter is based on the work of Ullah et al. (2017) and Shahinur et al. (2017). The remainder of this chapter is organized as follows. Section 3.1 and Section 3.2 describe the mechanical properties and sustainable properties respectively. Section 3.3 describes the primary production of jute. Section 3.4 describes the experimental description of tensile test performed to measure the mechanical properties of jute yarns. Section 3.5 describes the results associated with mechanical properties of jute yarn. Section 3.6 describes different types of quantification approaches to quantify the data. Furthermore, a comparison is also made to investigate a reliable approach for quantification. Section 3.7 draws the concluding remarks.

3.1 Mechanical Properties

In the strength and stiffness limited design, the *MI* is related to the mechanical properties such as *TS*, *E*, and ρ. The uncertainty related to this material properties are quantified in this chapter. The uncertainties related to these properties are considered for material selection which will be described in Chapter 4.

3.1.1 Tensile Strength and Modulus of Elasticity

A modulus of elasticity (E) is defined as the ratio of stress (σ) and strain (ε). Suppose the tensile force (F) is applied to a specimen and the instantaneous length (l) is recorded. The initial length of the specimen is l_0. Then, the strain is the change of length as shown in equation (3.1), and stress is the ratio of tensile force and area as shown in equation (3.2). Here, r is the radius of a specimen.

$$\varepsilon = \frac{l - l_0}{l_0} \tag{3.1}$$

$$\sigma = \frac{F}{\pi r^2} \tag{3.2}$$

If the stress verses strain graph is plotted, some specimens follow the linear and others follow nonlinear pattern. The typical natures of linear and nonlinear patterns are illustrated in Figure 3.1(a) and Figure 3.1(b) respectively.

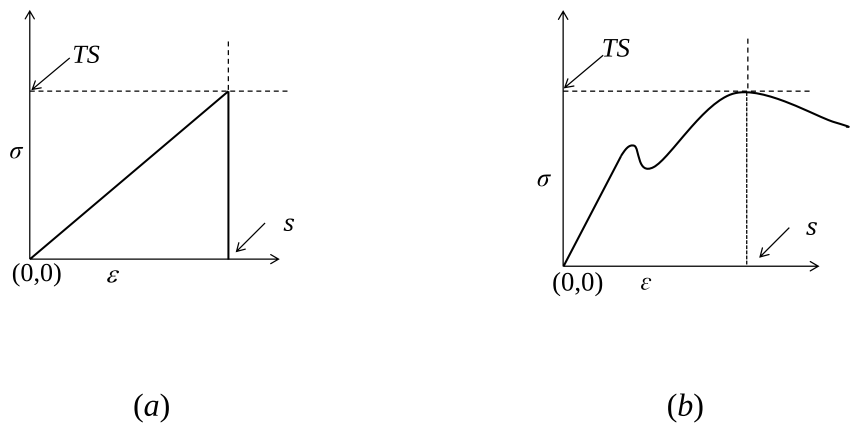


Figure 3.1: Data analysis of stress-strain curve for (a) linear and (b) non-linear.

In both cases, the maximum stress is considered as TS of the specimen. The strain corresponding to TS is considered strain to failure (s). In both cases, the E is determined by equation (3.3).

$$E = \frac{TS}{s} \tag{3.3}$$

3.1.2 Density

A density (ρ) is defined as mass per unit volume. It is mathematically expressed by equation (3.4), where m is mass and V is a specific volume.

$$\rho = \frac{m}{V} \tag{3.4}$$

From equation (3.4), to have low ρ, either m can be decreased or V can be increased. Thus, low-density materials are lighter than high density materials. For example, the density of the iron, Fe is 7-8 Mg/m^3 (here the phrase “Mg” is mega gram) whereas the density of Al is 2.5-2.9 Mg/m^3 (Ashby, 2007). Therefore, for light car body, Al will be a better option due to low fuel consumption.

3.2 Sustainable Properties

3.2.1 CO_2 Footprint

CO_2 foot print or CO_2 emission can be defined as the entire amount of greenhouse gases formed directly and indirectly by human activities, usually expressed in equivalent tons of carbon dioxide (CO_2) (Time for change, 2017). When we heat our house with coal, oil, or gas, it will emit CO_2. Even when the house is heated in cold regions with electricity, generation of the electrical power also releases a certain amount of CO_2. For each liter of oil heating consumption, 13 kg carbon dioxide (CO_2) is emitted. For liquid, the unit of the CO_2 footprint is liter or gallon-CO_2/ kg and for a metal it is kg-CO_2 / kg.

3.2.2 Recycle fraction

The *Recycle fraction* is a measure of the proportion of a material used in products which can economically be recycled (Material properties, 2017). *Recycled fraction* is a number that lies between 0 and 1 (or a percentage) and has no units. Re-melted (thermoplastics and metals) or shredded (wood and paper) materials are highly recycled. Natural materials are disposable, thus, the recycle fraction of natural material per year is 0%. However, the plastic is fully recyclable, thus, recycle fraction is 100%.

3.2.3 Water usage

Water usage or water footprint is the amount of water used for the primary production of a given component or crop or the amount allocated for a particular purpose (Water foot print network, 2017). Water usage is measured in cubic meters per ton or Kg of production, per hectare of cropland, per unit of currency and other functional units. The water usage of a 150 gram soy burger produced in the Netherlands is about 160 liters.

3.3 Production of Jute Material

As mentioned in Chapter 1, jute is one of the most widely used natural materials after cotton (Anon., 2017), and is of interest for the development of eco-products (Alves, et al., 2010). Accordingly, numerous products are made from jute fibers, yarns, fabrics, and composites. Such jute-based products are sacks, bags, corrugated sheets, carpets, shoes, sandals, and fabrics are now available in the market (Sobhan, et al., 2010). The demand for these products is growing significantly, as they are eco-friendly products. Figure 3.2 shows a typical scenario of primary production of jute fibers and yarns. As shown in Figure 3.2, jute is grown in plantations, where it is collected after it matures. The plants are then typically soaked in water so that the fibers can be more easily separated for collection. Other processes to collect the fibers without soaking may be used if preferred. The jute fibers are then dried and marketed for further processing. The jute yarn is often produced from jute fibers. Sometimes, yarns are chemically or physically (radiation, and light impose) treated (Jafrin, et al., 2014; Shahinur, et al., 2013) to enhance their material properties.

Figure 3.2: Primary production jute yarn.

In this study, the jute yarn is considered because it is more widely used primary material in the jute product compared to jute fiber. The mechanical test for jute yarn is performed and the data are quantified in a systematic manner (statistics, probability, and possibilistic approaches) for taking a decision. Therefore, the following section reports on the results of tensile tests were performed on jute yarn specimens.

3.4 Description of Experiment

A tensile test allows the determination of properties: *TS*, *E*, and *s*. Figure 3.3 schematically demonstrates the experimental procedure to know certain mechanical properties of the jute yarn. The procedure consists of the following four steps:

a) *Collection of jute yarn and diameter measurement*
b) *Specimen preparation with required length*
c) *Tensile testing* and
d) *Data analysis*

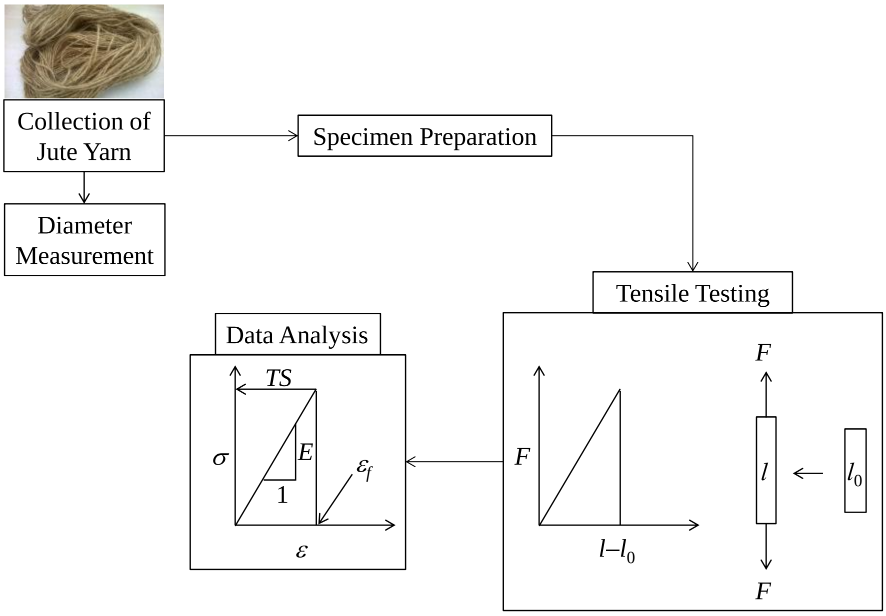
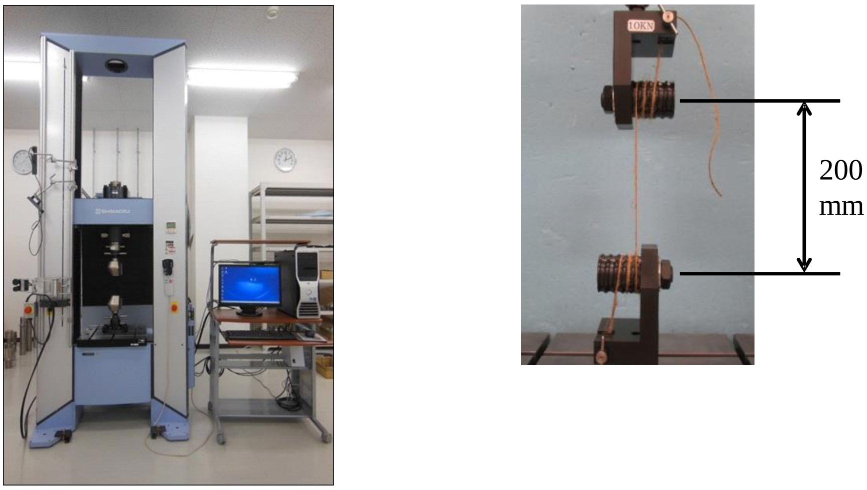


Figure 3.3: Schematic diagram of the experiment

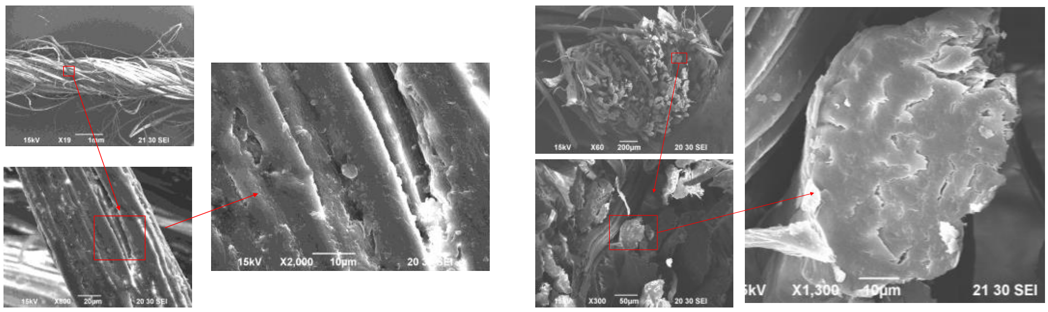
Jute yarn used in this study was collected from the Bangladesh Jute Research Institute (BJRI, http://www.bjri.gov.bd/) located in Dhaka (the capital of Bangladesh). The yarn count was 10^S (single ply) with a diameter of 1.7 mm as schematically shown in Figure 3.3. The diameter is measured by an ordinary micrometer. The value of the diameter is the average value of several trials (rounded to one decimal place). A universal tensile testing machine (Autograph AG-X; make: Shimadzu Corporation, Kyoto, Japan) was used to perform the tensile tests, as shown in Figure 3.4(*a*). A set of 15 specimens was prepared, each having a length of approximately 1.1 m. The grip-to-grip length of a specimen was fixed at 200 mm in the tensile tests, as shown in Figure 3.4(*a*) and (*b*). In these tests, the F is applied to each specimen and l is recorded. The maximum load (the load just before the failure) is divided by the cross-sectional area of the yarn to calculate the *TS*. The elongation just before the failure is divided by the initial length (200 mm) to calculate the s (equation 3.1). The *TS* was divided by the s to calculate the E. The results are summarized in Table 3-a



(*a*) (*b*)

Figure 3.4: (*a*) Experimental equipment and (*b*) Gripping.

Finally, the data (*F* -*l*) curve are analyzed to determine the material properties called *TS* and *E*. To obtain *TS* and *E*, the *F*–*l* curve is converted into $\sigma - \varepsilon$ curve using equations (3.2) to (3.4). To avoid deflection, a preload of 5 N is applied to the specimen. The elongation velocity is set to 100 mm/min in all tests. Figure 3.5 shows magnified images (SEM image) of the surface of yarn specimen used in this study.



(*a*) (*b*)

Figure 3.5: Magnified (*a*) front and (*b*) cross section view of yarn specimen.

3.5 Results

The results of the tensile tests and uncertainty in the data of mechanical properties on the base of jute yarn specimens are discussed in following sections.

3.5.1 Mechanical Properties

Figure 3.6 shows load versus elongation for the fifteen jute yarn specimens. As seen in Figure 3.6, the yarns failed at an elongation of an approximately 10–15 mm, and a load of approximately 75–115 N.

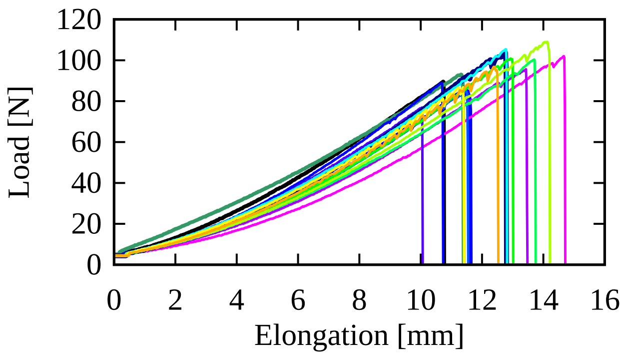


Figure 3.6: Load versus elongation plots of fifteen jute yarn specimens.

It is observed that there is a wide range of variation in the load as well as elongation before failure. Using equation (3.1) to (3.4), the *TS*, *E*, and *s* of each specimen are calculated, and the mechanical properties of jute yarn are summarized in Table 3-a.

Table 3-a. Mechanical properties of jute yarn.

Specimen No.	**Properties**		
	***TS* [MPa]**	***E* [GPa]**	***s* [%]**
1	42.65	0.68	6.27
2	38.24	0.67	5.71
3	48.04	0.68	7.11
4	44.44	0.68	6.51
5	44.22	0.64	6.88
6	46.43	0.72	6.41
7	45.70	0.71	6.40
8	37.39	0.65	5.78
9	39.20	0.73	5.37
10	33.87	0.67	5.03
11	42.13	0.62	6.74
12	44.94	0.61	7.37
13	41.00	0.72	5.70
14	37.46	0.64	5.82
15	39.48	0.73	5.38

3.5.2 Uncertainty in the Mechanical Properties

This section presents the uncertainty in the data of mechanical properties of jute yarn specimens. Figure 3.7 shows the plot of *E* vs. *TS*, *E* vs. *s*, and *s* vs. *TS*, which corresponds the variation of *TS*, *E*, and *s* of jute yarn. *TS*, *E*, and *s* of jute yarn are hardly correlated with each other. The uncertainty in the data of mechanical properties, *TS*, *E*, and *s* are quite high as detected in Figure 3.7. The value of *TS* varies from 25 to 50 MPa whereas *s* varies from 5 to 7.3%. Similarly, the *E* varies from 0.61 to 0.73 GPa. As there is a lack of consistency in the mechanical properties, it is not possible to calculate the *MI* of jute yarn.

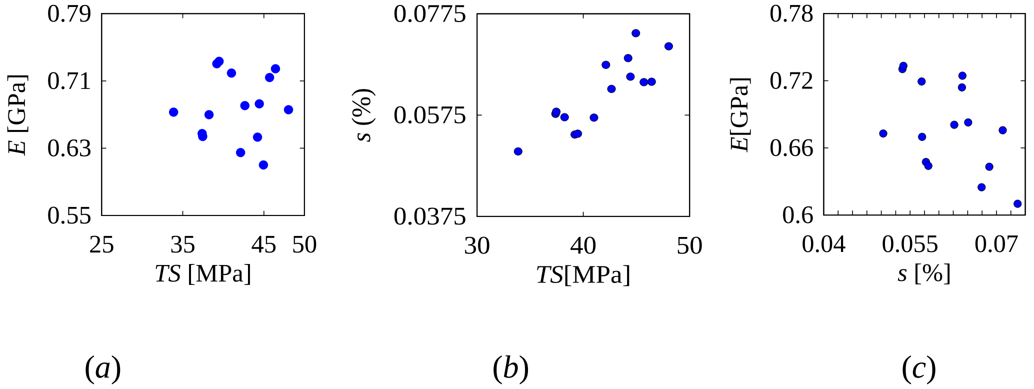


Figure 3.7: Uncertainty in the mechanical properties (*a*) *TS*, (*b*) *E*, and (*c*) *s* of the jute yarn.

To be benefitted from the experimental results as shown in Figure 3.6 and Figure 3.7, one needs to quantify the uncertainty associated with *TS*, *E*, and *s* in a systematic manner while selection of a material is important for product development.

3.6 Uncertainty Quantification

As mentioned earlier, degree of uncertainty is associated with the data of material properties. This uncertainty is even more prevalent for the material properties of natural materials, jute as shown in Figure 3.7. Therefore, uncertainty quantification is an important aspect of studying natural material properties. Without uncertainty quantification, eco-product development processes may not provide the desired outcome. Numerous studies have tried to quantify the uncertainty exhibited by the material properties of natural materials. In many of these studies (Fidelis, et al., 2013; Silva, et al., 2008; Defoirdt, et al., May 2010; Hossain, et al., 2014) significant variability was observed in the respective material properties. The remainder of this section uses statistical, probabilistic and possibilistic method to quantify the uncertainty in the data of mechanical properties of jute yarn.

3.6.1 Uncertainty Quantification by Statistical Method

Table 3-b shows the mechanical properties of the jute yarn for various statistical parameters, namely, minimum, maximum, $\bar{x}$, *sd*, and μ. From Table 3-a, the μ and *sd* of *TS*, *E*, and *s* for jute yarn are calculated using equation (2.9-10). The ranges of *sd* and μ at 95 % level of confidence are calculated using equation (2.11-12) and listed in Table 3-b.

Table 3-b. Statistical uncertainties of jute yarn.

Parameters	**Properties**		
	***TS* [MPa]**	***E* [GPa]**	***s* [%]**
Minimum	33.87	0.61	5.03
Maximum	48.04	0.73	7.37
Average ($\bar{x}$)	41.68	0.68	6.17
Standard Deviation (*sd*)	4.01	0.04	0.69
Expected Value (μ) at 95% Confidence interval	[37.76, 45.60]	[0.64, 0.72]	[5.53, 6.87]
Standard Deviation ($\sqrt{\sigma^2}$) at 95% Confidence interval	[2.94, 6.32]	[0.03, 0.06]	[0.51, 1.09]

In the case of *TS*, the upper expected value of *TS* is 45.60 MPa + 6.32 MPa = 51.92 MPa, which is quite high, given the maximum value of *TS* (48.04 MPa). The lower expected value of *TS* is 37.76 MPa − 2.94 MPa = 34.82 MPa, which is higher than the minimum value of *TS* (33.87 MPa). In case of *E*, the upper expected value of *E* is 0.72 GPa + 0.06 GPa = 0.78 GPa, which is quite high given the maximum value of *E* (0.73 GPa). The lower expected value of *E* is 0.64GPa − 0.03 GPa = 0.61 GPa, which is equal to the minimum value of *E* (0.61 GPa). In case of *s*, the upper expected value of *s* is 6.87% + 1.09% = 7.96%, which is quite high given the maximum value of *s* (7.4%). The lower expected value of *s* is 5.53% – 0.51% = 5.02%, which is lower than the minimum value of *s* (5.03%). If the confidence interval is increased and the ranges of expected values and variances of the respective material properties are recalculated, the ranges of the expected values for all properties become wider compared to those observed in the data points (Table 3-a). On the other hand, if the confidence interval is decreased, the range of the expected values of the respective material properties becomes narrow compared to those observed in the data points (Table 3-a). Therefore, when the uncertainty of a material property is quantified using an expected value, for a limited number of data points, there is a high possibility to have a strict or loose estimation, depending on the confidence interval.

Even if the above statistical analyses are avoided, and a more straight forward approach is used, similar results are obtained. For example, consider the following approach: uncertainty associated with x is estimated by $\bar{x} \pm sd$ as defined in Equation (2.11). Many authors have adopted this approach, as discussed in the literature review of Chapter 1. Based on this contemplation, the uncertainty associated with *TS* is a range [37.67, 44.69] MPa, a highly truncated range compared to the range derived from its minimum and maximum values, i.e., [33.87, 48.04] MPa. Similarly, the uncertainty associated with *E* is a range [0.64, 0.72] GPa that is highly truncated compared to the range derived from its minimum and maximum values, i.e., [0.61, 0.73] GPa. In addition, the uncertainty associated with *s* is a range [5.51%, 6.89%] that is highly truncated compared to the range derived from its minimum and maximum values, i.e., [5.03%, 7.4%].

3.6.2 Uncertainty Quantification by Probabilistic Method

The probability distribution is one of the widely used approaches to quantify the uncertainty and many authors have quantified the uncertainty of the natural material using this approach. In Chapter 2, the mathematical entities for the Weibull distribution have been discussed. This section describes the Weibull results of the jute yarn. In this study, Weibull distribution is used to quantify the aleatory uncertainty of the mechanical properties (*TS*, *E*, and *s*) of jute yarn. At first, the data (from Table 3-a) of *TS*, *E*, and *s* of jute yarn are rearranged in ascending order and each datum is indexed as shown in Table 3-c.

Table 3-c. Indexing of the *TS*, *E*, and *s* data for jute yarn.

Index Number (x_i)	**$F(x_i)$**	**Properties**		
		TS_i [MPa]	**E_i** [GPa]	**s_i** [%]
1	0.045	33.868916	0.610077	5.03%
2	0.110	37.389838	0.624817	5.37%
3	0.175	37.458465	0.643132	5.38%
4	0.240	38.238556	0.64395	5.70%
5	0.305	39.200715	0.647407	5.71%
6	0.370	39.478021	0.669835	5.78%
7	0.435	41.003195	0.67285	5.82%
8	0.5	42.125016	0.67575	6.27%
9	0.564	42.654415	0.680621	6.40%
10	0.629	44.217407	0.682742	6.41%
11	0.694	44.437294	0.714037	6.51%
12	0.759	44.944255	0.719315	6.74%
13	0.824	45.700576	0.724472	6.88%
14	0.889	46.428834	0.730405	7.11%
15	0.954	48.036641	0.733295	7.37%

3.6.2.1 Parameter Calculation

To determine the Weibull parameters, *m* and *λ*, the cumulative function (2.9) can be written as:

$$F(x) = 1 - e^{-\left(\frac{x}{\lambda}\right)^m}$$

$$1 - F(x) = e^{-\left(\frac{x}{\lambda}\right)^m}$$

$$\frac{1}{1 - F(x)} = e^{\left(\frac{x}{\lambda}\right)^m}$$

$$\ln\left(\frac{1}{1 - F(x)}\right) = \left(\frac{x}{\lambda}\right)^m$$

$$\ln\left(\ln\left(\frac{1}{1-F(x)}\right)\right)=m\ln\left(\frac{x}{\lambda}\right) \tag{3.5}$$

$$\ln\left(\ln\left(\frac{1}{1-F(x)}\right)\right)=m\ln x-m\ln\lambda$$

The plot of $ln\left(ln\left(\frac{1}{1-F(x)}\right)\right)$ vs. $ln(x)$ is shown in Figure 3.8.

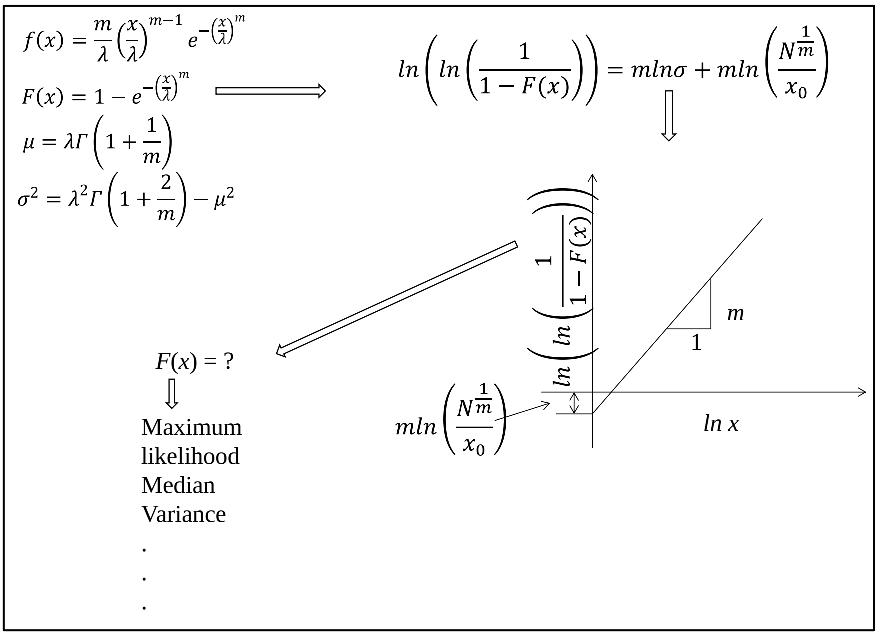
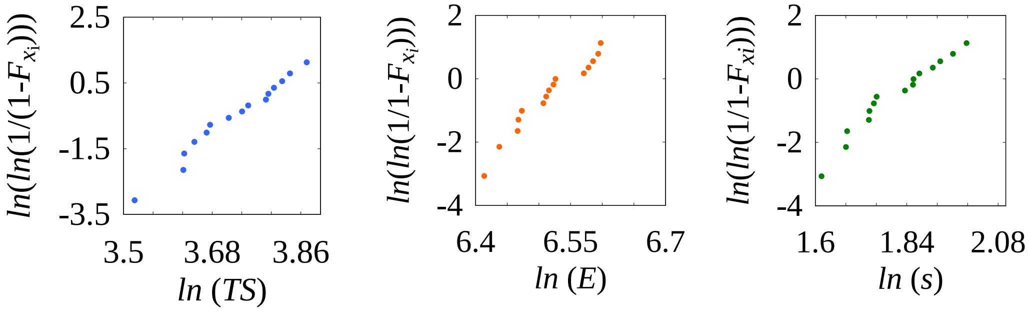


Figure 3.8: Estimation of Weibull parameters.

It is observed that the plot of $ln\left(ln\left(\frac{1}{1-F(x)}\right)\right)$ vs. $ln(x)$ is a straight line where m (the slop of the equation (3.5)) is equal to the shape factor and $c = mln\lambda$. Weibull parameter, m and λ can be estimated from Figure 3.8. Moreover, μ and sd can be determined using equations (2.12) and (2.14), respectively. However, $F(x)$ is still unknown which is calculated using equation (2.8).

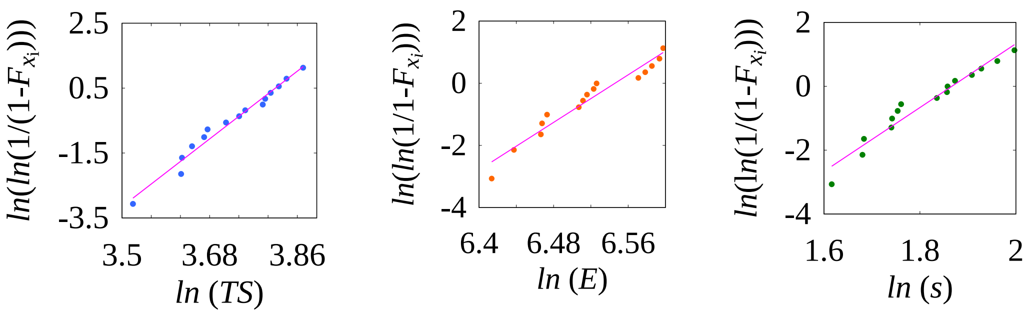
Using the data from Table 3-c, $ln(ln(1/1\text{-}F(x_i)))$ versus $ln(x)$ plots are drawn for TS, E, and s using equations (2.15) and (2.16) and shown in Figure 3.9, where $x_i \in \{TS_i, E_i, s_i\}$ and i = 1,..,15. Where, i is the i^{th} data point, $x(i)$ is the value of the data, and $F(x_i)$ is calculated using equation (2.16). In these plots as shown in Figure 3.9, x-axis represents $ln(x)$ and y-axis represents $ln(ln(1/1-F(x_i)))$. The dots of the curve represent the data of mechanical properties namely, *TS*, *E*, and *s*.



(*a*) (*b*) (*c*)

Figure 3.9: $ln(ln(1/1-F(x_i)))$ - $ln(x)$ plot for (a) *TS*, (b) *E*, and (c) *s* of jute yarn.

The least mean square lines of the plots (Figure 3.9) regarding *TS*, *E*, and *s* for jute yarn are shown in Figure 3.10. Suppose the equation of the plot for *TS* is $y = mx + c$, different parameters can be estimated from this straight line. In this particular case, slop (m) and intersection (c) are estimated and listed in Table 3-d.



(*a*) (*b*) (*c*)

Figure 3.10: Least mean square line plots of (*a*) *TS*, (*b*) *E*, and (*c*) *s* for the jute yarn.

Weibull parameter, shape factor (m) for *TS*, *E*, and *s* are estimated from the value of slop of the straight line for *TS*, *E*, and *s*, respectively as shown in Figure

3.10. The scale factor (λ) can be estimated using $c = m ln\lambda$ equation by placing the value of shape factors of the *TS*, *E*, and *s*. The value of the *m* for *TS* is 11.59, whereas it is 9.99 for *s* and 19.13 for *E* are listed in Table 3-d. As the $m \geq 1$ for *TS*, *E*, and *s*, the shape of the density function will follow the normal distribution pattern or hyperbolic. As the $m > 1$ for *TS*, *E*, and *s*, the failure rate of will increase with the time for respected properties as $s < TS < E$.

Table 3-d. Weibull parameters for *TS*, *E*, and *s* of jute yarn.

Parameters	**Properties**		
	TS	***E***	***s***
Shape factor (*m*)	11.59	19.13	9.997
Scale factor (λ)	43.46	697	6.466
Root mean square	0.97	0.94	0.94

From Table 3-d it is observed that the value of *m* for *E* is high compared to *TS* and *s*. A high value of *m* (for *E* = 19.13) means the low variability in the properties. Therefore, the *E* is varied in the low range compared to other two properties (*TS* and *s*) as shown in Figure 3.10. The value of the root means square is 97.75% for the tensile strength data, which indicates that the tensile strength property is linearly good fitted with the Weibull distribution (as shown in Table 3-d) compared to other parameters (*E* and *s*) for jute yarn.

The deduced Weibull functions of *TS*, *E*, and *s* from equation (2.4) for jute yarn denoted by *f*(*TS*), *f*(*E*), and *f*(*s*), respectively, as given by equations (3.7), (3.8), and (3.9). The Weibull distribution function for *TS*, *E*, and *s* are shown in Figure 3.11.

$$f_w(TS) = 1.1984 \times 10^{-18}[TS]^{10.59} e^{-[TS]^{11.59}} \tag{3.7}$$

$$f_w(E) = 7.77449 \times 10^{-54}[E]^{18.132} e^{-[E]^{19.182}} \tag{3.8}$$

$$f_w(s) = 7.86945 \times 10^{-8}[s]^{8.997} e^{-[s]^{9.997}} \tag{3.9}$$

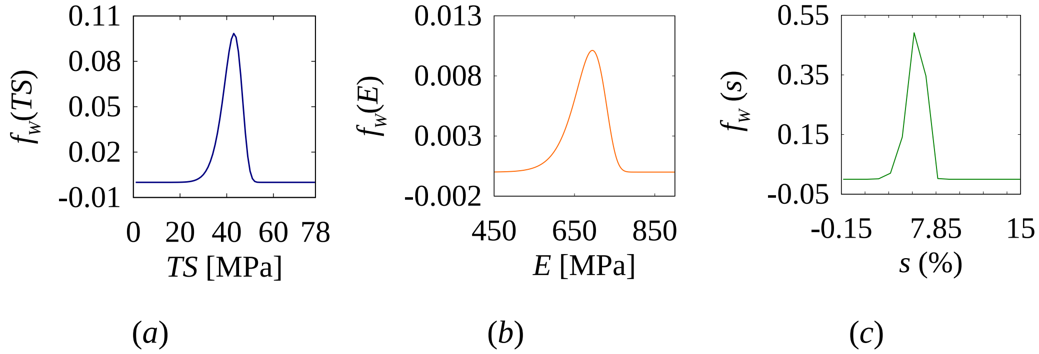


Figure 3.11: Weibull density functions of the (*a*) *TS*, (*b*) *E*, and (*c*) *s* for jute yarn.

Finally, μ and *sd* are determined using equation (2.6) and (2.7). Table 3-e summarizes the μ, *sd*, and σ^2 of *TS*, *E*, and *s* calculated using the data points shown in Table 3-b.

Table 3-e. Probabilistic uncertainties (Weibull distribution) of jute yarn.

Parameters	**Properties**		
	***TS* [MPa]**	***E* [GPa]**	***s* [%]**
Expected value (μ)	41.60	0.678	6.151
Standard deviation (*sd*)	4.35	0.43	0.704
μ at 95%confidence interval	[25.07, 69.00]	[0.408, 1.123]	[3.7, 10.20]
sd at 95%confidence interval	[2.62, 7.21]	[0.025, 0.071]	[0.44, 1.22]

In case of *TS* for jute yarn the upper expected value of *TS* is 76.21 MPa (69.00 MPa + 7.21 MPa). The lower expected value of *TS* is 22.45 MPa (25.07 MPa - 2.2.62 MPa). In addition, consider the case of *E* for jute yarn. The upper expected value of *E* is 1.19 GPa (1.123GPa + 0.71GPa). The lower expected value of *E* is 0.38 GPa (0.408 GPa - 0.025GPa). Finally, consider the case of *s*. The upper expected value of *s* is 11.43% (10.20% +1.22%). The lower expected value of *s* is 3.26 %(3.7% - 0.44%). This means when Weibull distribution is used to quantify the uncertainty of a material property from a limited number of data points, the ranges of the estimated value become wider.

3.6.3 Uncertainty Quantification by Possibilistic Method

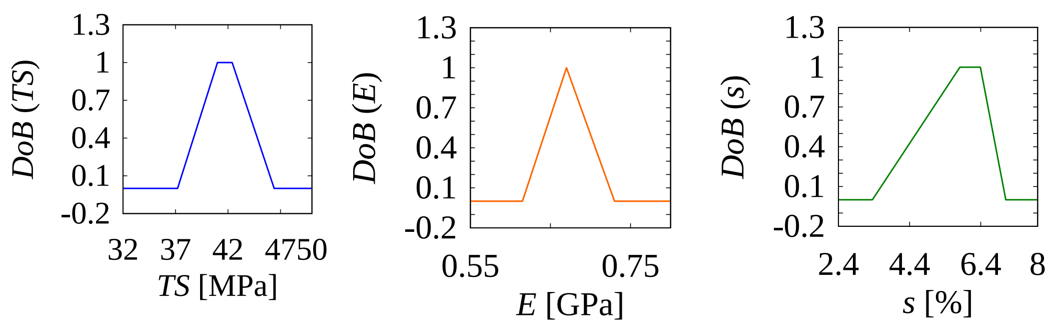
Apart from the statistical and probabilistic method to quantify the uncertainty associated with a material property of the jute yarn, other alternative is considered. As the distribution is not known, the possibility distribution is used to quantify the data of uncertainty associated with the material properties of Table 3-a.

In particular, a probability-possibility transformation is used as defined in (Ullah and Shamsuzzaman, 2013) to deduce three fuzzy numbers (or possibility distributions) which represent the uncertainty associated with *TS*, *E*, and *s*. The deduced membership functions of *TS*, *E*, and *s* from equation (2.17) and (2.18) for jute yarn are denoted by *DoB*(*TS*), *DoB*(*E*), and *DoB*(*s*), respectively. The possibility distribution functions of *TS*, *E*, and *s* for jute yarn are given by equations (3.10), (3.11), and (3.12), respectively.

$$DoB(TS) = \max\left(0, \min\left(1, \frac{TS - 37.2}{41 - 37.2}, \frac{46.4 - TS}{46.4 - 42.4}\right)\right) \tag{3.10}$$

$$DoB(E) = \max\left(0, \min\left(1, \frac{E - 0.615}{0.67 - 0.615}, \frac{0.73 - E}{0.73 - 0.67}\right)\right) \tag{3.11}$$

$$DoB(s) = \max\left(0, \min\left(1, \frac{s - 3.355}{5.815 - 3.355}, \frac{7.105 - s}{7.105 - 6.39}\right)\right) \tag{3.12}$$



(*a*) (*b*) (*c*)

Figure 3.12: Possibility distribution of (*a*) *TS*, (*b*) *E*, and (*c*) *s* of jute yarns.

From Figure 3.13 it is observed that *DoB*(*E*) takes the shape of a triangular fuzzy number, whereas the other two (*DoB*(*TS*) and *DoB*(*s*)) take the shape of trapezoidal fuzzy numbers. Using the membership functions defined in Equations (3.10) to (3.12), the uncertainty associated with the *TS*, *E*, and *s* of jute yarn can be estimated. In this case an operation called alpha cut (Ullah, 2016), is used i.e., the range corresponding to *DoB*(.) = alpha $\in$ [0, 1]. Some of the important alpha cuts are summarized in Table 3-f.

Table 3-f. Possibilistic parameters of jute yarn.

Alpha cut	**Properties**		
	***TS* [MPa]**	***E* [GPa]**	***s* [%]**
0	[37.2, 46.4]	[0.615, 0.73]	[3.355, 7.105]
0.5	[39.1, 44.4]	[0.6425, 0.7]	[5.585, 6.7475]
1	[41, 42.4]	0.67	[5.815, 6.39]
Expected values (centroid method)	41.75	0.67	5.67

The alpha cut at *DoB*(.) = 0 corresponds to the largest range that is called the support of a possibility distribution. Thus, the supports of *TS*, *E*, and *s* are [37.2, 46.4] MPa, [0.615, 0.73] GPa, and [3.355%, 7.105%], respectively. Note that a support does not contain the minimum and maximum values of the data points. This means that the deduced possibility distribution considers the extreme data points (minimum and maximum values) as outliers and excludes them from the uncertainty quantification. Therefore, minimum and maximum values are automatically truncated. The most possible value(s) correspond(s) to *DoB*(.) = 1, which is a point for *E* and are two ranges for the other two properties, as shown in Table 3-f. The ranges corresponding to *DoB*(.) = 0.5 are the logically consistent ranges. This means that any propositions made taking a range *DoB*(.) > 0.5 are true more than they are false. For example, the proposition is: "*TS* of jute yarn is 39.5 MPa." The truth-value of this proposition is 0.605263158 =

DoB(*TS* = 39.5) [refer to Equation (3.10)]. This means it is somewhat true that the *TS* of jute yarn is 39.5 MPa. Therefore, this value of *TS* can be considered while designing a product using jute yarns. Consider another proposition: "The *TS* of jute yarn is 38 MPa." The truth-value of this proposition is 0.210526316 = *DoB*(*TS* = 38). This means that it is somewhat false that the *TS* of jute yarn is 38 MPa. Therefore, this value of *TS* can be avoided, when designing a product using jute yarns. In synopsis, when a design range is required for a material property of jute yarns, the range corresponding to *DoB*(.) = 0.5 can be considered, because the points included in this range are true more often than they are false. In addition, a possibility distribution provides the μ of the underlying parameter. In this case, the centroid method (Ullah, 2016) is used. Table 3-f also shows the centroid method-based expected values of *TS*, *E*, and *s* calculated from the respective possibility distributions. The expected value for *TS* and *E* lies in the core region whereas it is in the alpha-cut region in case of *s*. When no other details are given or available, one can consider these expected values for design and manufacturing decisions. Note that an expected value may or may not be a point in the core of the possibility distribution. Here, a core means the range or point corresponding to *DoB*(.) = 1 (i.e., the most possible value (*s*)). The μ of *E* is same as its core. This is not the case for the other two properties (*TS* and *s*) as shown in Table 3-f. Moreover, the μ of *s* is not included in its core which is not the case for the other two properties (*TS* and *E*).

3.6.4 Comparison among the Different Methods

This section presents a comparison among the results of the statistical approach, probabilistic approach, and possibilistic approach. There are different approaches to quantify the uncertainty, namely, statistics, probability (Dempster, 1968), possibility (Zadeh, 1978; Dubois and Prade, 1988), imprecise probability (Walley, 1991; Walley 2000), evidence (Shafer, 1976; Klir, 1990), and random interval (Joslyn and Booker, 2004). As there are many methods to quantify the uncertainty, which method should be selected? In this study, uncertainty is quantified using three mostly used approaches and investigated a reliable approach to quantify the uncertainty in the data. And it is concluded that, possibility distribution is better method and reliable compared to other methods. The quantified data using three different methods are listed in Table 3-g.

Table 3-g. Probabilistic and possibilistic uncertainties of jute yarn.

Properties	**Parameters**	**Statistical Method**	**Distribution**	
			Weibull	**Possibility**
	Expected value (μ)	[37.76, 45.60]	41.60	41.75
TS [MPa]	Standard deviation (*sd*)	4.01	4.35	
	μ at 95% confidence level/ Alpha cut	[2.94, 6.32]	[25.07, 69.00]	[39.1, 44.4]
	Expected Value (μ)	[0.64, 0.72]	41.60	41.75
E [GPa]	Standard deviation (*sd*)	0.04	4.35	
	μ at 95% confidence level/ Alpha cut	[0.03, 0.06]	[25.07, 69.00]	[39.1, 44.4]
	Expected Value (μ)	[5.53, 6.87]	0.67	0.67
s [%]	Standard deviation (*sd*)	0.69	0.43	
	μ at 95% confidence level/ Alpha cut	[0.51, 1.09]	[0.408, 1.123]	[0.642, 0.7]

In case of statistical approach, the expected values for the *TS*, *E*, and *s* are in the form of range [37.76, 45.60], [0.64, 0.72], and [5.53, 6.87], respectively as exposed in Table 3-g. For example, the expected value of the *TS*, *E*, and *s* of jute yarn are [37.76, 45.6], [0.64, 0.72] and [5.53, 6.87], respectively as shown in Table 3-g. As previously explained when there is uncertainty, statistical analyses estimate both highly pessimistic and optimistic values of *TS*, *E*, and *s* of jute yarns. That means some data are highly emphasized however, all data are not equally emphasized. When the uncertainty is represented by the statistical method the confidence interval is considered as range. This range may or may not be included in the experimental data points. Though, statistical approach is easy and familiar, one cannot rely on the statistical approach for such types of uncertainty quantification. Moreover, statistical approach deals with finite data, however, for infinite series (general assumption) the probability distribution is an alternative option. Last but not least, the true behavior of a variable is described by its infinite statistics, finite statistics describe only the behavior of the finite data set (Figliola and Beasley, 2000).

Apart from the statistical approaches to quantify the uncertainty associated with material properties, an alternative method (Weibull distribution) is considered. Statistical method estimates the expected value as a range. However, the Weibull and possibility distributions estimate a specific value for a certain group of uncertainty in the data of the *TS*, *E*, and *s*. In addition, probability distribution estimated the expected value of the *TS* is 41.60 MPa, whereas the possibility estimated the most possible value (expected value) of *TS* is 41.75 MPa for of jute yarn. Both of the quantification approaches give the same result for *TS* and *E* except *s*. Therefore, it can be informed that possibility distribution can quantify the data as probabilistic approach. Thus, one can rely on the possibility distribution to quantify the uncertainty. Therefore, it can be inferred that probabilistic and possibilistic approaches are better compared to the statistical method.

Generally, the Weibull distribution is used to observe the life-cycle of the product. The Weibull distribution is one of the special forms of the binomial distribution, and follows the binomial theorem. Nevertheless, most of the researchers use this distribution to quantify the variability for natural materials. However, the Weibull distribution may not be the appropriate approach to handle the uncertainty of the natural material. Because before quantifying the uncertainty using probability distribution, one needs to know that the data are following a distribution. In this study, before calculating the Weibull distribution of jute yarn, it is assumed that data of jute yarn properties are following the Weibull distribution. In case of the jute fiber from jute growth to fiber collection, the growth and collection conditions are unknown because they grow naturally and rotted, washed, dried and finally collected by farmers. Thus, it is difficult to say which distribution they are following. In engineering practice, it is truly impossible to control the operation conditions of the growth in the field naturally, that means the distribution is not known. Therefore, it is difficult to consider that jute is following a specific distribution. Thus, Weibull distribution above all probability distributions is not appropriate for such types of uncertainties quantification.

Besides, when the distribution is unknown, it is required to check through different categories of probability distribution for curve fitting, which is time-consuming. As possibility distribution deal with the uncertainty, in that sense possibility distribution is a better choice to calculate the uncertainty compared to other methods.

In addition, a graphical representation is better for human understanding compared to the average value. However, for a small range of the data, it is tough to calculate and draw the probability distribution. For drawing histogram of the data, it is needed to obtain a minimum number of intervals (K) (Figliola and Beasley, 2000; Ashby, 2007) in the range (N) of data shown by equation (3.13).

$$K = 1.87(N-1)^{0.40} + 1 \tag{3.13}$$

If the number of data and interval are large, the histogram will be better. Therefore, large number data (N) are better or required for probabilistic approach. On the other hand, possibility distribution can be used for a small number of data as well as big data. This type of evidence is obtained in case of Weibull distribution as there is a limited number of data the error estimation is higher as shown in Table 3-h. From Table 3-h, it is clear that the error estimation is high for *TS* (10.74), *E* (17.4), and *s* (1.57) of the jute yarn.

Table 3-h. Error estimation of quantified data for mechanical properties of jute yarn.

Parameters	**Properties**		
	***TS* [MPa]**	***E* [GPa]**	***s*[%]**
Expected value (μ)	41.60	0.677	6.15
Standard Error	10.74	17.4	1.58
Standard deviation (*sd*)	4.35	0.043	0.74
Standard Error	1.123	0.011	0.19

As the distribution jute yarn is unknown, possibility distribution may be better option to quantify the uncertainty. Moreover, this is one of the better ways to handle the uncertainty in the natural material (jute fiber). In possibility distribution, it is not mandatory to know that they are following any distribution or not. Moreover, a small range of data is able to give us a distribution or conclusion in case of possible distribution. Moreover, since the number of data points was small (15 data points), error estimation behind the probability distribution (i.e., Weibull distribution) becomes high. In fact, when it is unknown which distribution should be used to quantify the uncertainty, or even when there is insufficient data to deduce a probability distribution, the answer is to use a probability distribution-neutral representation (Ullah and Shamsuzzaman, 2013) of uncertainty. This was quite relevant to the case in this study, because of the limited number of data points (15 data points for each property, as shown in Table 3-c). As such, the concept of possibility (Zadeh, 1999) or possibility distribution (Dubois, et al., 2004) could be used. A possibility distribution is popularly referred as a fuzzy number (see (Ullah, 2016) for a definition). A possibility distribution entails a family of probability distributions (e.g., a triangular fuzzy number can entail a set of unimodal probability distributions, e.g., normal distribution, triangular distribution, and uniform distribution, (Ullah and Shamsuzzaman, 2013; Masson and Denœux, 2006)). In addition, a possibility distribution can also be deduced from a limited number of data points

(Ullah and Shamsuzzaman, 2013; Masson and Denœux, 2006). Such a distribution also provides a reliable representation of the uncertainty, which is compatible with the general concept of uncertainty (Mauris, et al., 2001). Therefore, it can be suggested that possibility distribution is better to quantify the uncertainty associated with the natural material properties.

Furthermore, in case of possibilistic approach, the upper limits of the alpha cut ranges are greater than the expected value of the respective material properties whereas the lower limits of the logically consistent ranges are smaller than the values of the respective material properties. This means that the lower limit of a logically consistent range is the most conservative estimation of the underlying material property. Therefore, one may consider the lower limit of a logically consistent range to be the design limit of the material property.

3.7 Conclusion

The variability or uncertainty associated with the properties of a natural material must be known beforehand to ensure the reliability, durability, and sustainability of any eco-product. Therefore, the uncertainty associated with the material properties of a natural material called jute yarn has been studied. This study clearly identified the uncertainty in the mechanical properties, *TS*, *E*, and *s* of jute yarns. The variability has been quantified using the conventional approach (average, standard deviation, and skewness), probabilistic approach (particularly Weibull distribution), and the possibilistic approach. From the statistical method and probability distribution, the μ and *sd* are determined, and the most possible and μ of *TS*, *E*, and *s* are determined from the possibility distributions. The logically consistent ranges of the *TS*, *E*, and *s* have also been determined.

In case of probability approach, one needs to consider the distribution at the beginning. However, it is not an important issue in case of possibility distribution. Using the possibility distribution one can easily set the lower limit as a design limit at 50% alpha-cut because at this alpha cut the truth value is partially true and partially false. As the possibility distribution (especially trapezoidal) has sharp value, one can easily find the range from the distribution without checking the log book.

Possibility distribution is more reliable than other quantification methods. Because, in case of possibility distribution, initially, it is not necessary to know the distribution, that means there is aleatory uncertainty. Furthermore, possibility distribution can handle a small amount of data. As both of these issues are available in natural jute material possibility distributions is used and recommend this distribution to quantify the aleatory uncertainty.

The lower range of the alpha cut is partially true and partially false, as well as below the alpha cut, the membership functions are more false than true. Therefore, it can be inferred that, the lower ranges of *TS* and *E* for jute yarn can be used as the design limits for a jute product.

Chapter 4: Decision Model to Select a Material under Uncertainty

The materials are considered as key factor in managing the complexity while developing sustainable engineering products. Thus, material selection is very critical issue. The objective of this chapter is to develop a decision model under uncertainty and observe its effectiveness. This chapter is based on the Shahinur et al. (2017). The remainder of this chapter has been organized as follows: Section 4.1 describes the uncertainties (epistemic) in the product development using *MI*. Section 4.2 describes the proposed decision model that consists of four major steps, namely, decision formulation, information gathering, compliance calculation, and aggregation. Section 4.3 presents the results of a material selection problem, using the proposed decision model where a large number of alloys of Aluminum (Al), Magnesium (Mg), and Titanium (Ti) are evaluated. Due to lack of information of the natural material, jute, in this particular case, a metallic material is selected from the three metallic materials. Finally, Section 4.4 provides the concluding remarks on this chapter.

4.1 Epistemic Uncertainty in Product Development

Before designing a product, it is a prerequisite to select materials, to ensure product functionality, quality, durability, and reliability. Suppose a designer wants to select a material for a stiff, light, strong and sustainable vehicle. While assessing the appropriateness of a set of materials (using *MI*) for making the parts of a product (vehicle), it is likely to be the case that the designer encounters a certain degree of epistemic uncertainty, as schematically illustrated in Figure 4.1. There is uncertainty in *MI* calculation because *MI* itself is uncertain for a given product. The material properties which are required to calculate the *MI* is uncertain, sustainable properties are not included in *MI*, and *MI* cannot guarantee a single selection of a material.

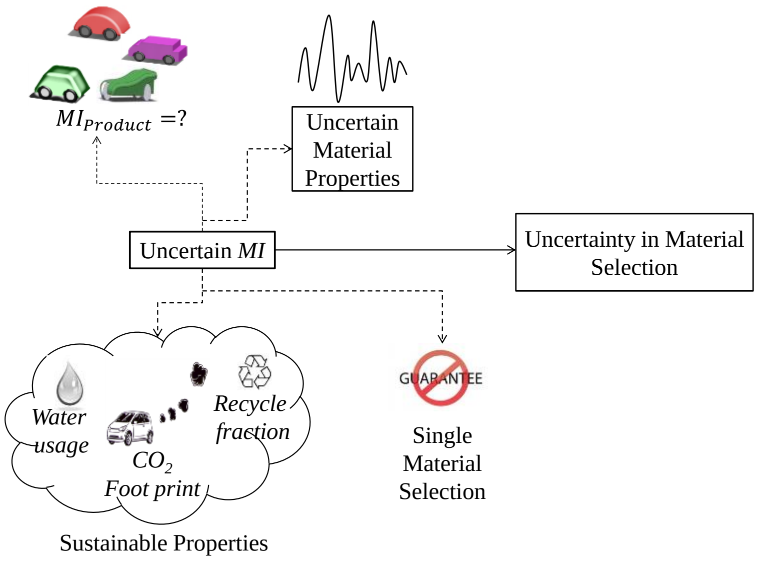


Figure 4.1: A scenario of epistemic uncertainty regarding a material selection of a product vehicle.

The shape and size (*MI)* of the car is customer dependent or it is unknown. It is worth mentioning that, the outline of a vehicle body depends on customer requirements (Sharif Ullah et al., 2016) and the outline is refined to get the final configuration using numerous engineering analyses where the materials must be known beforehand (Omar, 2011). Initially, the objectives or requirements (maximization or minimization) are known. Based on the customer requirement, if a designer prefers to maximize the structural integrity, it may create a conflict with the environmental impact. According to the general requirements, for above mentioned example, objectives can be expressed for strong material by maximizing *TS*, for light material minimizing ρ, for stiff material maximizing *E*. Sustainable products means CO_2 *Foot print* should be low, *Water usage* should be low and *Recycle fraction* should be high (say). Based on the requirement for a vehicle, it can be informed that, to select a material, designer needs to incorporate the sustainable properties in material selection procedure

(Muhammet Gul, 2017). Furthermore, the sustainable properties are not included in the material selection process *MI*. Based on above contemplation it can be concluded that, there is uncertainties in selection of materials for a vehicle using conventional graphical method *MI*.

If a designer prefers to select the optimal materials for making the body of a vehicle (say) at a very early stage of the design process, the designer needs to handle conflicting objectives. In this study the conflicting objective is represented by possibility objective function (marked as 1). Uncertainty in the material properties is needed to be quantified using three different approaches (marked as 2 to 4). Using compliance analysis (marked as 5) between the objective function and the quantified data of material properties, a material for vehicle is selected (marked as 7). The proposed model shown in Figure 4.2 is designed to handle epistemic uncertainty and conflicting objectives behind the objective of material selection.

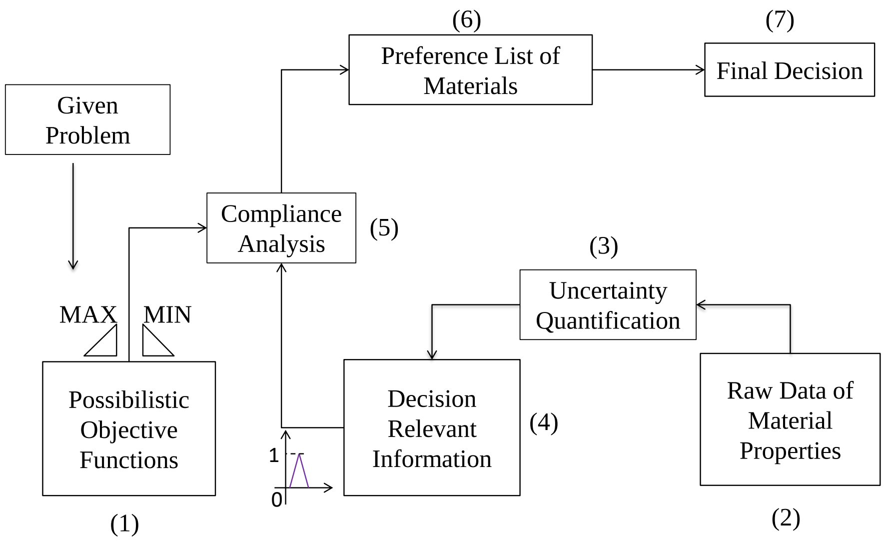


Figure 4.2: Schematic diagram of the decision making procedure under uncertainty.

4.2 Mathematical Description of the Model

This section describes the mathematical description of the proposed decision model to select the optimal alternatives. The proposed decision model employs the mathematical formulations described in Chapter 2 and helps users to make a decision under epistemic uncertainty, as described in Chapter 1. Figure 4.3 schematically illustrates the proposed decision model and its relationship with the decision-relevant (analytic and/or empirical) knowledge.

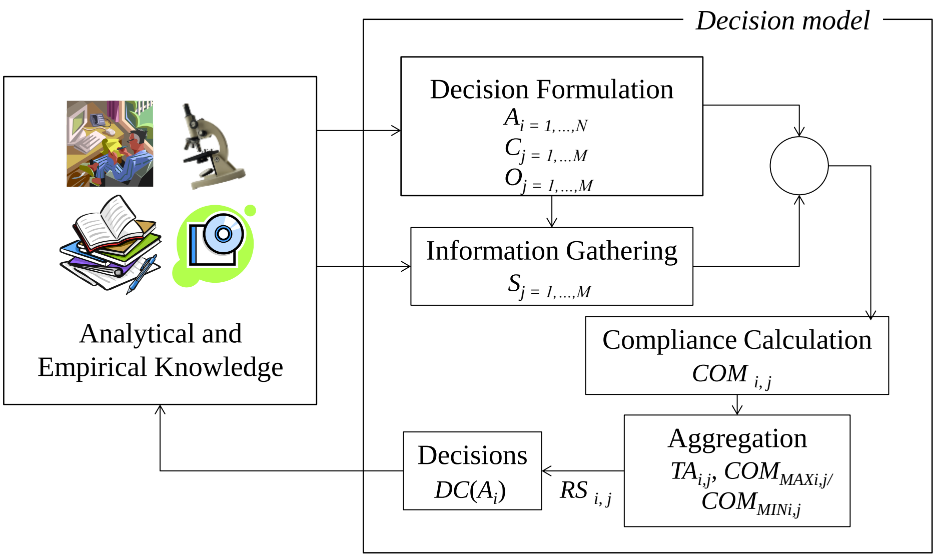


Figure 4.3: Proposed decision model.

As seen from Figure 4.3, the decision model consists of following five modules as listed below:

a) *Formulation module,*
b) *Information-Gathering module,*
c) *Compliance-Calculation module,*
d) *Aggregation module,* and
e) *Decision module*

The Formulation and Information-Gathering modules work in coordination with the decision-relevant knowledge. This means that the available knowledge regarding a given decision problem plays a vital role while performing the activities of Formulation and Information-Gathering modules. The output of the Formulation module serves as an input for the Information-Gathering module. The combined output of Formulation and Information-Gathering modules serves as the input for the Compliance-Calculation module. The output of the Compliance-Calculation module is the degrees of compliances for all alternatives for each criterion. Once the Compliance-Calculation module completes its function, the aggregation module makes a tradeoff among the compliances of some of the selected criteria based on the user-defined importance in order to rank the alternatives. The ranks of the alternatives help to make an informed decision. The decision made can be fed into the existing body of knowledge to enrich it, as schematically illustrated in Figure 4.3.

However, the above description of the proposed decision model is rather informal. A relatively formal description of the decision model is given, as follows:

4.2.1 Formulation Module

Let, $A_i = \{A_1, ..., A_N\}$ be the set of N different alternatives, $C_j = \{C_1,...,C_M\}$ be the set of M different criteria, and $O_j = \{O_1,...,O_M\}$ be the set of the states of the members in C for Formulation module, where, $\forall O_j \in$ {maximization, minimization}, $j = 1,...,M$.

The purpose of the Formulation module is to define A, C, and O. In order to define A, C, and O, the Formulation module relies on the analytical and empirical knowledge underlying the decision problem. It is illustrated in Figure 4.3. The Formulation module decides the natures of the objective functions for the criteria in C. Let $OB = \{OB_1,...,OB_M\}$ be the set of the objective functions of the criteria defined in C. As such, if O_j = maximization, then $OB_j = MAX_j$; otherwise $OB_j = MIN_j$.

4.2.2 Information Gathering Module

Information-Gathering module collects all sorts of data or information needed for determining the degree of compliances. It gathers the information to define the supports of the objective functions. Let, $S_j = [a_j, b_j]$ be the support of OB_j, $\forall j \in \{1,...,M\}$. Thus, $S_j = [a_j, b_j]$, $\forall j \in \{1,...,M\}$ can be a *deterministic*, *local*, *semi-global*, or *global* supports as described in Section 4.2.2.1. The other function of the Information-Gathering module is to gather the decision-relevant information on each alternative defined in A for all criteria defined in C. Here, a piece of decision-relevant information denoted as $DRI_{i,j}$ can be a set of numerical values $\{d^k_{i,j} \mid k = 1,2,...\}$, a set of real intervals $\{P^l_{i,j} \mid l = 1,2,...\}$, a set of triangular fuzzy numbers $\{D^r_{i,j} \mid r = 1,2,...\}$, and any combination of these. This implies that $DRI_{i,j} \subseteq \{DRI^k_{i,j}, DRI^l_{i,j}, DRI^r_{i,j}\}$, where $DRI^k_{i,j} = \{d^k_{i,j} \mid k = 1,2,...\}$, $DRI^l_{i,j} = \{ P^l_{i,j} \mid l = 1,2,...\}$, and $DRI^r_{i,j} = \{ D^r_{i,j} \mid r = 1,2,...\}$.

4.2.2.1 Determining the Supports

To define the maximization or minimization fuzzy number denoted as *MAX* or *MIN*, as described in the Chapter 2 Subsection 2.3.2, the support $[a, b]$ must be known beforehand. Despite the remarkable progress of fuzzy-number-based knowledge-based systems, it remains true that no unique, best-of-the-world solution exists for setting a support of a fuzzy number unless it is induced using a set of numerical data. Keeping this in mind, this section describes four types of supports, for defining *MAX* or *MIN*.

a) *Deterministic*,
b) *Local*,
c) *Semi-global*, and
d) *Global*

These supports are described below using numerical examples.

First, the support called deterministic support has been considered. Deterministic support means a support that is known to all without any controversy. In case of recycle fraction, it is customary to express the recycle fraction using a number taken from the interval [0, 1]. This means that if one defines *MAX* or *MIN* for maximizing or minimizing the recycle fraction, respectively, then the support [a, b] is equal to [0, 1], i.e., [a, b] = [0, 1]. The same argument holds for numerous physical quantities. It is worth mentioning that the compliances underlie a deterministic support that is equal to [0, 1]. If one is interested in seeing whether the compliances of an alternative to a set of criteria are being maximized, s/he obviously chooses an *MAX* for compliance maximization. In this case, the *MAX* underlies a support equal to [0, 1] because the values of the compliances always lie in the interval [0, 1] no matter the type of compliance (crisp, range, and fuzzy), as described in the Chapter 2 Section 2.4.

On the other hand, the local, semi-global, and global supports are somewhat subjective, and, thereby, depending on the user's judgment or the available numerical data. For example, consider the following scenario. As there are seven classes of engineering materials and assume that one is interested in maximizing or minimizing the ρ of the material. According to (Ashby, 2005, p.520-521) the ρ (Mg/m^3) of wood and wooden products, foams, rubbers, polymers, composites, ceramics, and metals and alloys lies in the interval [0.6, 1.05], [0.016, 0.47], [0.92, 0.955], [0.89, 1.58], [1.5, 2.9], [1.9, 15.9], and [1.74, 8.94], respectively.

Now, if one considers a class of materials, e.g., metals and alloys, as the alternatives, and wants to evaluate the materials in the class using density as one of the criteria, then an interval [1.74, 8.94], or even a larger one (e.g., [1, 10]), becomes the support of *MAX* or *MIN* because the suggested support subsumes the intervals representing the density of all materials belonging to the considered class according to the supplied data. This kind of support is called the local support in the sense the support focuses alternatives that belong to a single class.

On the other hand, if one considers two classes of materials, e.g., polymers and ceramics, as the alternatives, and wants to evaluate the materials of both classes using density as one of the criteria, then an interval [0.89, 15.9], or even a larger one (e.g., [0.5, 20]), becomes the support of *MAX* or *MIN* because the suggested support subsumes [0.89, 1.58] and [1.9, 15.9], i.e., the intervals underlying the two classes of materials considered in terms of the criterion called density according to the supplied information. This kind of support is called the semi-local support.

Moreover, if one considers all materials as alternatives, and wants to evaluate them using density as one of the criteria, then an interval [0.016, 15.9], or even a larger one (e.g., [0.01, 20]) becomes the support of *MAX* or *MIN*. The reason is that the suggested support includes all intervals for all the materials considered in terms of the criterion called density according to the supplied information. This kind of support is called the global support.

4.2.3 Compliance Calculation Module

The Compliance-Calculation module calculates the degree of compliance for each combination of alternative and criterion. A degree of compliance denoted as $COM_{i,j} \in [0, 1]$ is calculated by inputting each member of $DRI_{i,j}$ into CC_{MAX}, CC_{MIN}, RC_{MAX}, RC_{MIN}, TC_{MAX}, or TC_{MIN}, as defined in Chapter 2. If $DRI^{z}_{i,j}$ is a member of $DRI_{i,j}$, then the corresponding degree of compliance can be represented as $COM^{z}_{i,j}$

4.2.4 Aggregation Module

Finally, the Aggregation module aggregates the compliances of an alternative for some selected criteria in order to rank the alternatives so that one can make an informed decision. Let, $Y_{i,j} = \{COM^{z}_{i,j} | z = 1, 2, ...\}$ be the set of compliances of the *i*-th alternative with respect to *j*-th criterion. Using $Y_{i,j} \in [0, 1]$ as a triangular fuzzy number denoted as $TA_{i,j}$ can be induced. The induction process is described in Chapter 2 (Section 2.3). Let the support and core of the induced triangular fuzzy number $TA_{i,j}$ be $[t_{1ij}, t_{3ij}]$ and t_{2ij}, respectively. Since the values of the compliance lie in the interval [0, 1] and the compliance must be maximized, a special maximization fuzzy number denoted as COM_{MAX} can be considered where the support and core are $[a, b] = [0, 1]$ and $b = 1$, respectively. As a result, the compliance of $TA_{i,j}$ with respect to COM_{MAX} is the ranking score of the i-th alternative with respect to the j-th criterion denoted as $RS_{i,j}$. Recall the procedure of determining the compliance of a triangular fuzzy number with respect to a maximization fuzzy number described in Chapter 2 (as shown in Figure 2.11 and equations (2.22) to (2.25). This procedure is valid for $TA_{i,j}$ and COM_{MX}, too. The interaction between $TA_{i,j}$ and COM_{MAX} is schematically illustrated in Figure 4.4, which is a similar case illustrated in Figure 2.11.

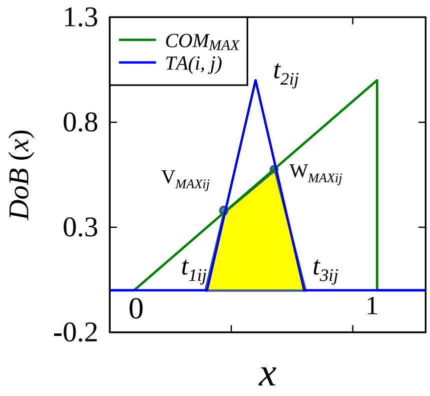


Figure 4.4: Determining the ranking of an alternative based on a criterion.

In Figure 4.4, the points of intersections of $TA_{i,j}$ and COM_{MAX} are $V_{MAXij}(V_{MAXxij}, V_{MAXyij})$ and $W_{MAXij}(W_{MAXxij}, W_{MAXyij})$. This yields ranking score denoted by $RS_{i,j}$ is expressed by following equation (4.1).

$$RS_{i,j} = \frac{V_{MAXyij}(W_{MAXxij} - t_1) + W_{MAXyij}(t_3 - V_{MAXxij})}{t_{3ij} - t_{1ij}} \tag{4.1}$$

This relationship is found by substituting 0, 1, t_{1ij}, t_{2ij}, t_{3ij}, V_{MAXxij}, V_{MAXyij}, W_{MAXxij}, and W_{MAXyij} for a, b, t_1, t_2, t_3, V_{MAXx}, V_{MAXy}, W_{MAXx}, and W_{MAXy}, respectively, in the equations (2.22) and (2.23). The V_{MAXx}, V_{MAXy}, W_{MAXx}, and W_{MAXy} can be expressed by equation (4.2) and (4.3).

$$V_{MAX_{xij}} = V_{MAX_{yij}} = \frac{t_{1ij}}{1-\left(t_{2ij}-t_{1ij}\right)} \tag{4.2}$$

$$W_{MAX_{xij}} = W_{MAX_{yij}} = \frac{t_{3ij}}{1+\left(t_{3ij}-t_{2ij}\right)} \tag{4.3}$$

Therefore, the ranking score $RS_{i,j}$ defined in equation (4.1) is calculated after calculating V_{MAXxij}, V_{MAXyij}, W_{MAXxij}, and W_{MAXyij} using equations (4.2) and (4.3), respectively.

4.2.5 Decision Module

In the decision module, to calculate the decision score, the ranking scores of an alternative A_i for all criteria can be added using the weighted importance. This yields a decision score denoted as DC (A_i), is given by equation (4.4) as follows:

$$DC(A_i) = \sum_{j=1}^{M} w_j RS_{ij} \qquad \text{so that} \quad w_j = \frac{IMP_j}{\sum_{j=1}^{M} IMP_j} \tag{4.4}$$

In equation (4.4), IMP_j is the importance of j-th criterion that is an integer in the scale 0 to 10, i.e., $IMP_j \in \{0,...,10\}$, $\forall j \in \{1,...,M\}$. It is clear from equation (4.4) that when IMP_j is greater, the importance of the criterion will become higher. Therefore, w_j represents the normalized weight of the j-th criterion, $\forall w_j \in [0, 1]$. Note that each IMP_j is assigned subjectively by the decision maker(s).

4.3 Implication of the Model: A Case Study

In this section, in order to show the effectiveness of the model, a material selection case study is presented, where the complete information about alternative materials and design specifications are not known. First, consider the Formulation module. Here, three alternatives ($\{A_i | i = 1,\ldots,3\}$) are considered as listed in Table 4-a based on the general knowledge regarding materials used for making vehicle parts (McDowell et al., 2010; Omar, 2011; Mayyas et al., 2012a-b; Poulikidou et al., 2015).

Table 4-a. List of Alternatives (A_i| i = 1,…,3).

A_1 = Aluminum Alloys (Al)	**A_2 = Magnesium Alloys (Mg)**	**A_3 = Titanium Alloys (Ti)**
In total 197 types of Al alloys are considered in A_1	In total 30 types of Mg alloys are considered in A_2	In total 45 types of Ti alloys are considered in A_3

The first alternative is a set of Al alloys that consists of 197 types of Aluminum-based alloys. The second alternative is set of Mg alloys that consist of 30 types of magnesium-based alloys. The last alternative is set of Ti alloys that consist of 45 types of titanium-based alloys. The number of alloys; 197, 30, and 45 of Al, Mg, and Ti, respectively, are considered based on the information available in a material database (CES Selector, Ref.).

The *MI* is used to select materials for engineering components (Ashby, 2005, p.509-512). The *MI* depends on the nature of a component (e.g., tie, shaft, beam, column, plate, and panel) and the objective (e.g., stiffness-limited design at minimum mass and strength-limited design at minimum mass). In these *MI*, the material properties such as ρ, *TS*, and *E* are involved. Therefore, when the nature of the component is unknown (shown in Figure 4.1), at least, three material properties (ρ, *TS*, and *E*) are needed to be considered to ensure the structural integrity of the component. In addition, according to the *MI* (Ashby, 2005, p.509-512) to achieve a given objective, the ρ must be minimized whereas the *TS* and *E* must be maximized. On the other hand, the environmental impact of a vehicle can be minimized by reducing its weight. Therefore, minimization of ρ helps to reduce the environmental impact, too. Moreover, to reduce the usages of material, i.e., to increase the material efficiency (Allwood, et al., 2011; Ullah et al., 2013; Ullah et al., 2014), the recycle fraction of materials must be maximized. At the same time, the primary production of materials must not produce a lot of greenhouse gasses (i.e., consume energy) and consume resources (e.g., water and land) (Rashid et al., 2011; Ullah et al., 2013; Ullah et al., 2014). Thus, besides ρ, *Water usage*, CO_2 *Footprint* of the primary production of materials, and the *Recycle fraction* must be considered in order to accommodate the issue of sustainability while selecting material for making the body of a vehicle.

Based on the above contemplation, a set of six criteria ($C = \{C_j \mid j = 1,\ldots,6\}$), namely, ρ, *TS*, *E*, *Water usage*, CO_2 *Footprint*, *Recycle fraction*, is considered to evaluate the alternatives called Al, Mg, and Ti. The decision-relevant information on these six criteria is shown by the min-max plots in Figure 4.5. Since the material property of an alloy is given by some numerical ranges or as crisp granular information see CES Selector database Ref. (Granta Company, 2017, CSE selector), the minimum and maximum values of each range can be plotted on the horizontal and vertical axis, respectively. For example, let the ρ of an alloy is [2.63, 2.78] Mg/m^3 (here the phrase "Mg" is mega-gram, not Magnesium). This piece of decision-relevant information is a point (2.63, 2.78) on the min-max plot. However, based on the decision-relevant information shown in Figure 4.5, the supports ($S = \{S_j \mid j = 1,\ldots,6\}$), of the respective criterion are determined, as summarized in Table 4-b.

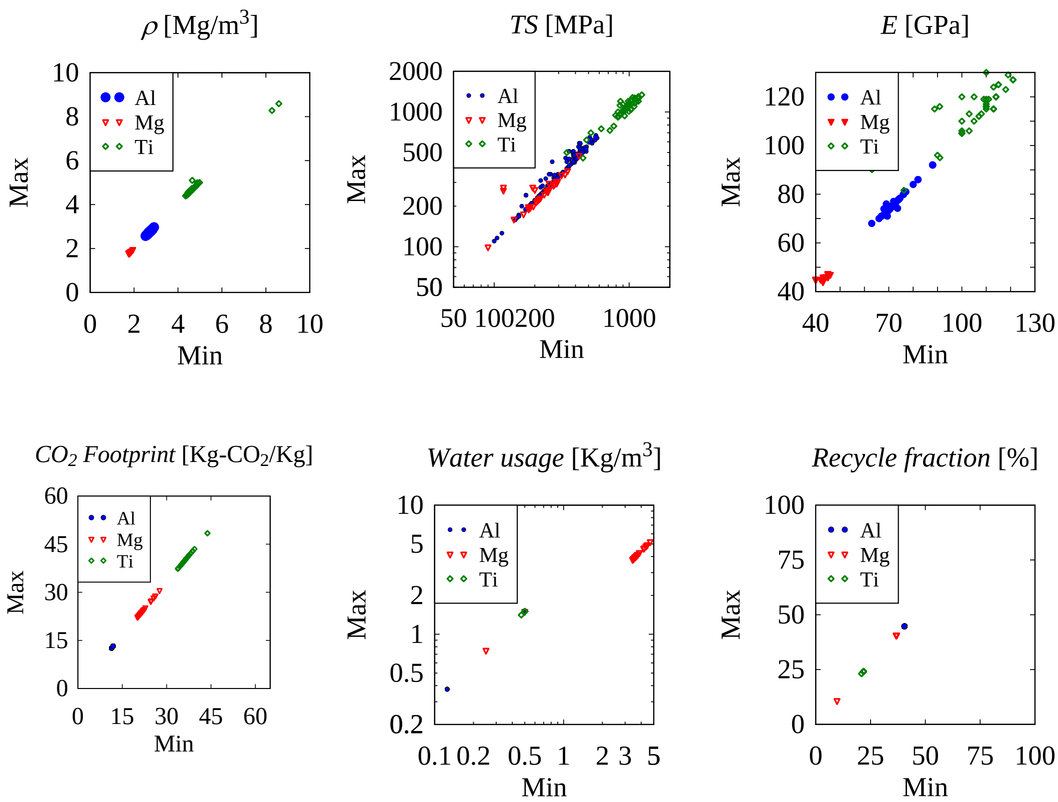


Figure 4.5: Decision-relevant information for three different categories of metal alloys.

Table 4-b. States of criteria and their supports.

Items (↓)	Criteria (C_j|j= 1,…,6)					
	$\boldsymbol{C_1 = \rho}$	$\boldsymbol{C_2 = TS}$	$\boldsymbol{C_3 = E}$	$\boldsymbol{C_4}$ **= *Water usage***	$\boldsymbol{C_5}$ **= $\boldsymbol{CO_2}$ *Footprint***	$\boldsymbol{C_6}$ **= *Recycle fraction***
Units	[Mg/m^3]	[MPa]	[GPa]	[m^3/kg]	[kg-CO_2/Kg]	[%]
States	minimize	maximize	maximize	minimize	minimize	maximize
Supports[a,b]	[1, 15]	[5, 1800]	[10, 250]	[0.1, 10]	[1, 65]	[0, 100]

According to the customer requirement, the optimization states of the criteria are also listed in Table 4-b. From Table 4-b it is observed that, the *TS*, *E*, and

Recycle fraction must be maximized whereas the ρ, *Water usage*, and CO_2 *Footprint* must be minimized, as described early. Based on this, the objective functions ($OB = \{OB_j \mid j = 1,\ldots,6\}$) of the six criteria are plotted and shown in Figure 4.6.

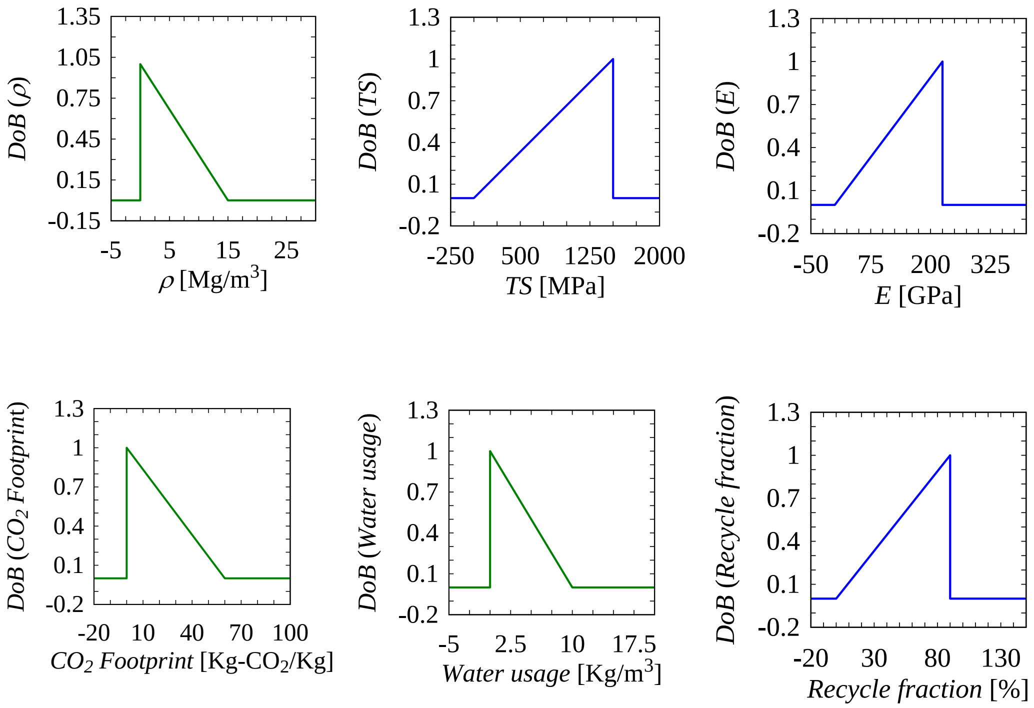


Figure 4.6: Objective functions of six criteria.

The supports (x-axis) are chosen from plots as shown in Figure 4.5 for objective functions of six criteria. As such, all supports here are local supports except the support of *Recycle fraction* that is a global support. According to the fuzzy theory, the range of *DoB* is 0-1 (y-axis). When the requirement is to maximize the criteria the function becomes ramp up ($OB_j = MAX_j$) for *TS*, *E*, and recycle fraction and vice versa. The objective function of *TS* is maximization as shown in Figure 4.7 (a). The support of the objective function is taken from the plot of minimum and maximum range of *TS* as shown in Figure 4.5 and Table 4-b. Compliance is calculated using the interaction between objective function and quantified data. The interaction between the objective function and the crisp granular information of Al 2014, wrought T4 is shown in Figure 4.7(*b*). The compliances of alternative are determined using the procedure described in the Chapter 2. The maximum and minimum values of *TS* for Al 2014, wrought T4 are 350 MPa and 440MPa, respectively. Thus, p = 350 MPa and q = 440 MPa and the objective function of *TS* is maximization. Therefore, using equation (2.23) the compliance of Al 2014, wrought T4 becomes 0.21.

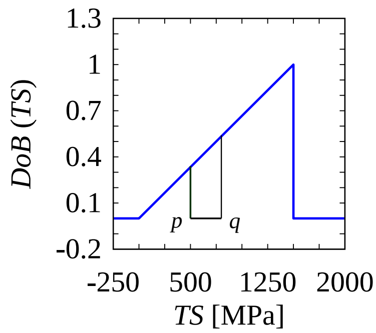


Figure 4.7: Interaction between crisp granular information with the objective function for Al 2014, wrought T4 alloy.

Using similar procedure, the compliances of 197 types Al alloys are calculated for criteria, *TS*. Similarly, the compliances for other two alternatives (30 types of Mg alloy and 45 types of Ti alloy), six criteria are calculated and shown in Figure 4.8. From the Figure 4.8, it can be observed that the order of preference list of material in terms ρ is Mg > Al > Ti, *TS* is Ti > Al > Mg, *E* is Ti > Al > Mg, *Water usage* is Al > Ti > Mg, *CO_2 Footprint* is Al > Mg > Ti, and *Recycle fraction* is Al > Mg > Ti. The values of the compliances for each criterion and alternative have uncertainty. This uncertainty is quantified by possibility distribution.

The Possibility distribution of a criterion for each alternative is determined using the procedure described in the Chapter 2 Section 2.2.3. The possibility distributions for compliance of the alternatives namely, Al, Mg, and Ti for different criteria (ρ, *TS*, *E*, *Water usages*, *CO_2 Foot print, and Recycle fraction*) are calculated and shown in Figure 4.9.

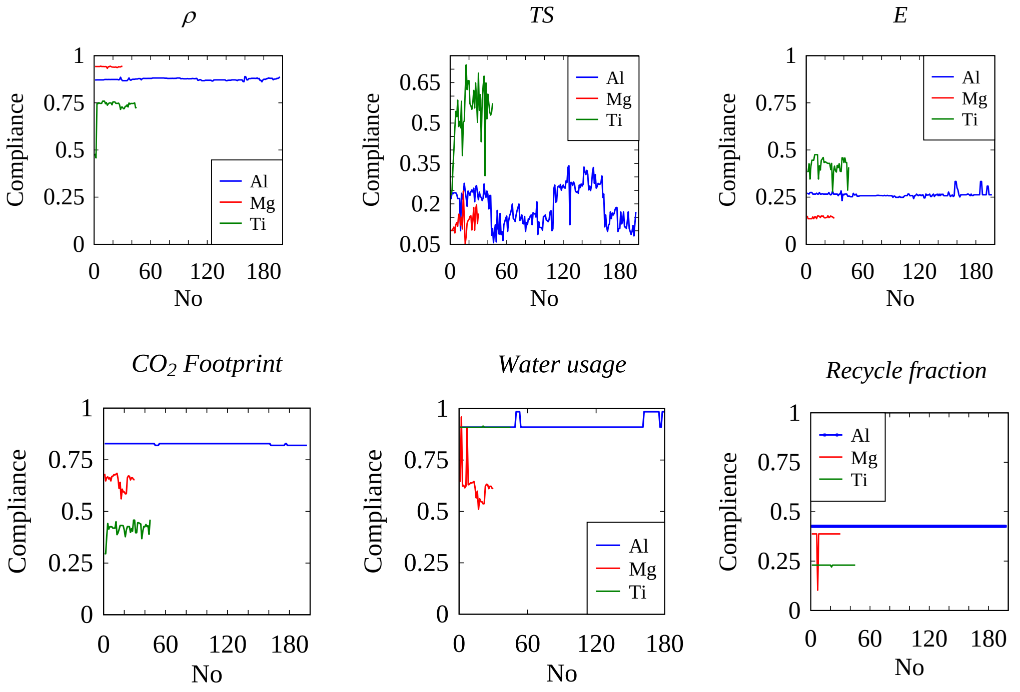


Figure 4.8: Compliances of the alternatives for a respective criterion.

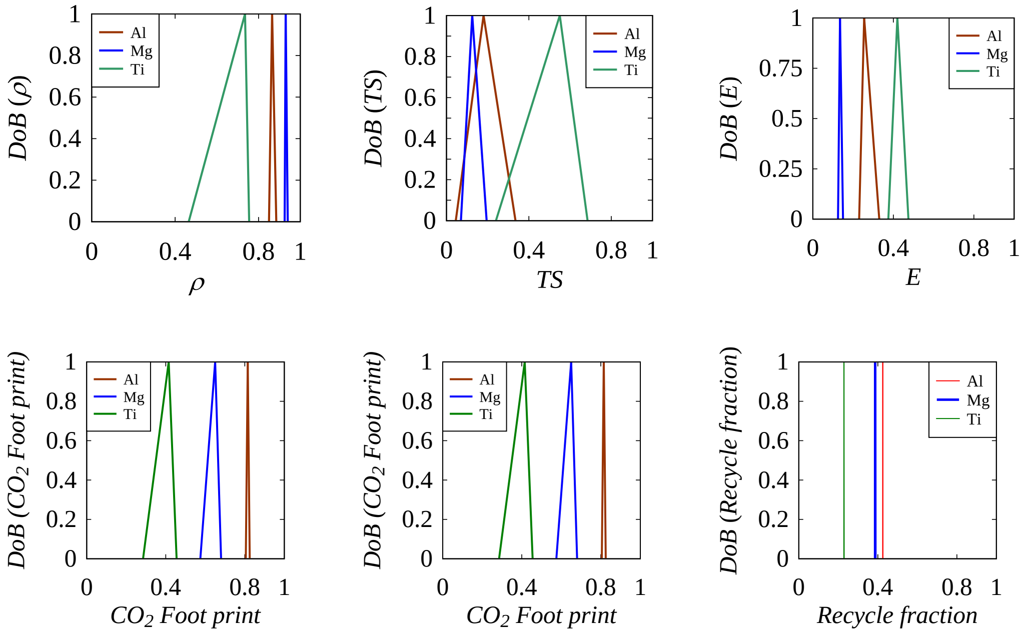


Figure 4.9: Possibility distribution of the alternatives for the respective criterion.

In case of vehicle design, to choose the best material *TS*, *E* should be high, whereas ρ, *CO_2-Foot print*, and *Water usage* should be low. As shown in Figure

4.9, it is clearly understood, which alternatives comply more with the objective functions according to criteria because the plots are the possibility distribution functions of the compliances. For example, in case of ρ the possibility distribution function of Mg lies near the value of 1 according to x axis. Therefore, the preferential list of material according to ρ is Mg > Al > Ti. The preference of lists of materials based on the ranking score and according to *TS*, *E*, *CO_2-Footprint*, *Water usage* are Mg < Al < Ti, Ti > Al > Mg, Ti < Mg < Al, Mg < Al and Ti < Mg < Al respectively. That means graphically, it is possible to compare the alternatives and possible to make a preferential list of material. However, for exact decision making, it is required to go through the compliance calculation again. In case of *Water usage* as shown in Figure 4.9 there is no possibility graph for Ti because the variability in the data on *Water usage* for Ti is absent.

The interaction between the objective function and the possibility distribution functions of six criteria and three alternatives are shown in the Figure 4.10. The value of the compliance always lies between 0 and 1. As the requirement is to maximize the compliances value, the objective function need to be maximized, however, the support is lies between 0 and 1.

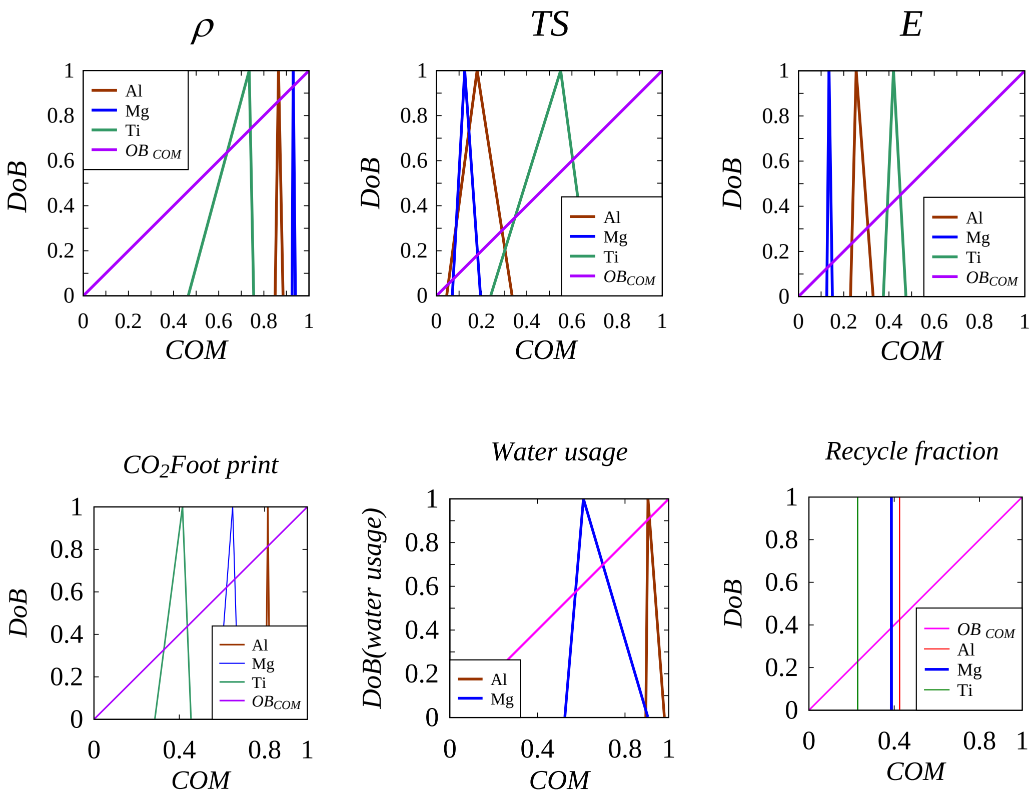


Figure 4.10: Interaction between the objective function and the possibility distribution for five criteria, three alternatives.

The points of the interaction between possibility distribution of compliances of ρ for Al alloys and the maximization objective function *are* $V_{MINx} = V_{MINy} = 0.862$, $W_{MINx} = W_{MINy} = 0.867$. The support of the possibility distribution function of compliances for ρ of Al is [0.85, 0.885] that is $t_1 = 0.85$, $t_2 = 0.86$ and $t_3 = 0.885$. In case of ρ, using equation (2.33), the ranking score is calculated for Al alloys and it is 0.982. Similarly, the interaction between the objective function and the possibility distribution function of compliances for three alternatives and six criteria are calculated using equation (2.29) and equation (2.25). This ranking score also preserves the above mentioned order of preferences, as indicated in the last row in Table 4-c. This means that, the ranking score is an effective mean of aggregating the uncertainty associated with an alternative for a given criterion.

Table 4-c. Ranking scores of the alternatives.

Alternatives (A)	Criteria (C)					
	$C_1 = \rho$	$C_2 = TS$	$C_3 = E$	C_4 = *Water usage*	C_5 = CO_2 *Footprint*	C_6 = *Recycle fraction*
A_1 = Al	0.982	0.327	0.470	0.992	0.966	0.670
A_2 = Mg	0.995	0.243	0.255	0.872	0.871	0.624
A_3 = Ti	0.906	0.741	0.666	0.915	0.780	0.405
Preferential order	Mg > Al > Ti	Ti > Al > Mg	Ti > Al > Mg	Al > Ti > Mg	Al > Mg > Ti	Al > Mg > Ti

Once the ranking scores are known, the decision-score (*DC*) can be calculated as described in the previous Section 4.1.5. In doing so, the importance of the criteria must be set. For this particular case, the criteria called *ρ*, *Water usage*, CO_2 *Footprint*, and *Recycle fraction* are useful in assessing the sustainability of material, and, thereby, the sustainability of vehicles, as described above. The other two criteria, namely, *E*, and *TS* are useful for ensuring the structural integrity of the body of a vehicle. One can determine the decision-scores of the alternatives for different sets of importance as shown in Table 4-d. In particular, three sets of importance are chosen here for determining the decision-scores. In the first set, both sustainability and integrity criteria are considered equally important. For example, the importance of the criteria are *ρ* = *TS* = *E* = *Water usage* = CO_2 *Foot print* = *Recycle fraction* = 10. According to this requirement the value of compliances for *ρ*, *TS*, *E*, *Water usage*, *CO2 Foot print*, *Recycle fraction* are 0.982, 0.327, 0.47, 0.992, 0.966, and 0.426, respectively, for Al. Thus, the total value of the compliance is 0.694 for Al. Similarly, the compliances for Mg and Ti are 0.609 and 0.735, respectively, as shown in Table 4-d.

Table 4-d. Decision score for Set-1 based on six criteria of three alternatives.

		Criteria(C)						
		ρ	***TS***	***E***	***Water usage***	***CO_2 Foot print***	***Recycle fraction***	**Total**
Set-1	Importance	10	10	10	10	10	10	60
	Weight	0.166	0.166	0.166	0.166	0.166	0.166	1
	Al	0.163	0.054	0.078	0.165	0.161	0.071	0.694
	Mg	0.165	0.040	0.042	0.145	0.145	0.104	0.640
	Ti	0.151	0.123	0.111	0.152	0.13	0.067	0.735

The decision-score of Ti becomes the maximum, followed by Al and Mg, respectively. Thus, when both sustainability and integrity criteria have the same degree of importance, the list of preferences is Ti > Al > Mg. Similarly, the other decision formulation are made for Set-2 and Set-3 and shown in Table 4-e.

In the second set, sustainability criteria are considered relatively more important than the integrity criteria. This makes Al's decision-score the maximum followed by those of Ti and Mg, respectively, shown in Table 4-e. Thus, when the sustainability criteria are more important than the integrity criteria, the list of preferences is Al > Ti > Mg. This means that Ti and Al alternate their positions once the integrity criteria lose their importance compared to those of sustainability.

In the last set, the sustainability criteria are considered very important compared to those of integrity. This makes Al's decision-score the maximum followed by those of Mg and Ti, respectively. Therefore, when the integrity criteria are somewhat insignificant compared to those of sustainability, the list of preferences is Al > Mg > Ti. This means that Al and Mg are the preferred materials when the sustainability is a key concern.

Table 4-e. Decision scores of the alternatives.

Criteria (*C*)	**Importance**		
	Set 1	**Set 2**	**Set 3**
$C_1 = \rho$	10	10	10
$C_2 = TS$	10	5	1
$C_3 = E$	10	5	1
C_4 = *Water usage*	10	10	10
C_5 = *CO_2 Footprint*	10	10	10
C_6 = *Recycle fraction*	10	10	10
Alternatives (*A*)	Decision scores		
A_1 = Al	0.694	0.753	0.820
A_2 = Mg	0.643	0.722	0.812
A_3 = Ti	0.735	0.742	0.749
List of Preferences	Ti > Al > Mg	Al > Ti > Mg	Al > Mg > Ti

4.4 Conclusion

- A new pragmatic decision model has been developed to select a material under the epistemic uncertainty. The effectiveness of the proposed decision model is observed to select a material.
- An optimal material is selected from three metallic materials for a vehicle body. In this particular case it is considered that there is no information of *MI*, no information of the shape of components of vehicle body. To make a strong, stiff, light, and sustainable car body from Al, Mg, and Ti, a material is selected through the proposed model. Finally, based on three different customer (e. g. Car Company) requirements, decision is made from limited information (design configuration, data uncertainty, and conflicting issue under sustainability). Thus, this type of model can be used in different manufacturing companies, especially; it

can be used at the starting of the company to reduce the complexity of the infrastructure.

- Limitation of the model is that the designer needs to take care of alternatives selection. All types of the criteria should be known at the beginning, regarding the objective function; otherwise, requirement will not be accomplished.
- When the Material Index for selecting an optimal material is unknown, a set of possibilistic objective functions can be used, instead.
- The possibilistic objective functions can be used without any complicacy to incorporate the aspect of sustainability in a material selection process.
- The interaction between the uncertainty of a material property and relevant possibilistic objective function can be quantified by a concept called compliance.

  The compliance can be aggregated for all objective functions in order to rank a material.

Chapter 5: Discussions

Selection of an optimal material is a critical as well as important issue in the product development process. It is critical in the context that around 80% cost and energy consumption of a product is required for material and to support its life cycle. The optimal material is required for a component of the product in view of sustainability, durability, and reliability. Otherwise, it is nothing, but a waste of material, money, and energy. Additionally, the material properties of the alternative resources vary to a large extent which leads uncertainty in the material selection method. In this study, uncertainty of a material property has been quantified using a reliable approach, the possibilistic distribution. When the material selection is required at an early stage of the product development, there is uncertainty in material selection using *MI*. In this study, using an alternative representation of the *MI*, a decision model is developed to select a material based on compliance. However, still, no one has considered these types of uncertainties at the initial stage of product development. To shed some light on the issues of uncertainty quantification and material selection, this thesis poses and answers the following questions:

a) *How will the findings of this research help others?*
b) *What will be the further research based on the output or methods?*

Due to lack of information of sustainable properties, jute material was not incorporated in the proposed decision model to see the effectiveness. This study can link the different stakeholders of the eco-product frame-work. A framework for eco-product development based on the different stakeholders may be introduced as Figure 5.1

This study has an influence on the different components of the framework. The following framework will give a clear idea of how the research output of this study will help others.

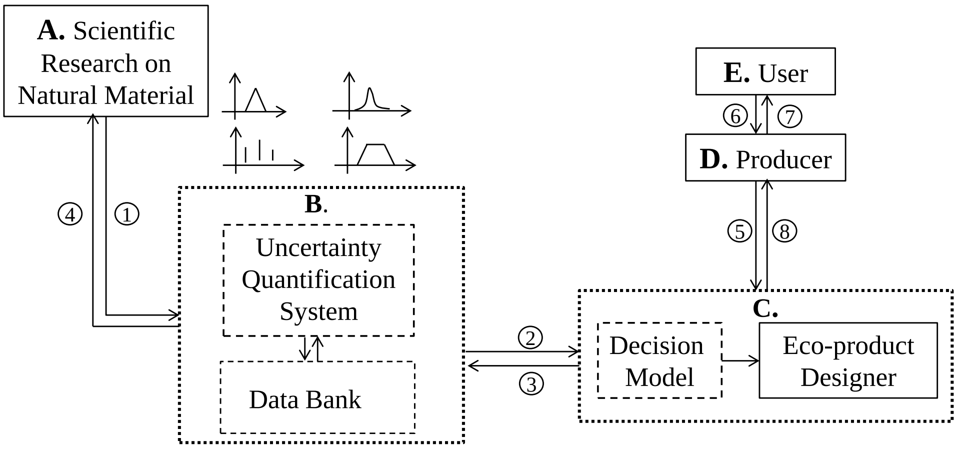


Figure 5.1: A Complete framework of eco-product development.

The important stakeholders of the eco-product framework are Researchers (A), Uncertainty Quantification System (B), Eco-Product Designer (C), Producer (D), and User (E) as shown in Figure 5.1. Diversified researches (A) have been developing on the material (e.g. characterization) since last few decades. The outcomes of these researches are needed to be stored for future use. due to various reasons, there is variability or uncertainty in the data of research natural material properties which are needed to quantify (B). Thereafter, based on that decision-relevant information, eco-designer can compare one material with another using proposed decision model (C) and select a material. This type of decision model is essential for the designer at the initial stage of the eco-product design. First, a designer designs a product on the basis of selected material, next the producer (D) produces a product using that designer specification (marked as 5), and then the user (E) uses (marked as 6) that product. Depending on the user requirement or feedback (marked as 7), the eco-designer gets information (marked as 8) from the producer and the eco-designer will redesign the product by changing the input of the decision model. The decision of the proposed model can be stored as information in the system B for further use. The initial or former decision (marked as 2) and modified (marked as 3) decision can be also stored as information in the stored system (B).

Different types of research work have been going on the natural materials (e.g. characterization), during the last few decades by different researchers (shown in Figure 5.1 marked as A) of different countries (Bangladesh, India, China, Brazil, and so on). The designer designs the products based on the research information and knowledge. However, there is no link between the researchers (A) and eco-designers (C) to transfer or share their knowledge. As a result, researchers, as well as designers, are not benefited from each other. Based on the above contemplation, different collaboration can be made between the different organizations of different countries for diversification of the eco-product. Moreover, this study can integrate A and C as shown in Figure 5.1. Based on this study, the outcome of scientific research (A) can be stored in the Data Bank (B). That data can be quantified through the proposed quantification method. Based on quantified data, the designers can select material from different alternatives, using the proposed decision model, at the beginning of the product designing.

There is uncertainty in the data of material properties and dissimilarities among the research outputs. In addition, research outputs of the natural materials are not stored in a systematic manner that means there is no specific system or source to store the scientific results, for further use. Therefore, designers cannot rely on the existing system for eco-product development. Now, if eco-designers (C) want to design an eco-product, using natural material, they need a reliable source of information (like B). Using that information, designer can select an optimal natural material for eco-product. In this context, the proposed quantification approach (possibility distribution) can be used to quantify the uncertainty and store the data in the data bank. Thus, the eco-designer will be benefited from this study and will be able to quantify the uncertainty using proposed quantification system. Hence, the designer will be able to develop diversification in the eco-product using that quantified data. Furthermore, small and large number of data can be stored using the possibility distribution. Therefore, this study will be helpful to store and to quantify the uncertainty (B) in the data for further uses. Besides, the proposed quantification approach will enrich the data bank by storing the data using the possibilistic method. To select a material for an eco-product, the data of material properties are required (discussed in Chapter 1), which now can be collected from the B. The designer (C) will able to design an echo product using the data or information from B. Moreover, a designer using the proposed decision model can select a material before designing a product.

In eco-product, the natural materials (particularly jute fiber) are sometimes used as a raw or after modification. Jute material namely, jute fiber, yarn, and jute fabrics (alternative form of jute) are used in the jute based eco-products development shown in Figure 5.2.

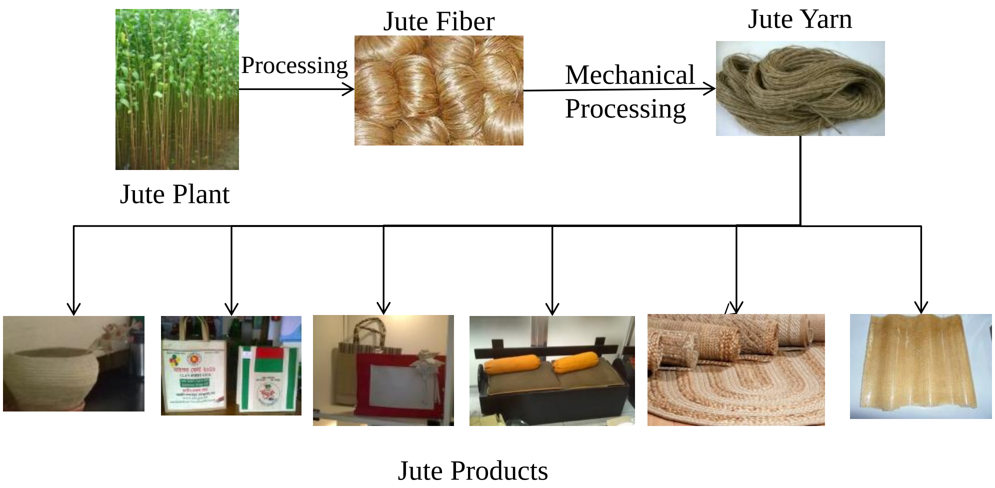


Figure 5.2: Jute product made from jute yarn.

Jute fiber is a primary material of the jute product and the data of jute fiber is used for the Jute-product designing. However, most of the jute based products are made from jute yarn. From this study, it is observed that yarn data can be used for product designing compared to the fiber data. When the jute fibers properties, *TS* and *E* (see Shahinur and Ullah, 2017; Defoirdt, et al., May 2010;

Biswas, et al., 2013; Biswas, et al., 2011; Shahinur, et al., 2015; Jafrin, et al., 2014; Hossain, et al., 2014), are considered, their values were too high in comparison to those for yarn properties. For a better understanding, the possibility distributions of jute yarn and jute fibers are shown side by side in Figure 5.3.

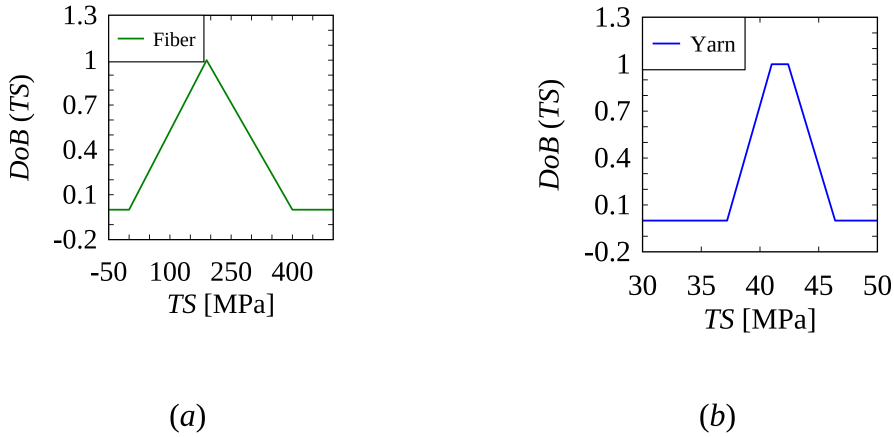


Figure 5.3: Comparison between jute fiber and yarn in terms of uncertainty in *TS* (*a*) fiber and (*b*) yarn (copied from Figure 3.12).

The possibility distribution for the *TS* of jute fibers shown in Figure 5.3(*a*), taken from (Shahinur and Ullah, 2017), was determined by the same methodology as described in Chapter 2 (Dubois, et al., 2004). As seen from the *TS* possibility distribution for jute fibers shown in Figure 5.3(*a*), the degree of the associated uncertainty is very high. That means fiber data are unreliable compared to jute yarn as shown in Figure 5.3(*b*). However, the data of jute yarn on the basis of possibility distribution are more pragmatic because the range of the jute yarn is narrow compared to jute fiber data. This means the combined strength of strong and weak jute fibers creates the resultant strength of the jute yarn. Since a bundle of jute fibers (not a single fiber) supports the strength of a product, the material properties of jute yarn can be used for making various design decisions. The proposed quantification approach can also be used to take a decision. For example, using possibility distribution, it can be concluded that yarn data are best for calculating the design limit compared to jute fiber. The decision maker can keep these types of information and distribute to the company for eco product diversification.

Furthermore, the proposed possibility distribution approach will not only be used to quantify the uncertainty, it can be also be used to see the effect of the controller (chemical or physical) on the material performance (output) while there is uncertainty. This method can also be used to identify the suitable commercial manufacturer for a specific production. These types of investigation need be proved using the proposed decision model.

This model is not only usable for material selection, but can also be used in the selection of company at the starting of the signing from different companies or suppliers. Due to some problem if a supplier (e.g. Dayal) discontinues its business, in that case, customer (TOYOTA, Car Company) needs to select a supplier. Thus, the decision can be taken under this proposed model. The proposed model can be used to select which material (Indian jute, Bangladeshi Jute, Chinese jute, and Brazilian jute) will be used from which country (Bangladesh, India, China, and Brazil) for a product (e.g. Jute Carpet). Furthermore, there are different approaches (e.g. DNA based, point cloud-based, if then else, AHP, TOPSIS, and PROMETHEE) to take a decision which can be compared with the proposed decision model.

If a designer wants to select a material for a particular product, the designer may need to follow the proposed decision model. In addition, Decision model (C) is required when there is epistemic uncertainty, material properties are uncertain and only objectives (maximization and minimization) are known. The objectives are maybe maximization or minimization or both. To differentiate the optimal material, their mechanical and sustainable properties are needed to be emphasized for the sustainability, reliability, and durability of the product. In this study, emphasis is given on the mechanical properties (like *TS*, *E*, and *s*) and sustainable properties (CO_2 *Footprint, Recycle fraction*, and *Water usage*) of Al, Mg, and Ti. To observe the effectiveness of the proposed decision model a metallic material is selected. On the other hand, there is a lot of information of metallic materials (for example *CO_2 Footprint*, *Water usage*, and *Recycle fraction*) and their information is well established. Moreover, these data are used in the factory for production. However, due to lack of life cycle information on natural material, jute, it is not possible to select a natural material from the Material Universe. Thus, the sustainable properties (for example *CO_2 Footprint*, *Water usage*, and *Recycle fraction*) of the natural material are required for sustainability calculation which is an open topic for future research.

Shape of Objective Function

This section describes the reason behind the selection of simple shape for objective function. Desire or requirement of the decision maker is termed as objective function. In this study, the objective functions are represented by two types of possibility distributions and the membership value of the objective function lies between 0 and 1. They are neither linear nor nonlinear, they follow the fuzzy logic and the function is fuzzy membership function. Moreover, linear and nonlinearity are considered in the hard computation world, whereas the proposed decision model is developed under soft computing. However, the shape of the proposed possibility objective function may be non-linear such as Gaussian distribution, convex and quadratic and polynomial function as shown in Figure 5.4.

As in this study it is considered that the objective functions are fuzzy functions, meaning the information of the objective functions is not clear or fuzzy or uncertain. Thus, it can be said that, it is unknown which nonlinear shape is appropriate for objective function for individual criteria. In this study, to represent the objective function, for simplicity, the rule of thumb has been followed. That is why in this study simple shape (ramp up and ramp down) is considered for objective function representation.

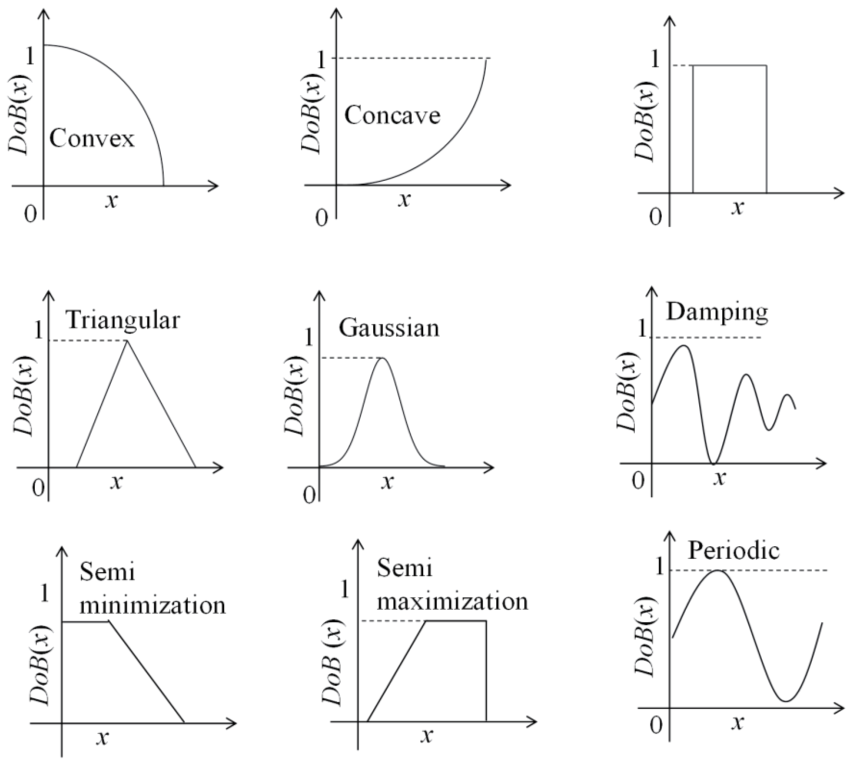


Figure 5.4: Different shapes of objective functions.

If the shape of the objective function is changed according to designer preference, the decision will be changed accordingly.

If the objective function is denoted by F: X→Y, the domain of X is the support and Y is the membership value of possibility objective function. X∈ℜ and Y∈[0,1]

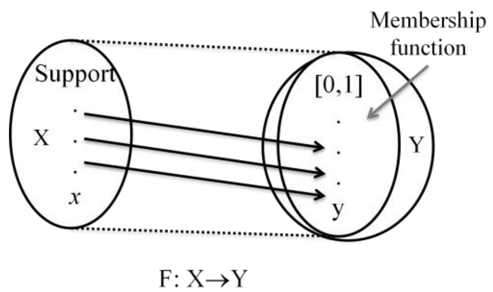


Figure 5.5: Domain of possibility objective function.

Thus, the support selection is an important issue to obtain the membership function for a specific criterion. The following section discusses the support selection.

Support Selection

Suppose two materials A and B from the Material Universe are needed to rank under a criteria ρ (data is a range for each alternative, say) as shown in Figure 5.6. Based on the selection of x-axis range according to designer, the support will be local, semi-local, global, and deterministic as shown in Figure 5.6.The data of ρ for A and B lies in the natural material group. Thus, when a designer selects a range regarding the value of ρ for A and B as [ρ_A, ρ_B] the support will be considered as local support. When the range of density of natural material is considered, then, it will be named as local support. When the whole range of density from the Material Universe is considered, they will be named as global support as shown in Figure 5.6. Meanwhile, the deterministic support for ρ is unknown, which cannot be represented by conceptual Figure.

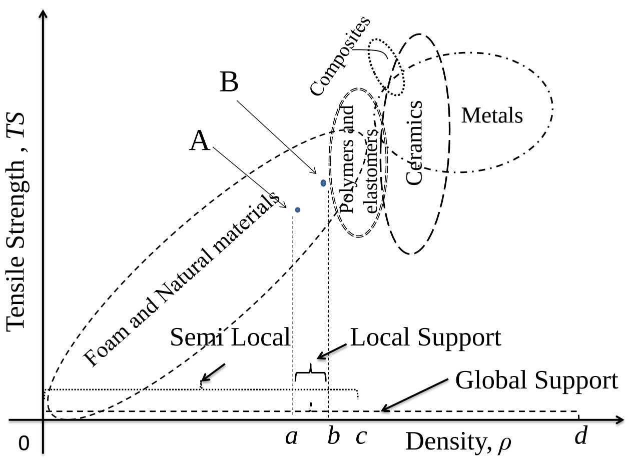
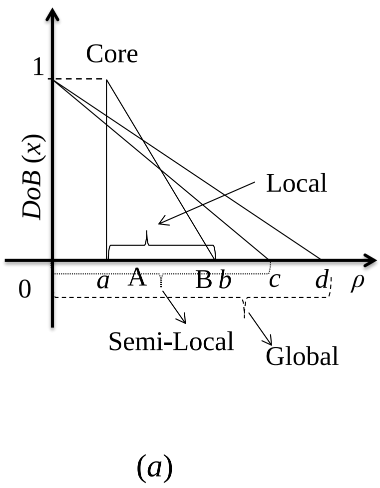


Figure 5.6: Support consideration to select material A and B from the Material Universe.

In this study, the objective function is represented by two types of possibility functions such as minimization and maximization. When one designer wants to minimize ρ, the possibility objective function for ρ can be represented by Figure

5.7 (*a*) based on the different supports consideration (say, data is CRISP). However, if other designer wants to maximize the ρ, the objective function can be represented by different maximization functions as shown in Figure 5.7 (*b*) based on support.



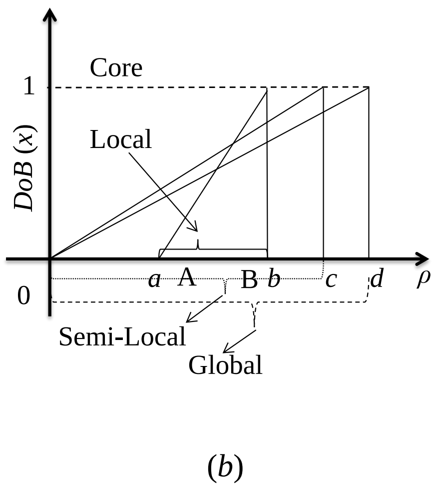


Figure 5.7: Determination of (*a*) minimization and (*b*) maximization objective function based on different supports.

The density of the natural material is comparable to the member of natural material. However, the density of natural materials is not comparable to the density of metallic materials. Thus, local support is the better option than other support to formulate the objective function. However, a designer can choose or select any type of support according to the requirement.

Support Calculation for Different Information

As the data has different categories, support selection based on the data type also varied. If data is CRISP, support will be calculated from the minimum and maximum value regarding a criterion for all alternatives as shown in Figure 5.8(*a*). For example: When $\rho_{Al} = a$, $\rho_{Ti} = b$, $\rho_{Mg} = c$, if $a < b < c$ support will be $[a, c]$. If $a < c < b$ support will be $[a, b]$ and if $b < a < c$ support will be $[b, c]$. For example, $\rho_{Al} = 0.5$ Mg/m^3, $\rho_{Ti} = 0.6$ Mg/m^3, $\rho_{Mg} = 1.2$ Mg/m^3, the local support will be [0.5, 1.2].

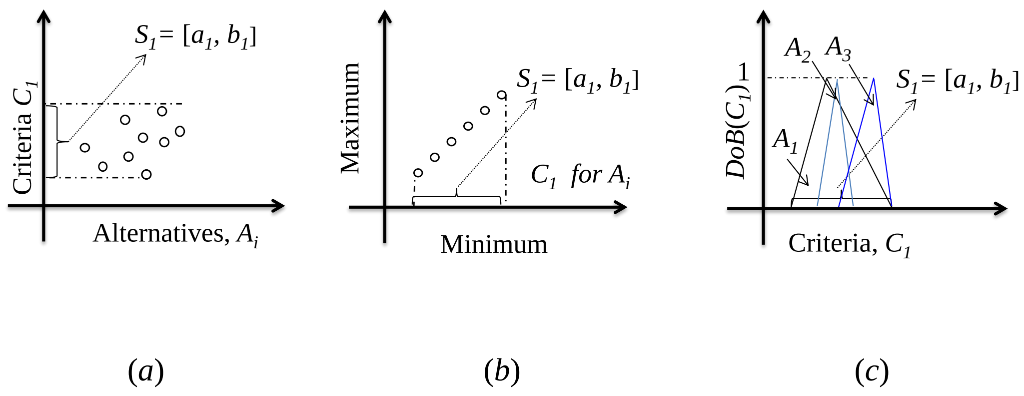


Figure 5.8: Support (local) selection for the possibility objective function of (*a*) CRISP (*b*) range and (*c*) uncertain criterion.

If data is a range, the range of *x*-axis of the minimum-maximum plot of the criteria for all alternatives will be selected as local support (discussed in Chapter 4 as shown in Figure 5.8(*b*)). If the data is uncertain (probability granular) for a criterion, that uncertainty can be represented by the possibility distribution. The possibility distribution for all alternatives if accumulated for that criterion, the range of *x*-axis will be selected as local support as shown in Figure 5.8(*c*).

Ranking of Alternatives for Different Supports

Support is an important issue to build the objective function. If a support is changed, the ranking remains same for same criteria but the value of the compliance is changed according to support. For example, consider the case of a criterion, ρ. The objective function (minimization, say) will become as shown in Figure 5.9 for different supports. The support is selected as local (Figure 5.9(*a*)), semi-local (Figure 5.9(*b*)), deterministic (Figure 5.9 (*c*)) and global (Figure 5.9(*d*)) (say).

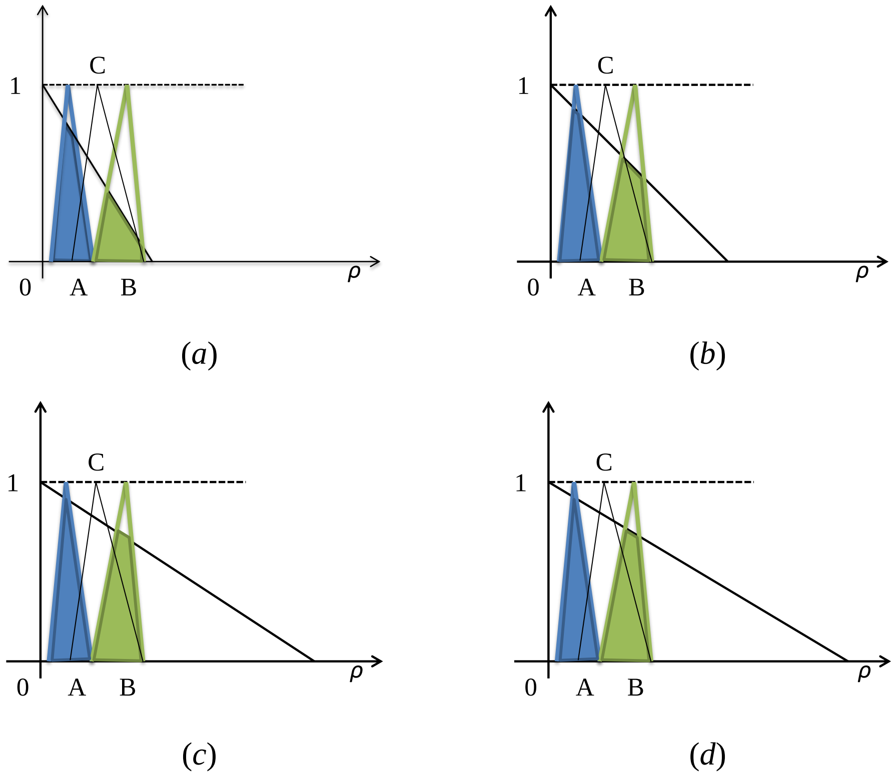


Figure 5.9: Interaction of objective function and the uncertainty of criteria for (*a*) local, (*b*) semi-local, (*c*) deterministic, and (*d*) global support (objective function is to minimize the criteria).

The ranking of three materials (Al, Mg, and Ti) is shown in the Figure 5.10 for the different ranges of support. Let us consider local, global, semi-local, and deterministic support of the ρ are [1, 15], [0, 100], [1, 20], and [0.01, 30], respectively. The ranking of alternatives remains same for different supports, however, value of the degree of compliance is changed accordingly as shown in Table 5-a. Therefore, it can be confirmed that if the support is changed the ranking of alternatives will not change. As the value of degree of compliance is changed, the decision scores would be changed.

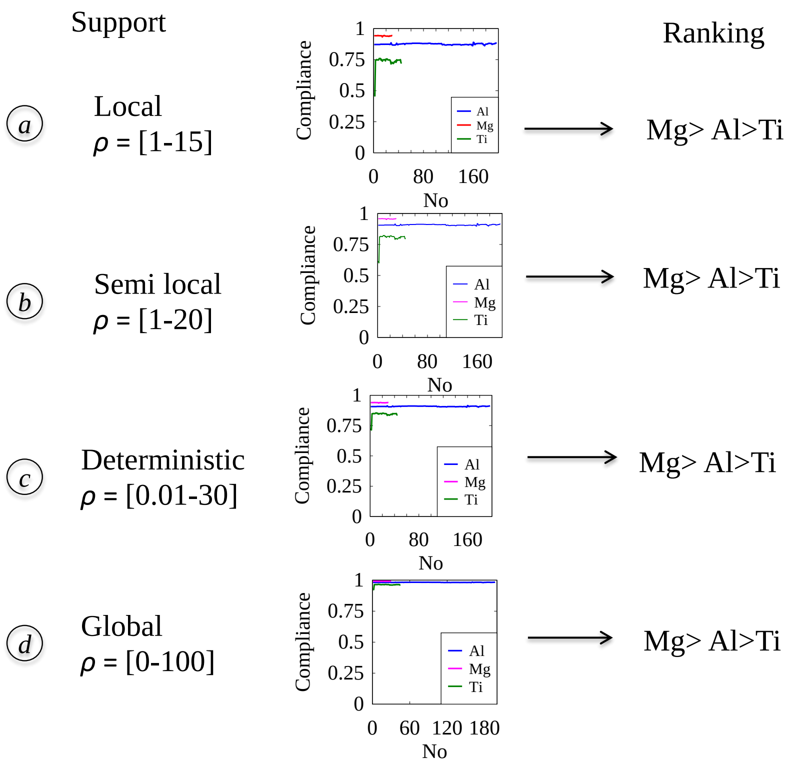


Figure 5.10: The ranking of alternative for different supports (*a*) local [1, 15] (*b*) global [1, 100] (*c*) semi-local [1, 20] and (*d*) deterministic [0, 30] in case of criteria (ρ).

Table 5-a. Degree of compliance for Al alloy based on different supports (ρ).

Support	**Local**	**Semi-local**	**Global**	**Deterministic**
Numerical Value	[1, 15]	[0, 20]	[0, 100]	[0.01, 30]
Value of Compliances	0.982	0.979	0.999	0.990

Thus, it can be said that there is some constraint for the proposed model regarding the support selection. In case of objective function formulation, the consideration of support should be local because similar categories of alternatives can be compared with each other. For example, speed of the car can be compared with car speed which has significant meanings. If a car speed is compared with the airplane speed it becomes illogical or insignificant. However, theoretically they can be compared, that is why there is option of semi-local, global and deterministic support.

Approaches of Compliance Calculation

Compliance of a criterion for an alternative means how far or close a criterion is from the desired objective function. The degree of compliance can be calculated in different ways based on the data pattern (or types of information). In this study, to observe the effectiveness of the proposed decision model, it is considered that each alternative comes from a group of alloy and the data of the criteria has individual range. Thus, at first, compliance is calculated for a criterion of each member of the alternative using crisp granular information theory. From that big data of compliances a possibility distribution is calculated for the group of members of an alternative. After that, the degree of compliance is calculated between that possibility distribution and maximization objective function for all alternatives as shown in Figure 5.11 (*a*). The approach to calculate the degree of compliance is different for different categories of data. Consider the data of criteria is uncertain (probability granular) or point cloud. Frist, by calculating possibility distribution of each criterion, the degree of compliance can be calculated for ranking between the alternatives and objective function as shown in Figure 5.11(*b*). However, when the range of data can be represented by point cloud, the ranking of alternatives can be made using this approach also.

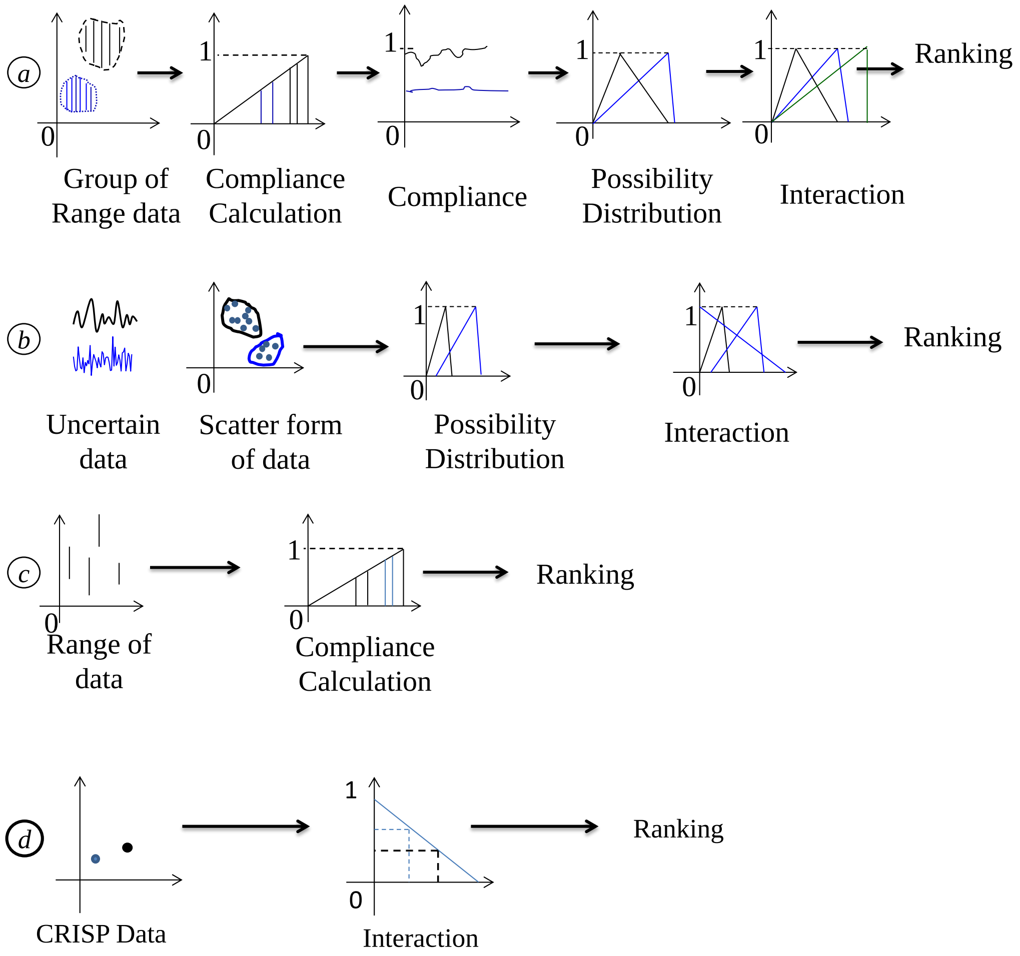


Figure 5.11: Different approaches for ranking under compliance (minimize a criterion).

Consider the data is a range. The degree of compliance can be directly calculated by interacting between the range of data and possibility objective function (minimization, say) as shown in Figure 5.11. When an alternative has many members and each member's criteria is in the form of range the group of alternatives are needed to be ranked then the method (*a*) should be followed. When information of the criteria is crisp, method (*d*) should be followed. If the information of criteria is hybrid, the hybrid concept can be used to rank the alternatives. Therefore, this is again an open topic for research. The proposed decision model can be used to take decision for different conditions or situations behind the multi-objective.

Difference between Pareto Optimal and Proposed Decision Model

This section describes the difference between the Pareto optimal and the proposed model. The Pareto optimal theory is developed for multi criteria. The criteria are conflicting (minimization and maximization) in nature and the units of the criteria are different (for example MPa, GPa, Mg/m^3, and %). In such case conventional linear and nonlinear solution are not applicable. That is why Pareto optimal is developed for multi-criteria solution. At first, in this case Pareto frontier or non- dominate line are found. In the non-dominated line one objective function (maximization or minimization) of a criterion is not dominated other objective function (maximization or minimization) of another criterion. The non-dominated curve may be convex or concave or any other shape as shown in Figure 5.12. If the solutions are in the non-feasible area, they are ignored. The possible solutions are searched in the non-dominated area or line or boundary and optimized. If the solutions are not in the non-dominated line but in the feasible area, then the distance between them are calculated. The solution, for which the lowest distance is obtained, is considered as optimum or decision.

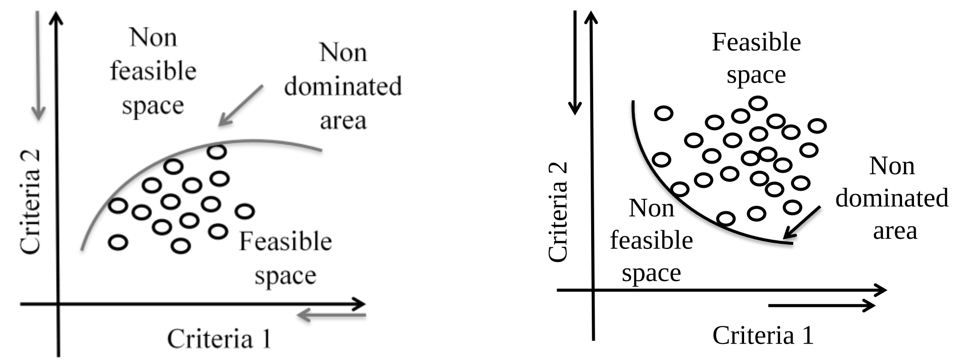


Figure 5.12: Non-dominated line for the solution under Pareto optimal.

In case of Pareto optimal the data of the criteria is considered as CRISP value. In case of two criteria the Pareto graphical representation is easy, however, for *n* types of criteria, it is tough to represent. Computationally this boundary is complex. Based on the linear-nonlinear system, linear and non-linear objective function, linear and nonlinear constrains different methods are developed to obtain the solution at Pareto frontier line. Uncertain data, granular data, range of data, means all the information or data are not known, unclear or partially known. Thus, in such case, how the Pareto frontier (non-dominated area) will be plotted and how the decision will be taken is not clear. As in this study, the information is fuzzy, the uncertainty in data of the criteria are represented by fuzzy function and interacted with the fuzzy objective function. The interacted (compliance) values are also fuzzy membership function, and it is considered that they all are possible solution or results. Among them, maximum membership value is considered as a decision.

In the proposed decision model from graphical presentation, it is easily comprehensible. The compliance of all criteria is calculated and ranking (based on membership function) is made. Based on the decision maker's importance on the criteria final decision is taken and the material is selected.

The similar procedure is followed for range and crisp data but calculation procedures are different [see Chapter 2].

Furthermore, each stage of the proposed decision model is graphically presentable (see Figure 5.13) which is easily understandable by human perception. In the proposed model all data are equally emphasized, however, the solutions are searched in all membership functions for all criteria for all alternatives, rather than the boundary.

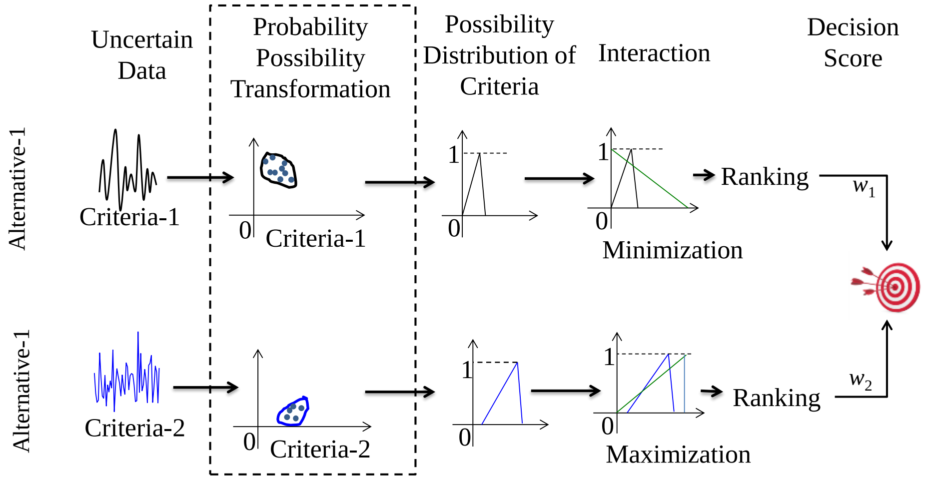


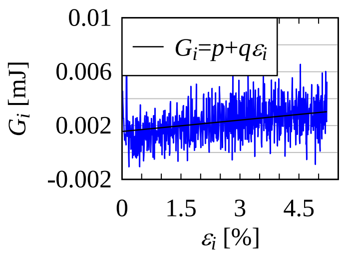
Figure 5.13: Decision making among two criteria under minimization and maximization objectives.

Limitation of the Proposed Model

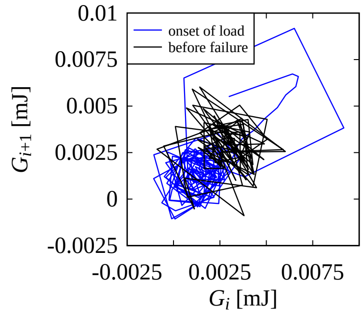
If the decision score for two or more alternatives becomes same, in that case single decision will be uncertain. In such case, the decision will be a set of alternatives and all the members of that set will be equally treated for decision. Therefore, this area is for future research.

Energy Absorption Properties

Energy absorption (as shown in Figure 5.14) can be considered as a criterion of material selection. The reason behind the failure of the jute yarn and fiber is the energy absorption as shown in the Figure (Ullah, et al., 2017). The energy absorption is like a funnel in case of fiber, whereas, in case of yarn the absorption and release of the energy pattern is a different shape. The variability of the energy absorption is controlled in yarn compared to jute fiber. The variability may be more controlled in jute products. Hence, this can be considered as a product development criterion. Before failure, the rate of energy absorption and release is high as shown in Figure 5.14(*b*) in the return map of the instantaneous energy absorption.



(*a*)



(*b*)

Figure 5.14: (*a*) Energy absorption pattern and (*b*) return map of jute yarn.

In this study to select a material for sustainable product it was focus on the mechanical properties (TS, E, and ρ) and sustainable properties under stiffness and strength limited design. In this study, 6 criteria and three alternatives are considered. Any other criteria can be incorporated in the proposed model. For example endurance limit, fracture toughness, failure strength, cost, reserve, safety factor, and availability of the resources. If someone wants to select a material for vibration limited design then they will consider loss co-efficient of the material. If a person wants to select a material for damage tolerance design then they will consider fracture toughness. If a person wants to select a material for strength limited and damage tolerance limited design then they will consider *E,* fracture toughness and failure strength of the material. The incorporation of the criteria is totally based on the designer's requirement. Energy absorption can also be a criterion to select a material for product development.

Chapter 6: Concluding Remarks

The materials have been considered as a key factor for managing the complexity while designing engineering products. Hence, optimal material selection is essential for sustainable product development. Material selection behind the epistemic uncertainty is a difficult issue. To shed some light on this issue (selection of a material under uncertainty) this thesis poses and answers the following questions:

How a material will be selected when the product specification is unknown?

What will be the tool to quantify the uncertainty?

What will be the method to select a material?

How the sustainable properties can be incorporated in the material selection procedure?

How to deal the uncertainties of the material properties?

Nevertheless, the following remarks can be made on the findings:

On Uncertainty of Material Index

1. Uncertainty is an integrated part of material universe, material properties, and material selection process.
2. When the Material Index for selecting an optimal material is unknown, a set of possibilistic objective functions can be used, instead.
3. The possibilistic objective functions can be used without any complicacy to incorporate the aspect of sustainability in a material selection process.
4. The interaction between the uncertainty of a material property and

relevant possibilistic objective function can be quantified by a concept called compliance.

5. The compliance can be aggregated for all objective functions in order to rank a material
6. Requirement or objectives of the customer can be represented by the possibility objective function.
7. A case study is made for a light, strong, stiff and sustainable car body. A material is selected from Al, Mg and Ti for three different conditions.
8. At initial case of the product designing, a sustainable material is selected under uncertainty; this selection of material will reduce the cost and complexity of the product.

On Uncertainty Quantification of Natural Material

1. The variability or uncertainty associated with the properties of a natural fiber-based material must be known beforehand to ensure the reliability of any eco-product made from it. Therefore, the uncertainty associated in the material properties of a natural material, called jute fiber, has been studied. In particular, tensile test is performed to determine the tensile strength and modulus of elasticity of jute yarn. The experimental results show that the tensile strength and modulus of elasticity of jute yarn vary significantly. The variability in the properties has been quantified using the conventional statistical approach (average, standard deviation, and skewness), widely used Weibull distribution and using the possibility distributions (a possibility distribution is probability-distribution-neutral representation of the uncertainty associated with a physical quantity). From the possibility distributions, the most possible and expected values of tensile strength modulus of elasticity and strain to failure have been determined.
2. The logically consistent ranges of tensile strength and modulus of

elasticity have also been determined. The lower limits of the ranges of jute yarn properties can be used as the design limits for jute product designing.

3. To quantify the uncertainty of material property, possibilistic approach is the most reliable one among the three. Therefore, it is recommended to use the possibility distribution for quantification of the uncertainty when the uncertainty is unknown, and the number of data is limited.

On Proposed Decision Model:

1. Using the proposed model, one can select a material for a component of the product, from the material universe. In addition, the model can be used to select a material.
2. Selecting appropriate materials at an early stage of a design process helps manage the complexity in the subsequent steps of product realization (detailed design, manufacturing, assembly, and operations management). Therefore, material selection entails a great deal of significance in engineering design.
3. The early stage of a design process means that the design specifications and requirements are not known. Therefore, conventional material selection procedures are not applicable for selecting materials at an early stage of a design process. This study sheds some lights on this issue by developing a novel decision model that helps make a decision even though the design specifications and requirements are still evolving.
4. In the presented decision model, the mathematical entities called triangular fuzzy number, compliance, and decision-score play a vital role. They are helpful for assessing and managing the heterogeneous decision-relevant information and conflicting objectives. The participation of a decision maker is also assured by introducing the user-defined importance in the calculation process of the decision-score.
5. Although a set of six criteria (*ρ, TS, E, Water Usage, CO_2 Footprint, and Recycle fraction*) is used in selecting materials for the body of a vehicle

under epistemic uncertainty, one can add other criteria (e.g., cost, reserve, thermal property, and alike) if needed. Adding criteria will enlarge the set of the degrees of compliances without adding any additional information processing steps in the decision making process. Therefore, the presented decision model possesses a great deal of scalabilities.

6. The advanced outlook on design process states that a design process is not just a knowledge-using process, but also a knowledge-creation process; the creation of knowledge takes place if one can handle the epistemic uncertainty in a systematic manner. As demonstrated in this study, the presented decision model can handle epistemic uncertainty in a systematic manner. It is also shown to be useful in creating new knowledge (e.g., it can create a list of material preferences even though the required design knowledge is not available). Therefore, the presented decision model can be integrated with a design process when knowledge-creation is preferred over knowledge-use. This is particularly true when a problem-based design is transformed into a solution-based design.
7. The variability or uncertainty associated with the properties of a natural fiber-based material must be known beforehand to ensure the reliability of any eco-product made from it. Therefore, the uncertainty associated in the material properties of a natural material, called jute fiber, has been studied. In particular, tensile test is performed to determine the tensile strength and modulus of elasticity of jute yarn. The experimental results show that the tensile strength and modulus of elasticity of jute yarn vary significantly. The variability in the properties has been quantified using the conventional statistical approach (average, standard deviation, and skewness), widely used Weibull distribution and using the possibility distributions (a possibility distribution is probability-distribution-neutral representation of the uncertainty associated with a physical quantity). From the possibility distributions, the most possible and expected values of tensile strength modulus of elasticity and strain to failure have been

determined. The logically consistent ranges of tensile strength and modulus of elasticity have also been determined. The lower limits of the ranges can be used as the design limits for jute product designing.

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List of Research Achievements

The following research publications have been submitted to defend the thesis

- Shahinur, S., Ullah, A.M.M.S., Alam, M.N.E., Haniu, H., and Kubo, A. A decision model for making decisions under epistemic uncertainty and its application to select materials. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing*, 31(3), 298-312, 2017, doi: 10.1017/S0890060417000191. Cambridge University Press. IF: 0.803, Cite Score: 1.06.
- Ullah, A.M.M.S., Shahinur, S., Haniu, H. On the Mechanical Properties and Uncertainties of Jute Yarns. *Materials*, 10(4), 450, 1-14, 2017, doi: 10.3390/ma10050450. MDPI. IF: 2.654, Cite Score: 3.26. Open Access.
- Shahinur, S., Ullah, A.M.M.S. Quantifying the Uncertainty Associated with the Material Properties of a Natural Fiber. *Procedia CIRP*, 61(2017), 541-546, 2017, doi: 10.1016/j.procir.2016.11.227, Elsevier. Cite Score: 1.6. Open Access.

International Conferences

- Shahinur, S., Ullah, A.M.M.S. Quantifying the Uncertainty Associated with the Material Properties of a Natural Fiber. Presented in The 24th CIRP Conference on Life Cycle Engineering, 8-10 March 2017, Kamakura, Japan. In Session: Sustainable Production Processes, Paper ID C13-3. [Digital Proceedings]

The other related publications of this study are as follows

- Shahinur, S., Hasan, M., Ahsan, Q. Physical and Mechanical Properties of Chemically Treated Jute Fiber Reinforced MAgPP Green Composites. *Applied Mechanics and Materials*, 860(2017), 134-139, 2017. doi: 10.4028/www.scientific.net/AMM.860.134. Trans Tech Publisher.
- Shahinur, S., Hasan, M., Ahsan, Q., Jafrin, S. Effect of Rot-Retardant Treatment on Properties of Jute Fiber. *Journal of Natural Fibers*, 14(2), 205-216, 2017. doi: 10.1080/15440478.2016.1193085. Taylor & Francis. IF: 0.974.
- Shahinur, S., Hasan, M., Ahsan, Q., Saha, D. K., & Islam, M. S. (2015). Characterization on the properties of jute fiber at different portions. *International Journal of Polymer Science*, vol. Article ID 262348, 6 Pages, 2015. doi: 10.1155/2015/262348. Hindawi. IF: 1.077, Cite Score: 1.02. Open Access.

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