Paper

Large-Scale FDTD Computation for Computational Electromagnetics

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Our recent studies related to the large-scale FDTD computation for computational electromagnetics are presented as an activity report. In this paper, high accuracy FDTD methods and the FDTD parallel computaion are described. These are applied to large-scale problems of electiromagnetic waves.

Keywords : FDTD method, high accuracy FDTD method, parallel computation, MPI, phase error

1. Introduction

The finite difference time domain (FDTD) method has been used widely to solve electromagnetic wave problems due to its versatility(1)~(3). Development of the high-speed large storage computer enables to analyze large-scale problems. In particular, modern supercomputers have the computational speed of several TFLOPS and a main memory of several gigabytes. Therefore, the FDTD computation are becoming possible for the analysis of problem such as scattering, indoor radio propagation, vehicles in the microwave frequency range for communications, broadcasting, and intelligent transport systems (ITS) by using these high-speed computers.

When we carry out large-sale FDTD analysis of a system much larger than the wavelength, the following problem and necessity arise, (1) accumulation of phase error due to numerical dispersion in the simulation method as waves propagate long distance, (2) development of a program suitable for high-speed computer.

(1) In order to reduce phase errors of the FDTD method, several methods have been proposed(4)~(19) such as the method of modifying the speed of light, the FDTD(2,4) method and the nonstandard FDTD (NS-FDTD) method. The method of modifying the speed of light is based on the principle that the numerical phase velocity, which is slower than the physical phase velocity, can be reduced by the increase of numerical phase velocity(4)~(8). The FDTD(2,4) method is second- and forth-order accuracy in time and space(9)(10). In the method, the maximum time step we can take is smaller than the usual FDTD method. The NS-FDTD method is developed to obtain highly accurate result at one fixed frequency(11)~(19). It is possible to reduce phase error to less than 1/1000 compared to the FDTD method.

(2) The high performance of supercomputers is achieved by parallel computation and the use of distributed-memory type architecture. The FDTD method is an explicit three-dimensional analysis method and a value of an electromagnetic field component can be calculated by using the values at the corresponding node and the neighbor nodes. In the method, the amount of data to be transferred between neighboring subregions is small. Therefore, the method is suitable for a parallel computing with the distributed-memory architecture. In distributed-memory computers, a parallel computation should be carried out using software such as parallel virtual machine (PVM) library or message passing interface (MPI)(20)~(22). Especially, the MPI library was developed by MPI forum in 1995, and is becoming the new international standard for parallel programming.

In this paper, our recent studies related to above topics are presented as an activity report. First, the characteristics of high accuracy FDTD methods are investigated and compared. Next, an FDTD parallel computation system we have recently developed using the MPI library for large-scale problems is described. Using the system, we analyzed, for the first time, the radiation characteristics of an antenna mounted on a realistic vehicle model in the microwave frequency range.

2. The Reduction of Phase Error of the FDTD Method

In the FDTD method, numerical results converge to true one as time and spatical increment decrease. However, it is impossible to decrease these increments small enough in a actual analysis. At this time, the phase error is generated in the method due to numerical dispersion caused by discretization.

In this section, the spatial increment is assumed to be $\lambda/10$. The time increment Δt is set to the maximum value we can take by the Courant stability condition for all methods. The Yee cell is assumed to be a cubic cell for simplicity. The results have the generality, so the results can be applicable to the case of a rectangular cell.

Fig. 1 shows the propagation wave form of electric fields E_y in the waveguide near the place 50λ apart from the source. The phase of the FDTD method is delayed more than $\lambda/2$ compared with the theory. The reason why the phase delays is that the numerical

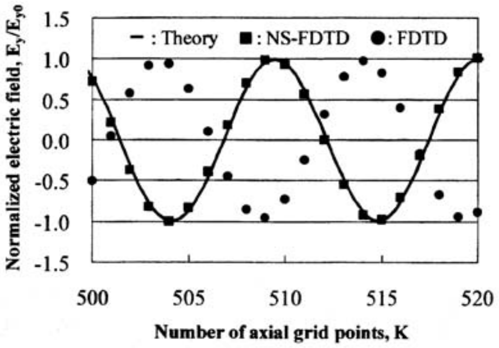

Fig. 1. Propagation waveforms on the waveguide axis in the FDTD method(15)

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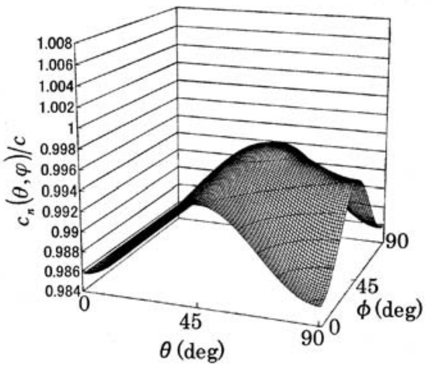

Fig. 2. Numerical phase velocity $c_n(\theta,\phi)$ of the FDTD method for $\Delta x=\Delta y=\Delta z=\lambda/10$(7)

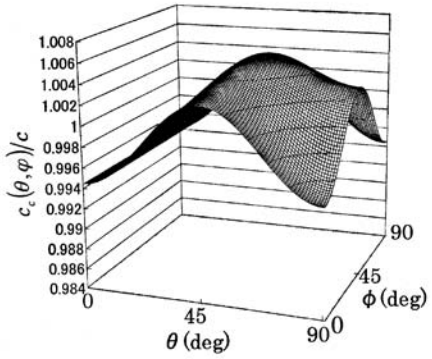

Fig. 3. Numerical phase velocity $c_n(\theta,\phi)$ of the FDTD method by modified the speed of light for $\Delta x=\Delta y=\Delta z=\lambda/10, \gamma_r= 1.00883$(7)

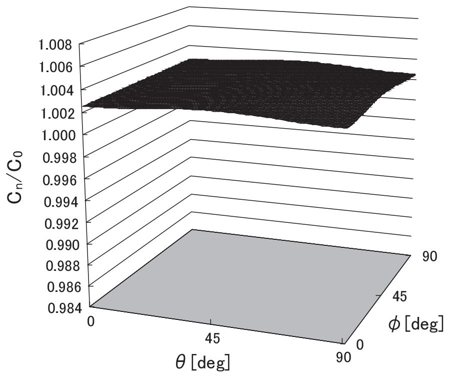

Fig. 4. Numerical phase velocity c_n of the FDTD(2, 4) method for $\Delta x=\Delta y=\Delta z=\lambda/10$

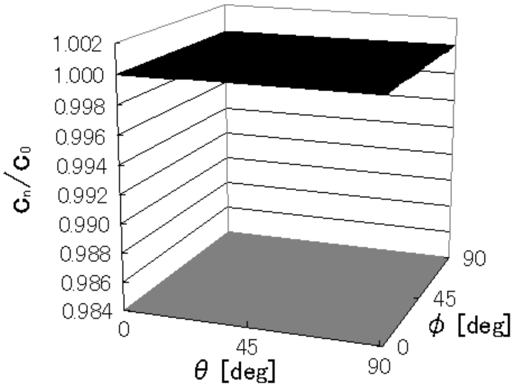

Fig. 5. Numerical phase velocity c_n of the NS-FDTD method for $\Delta x=\Delta y=\Delta z=\lambda/10$(14)

phase velocity of the FDTD method is slower about 1.2 % than the physical one. Fig. 2 shows numerical phase velocity c_n in the free space normalized by the physical one c. θ and ϕ are coordinates in the spherical coordinate. The figure shows that the value of c_n is less than c in every direction. Especially, c_n is the slowest in the x, y, and z directions and 1.4 % under.

2.1 Modification of the Speed of Light in the FDTD Method(4)~(8)

The simplest method to reduce the phase error is the method of modifying the speed of light in the FDTD method(4)(5)(7). The method is for one fixed frequency. In the method, the values of permittivity ε_0 and permeability μ_0 of vacuum are reduced by a coefficient γ_r. Then, the c_n can be increased by γ_r. Therefore, the error of phase velocity can be reduced to 0.6 % which is half of the error of the conventional FDTD method as shown in Fig. 3(7). However, the anisotropic characteristic of phase error cannot be corrected. When a cell is a rectangular, phase errors can be reduced by modifying the speed of light anisotropically(6)(8). That is, it is realized by modifying the permeabilities ε_ζ and the permeabilities μ_ζ (ζ=x,y, and z), similarly in the case of cubic cells.

2.2 FDTD(2, 4) Method(9)(10)

In this method, the time difference operator with the same accuracy as that in the FDTD method is used, but the spatial difference operator with second-order accuracy in the conventional FDTD method is replaced with one with fourth-order accuracy. Hence, the present method has second-order accuracy in time and fourth-order accuracy in space(9)(10). Fig.4 shows numerical phase velocity c_n of the FDTD(2, 4) method. The maximum time step for stable analysis in the FDTD(2, 4) method is 6/7 that of the FDTD method. Similarly, the symplectic fourth-order time-domain scheme has been proposed(11).

2.3 Non-Stanldard FDTD Method(12)~(19)

The most accurate method is the nonstandard FDTD method(12). The method is proposed by Cole for cubic cells. The method is second- and sixth-order accuracy in time and space. However, the application of the method is restricted to sine wave input(17)(19). Fig. 5 shows the numerical phase velocity(14). The numerical phase velocity c_n agrees well to the theoretical value in the every direction. It is found that the order of error is less than 10^{-5} % regardless of the angle. So, the phase error is extremely small. Recently, we have extended the method to the case of rectangular cells. The error characteristic is similar to the case of cubic cells. Furthermore, the method has been extended to the case of lossy media(13)(17).

2.4 Comparison of Maximum and Minimum Values of Numerical Phase Velocity of Methods in Each Frequecy(18)

In the following, cubic cells are assumed for simplicity. In the case of rectangular cells, the results are similar to the case of cubic cells. The results depend on the time increment Δt. However, fundamental characteristics do not depend on the time increment. In this study, the time increment Δt is set to the maximum value we can take by the Courant stability condition for all methods.

In this section, the maximum and minimum values of phase velocity are shown for each frequency instead of the usual global error expression. The usual global error expression cannot indicate

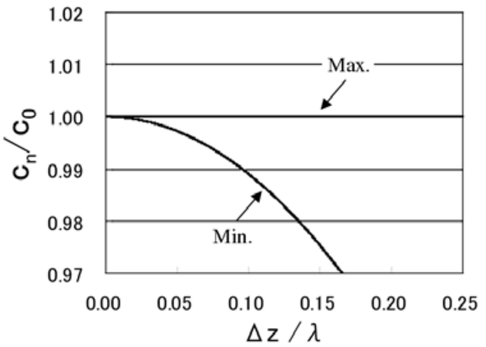

Fig. 6. Numerical phase velocity of the FDTD method in the case of cubic grids(18)

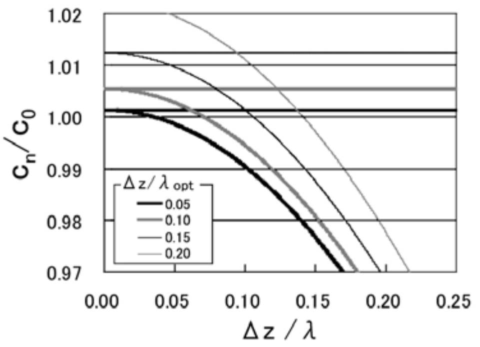

Fig. 7. Numerical phase velocity of the FDTD method by modifying the speed of light in the case of cubic grids(18)

anisotropic characteristics of phase error. To the contrary, the expression used in the paper can indicate those.

Fig. 6 shows numerical phase velocity of the FDTD method. Fig. 7 shows numerical phase velocity of the FDTD method by modifying the speed of light. λ_{opt} in the figure is the desired correction wavelength. The phase velocity errors are reduced at each desired frequency. Fig. 8 shows numerical phase velocity of the FDTD(2,4) method. Fig. 9 shows numerical phase velocity of the NS-FDTD method.

Here, characteristics of each method are compared and summarized. The FDTD method by modifying the speed of light is the simplest method, but the method has limitations in reducing the phase errors. The FDTD(2,4) method has wide-band characteristics, but the stability condition is rather severe compared with the usual FDTD method. The NS-FDTD method requires many arithmetical steps, but has extremely high-accuracy characteristics near the desired frequency.

2.5 Application of the NS-FDTD Method to Large-Scale Scattering Analysis(19) The radar cross section of the large cavity shown in Fig. 10 is computed. Fig. 11 shows the numerical results for the monostatic radar cross section of the cavity. Also shown in Fig. 11 are measured results and the numerical results obtained by the SBR method and the IPO method, which are high-frequency approximation methods. The results of the NS-FDTD method agree well with the measured results and with those obtained by the IPO method. The results obtained by the FDTD method are close to the measured results except near $\phi^i = 50$ [deg.] . Since only the magnitude is plotted in Fig. 11, the results

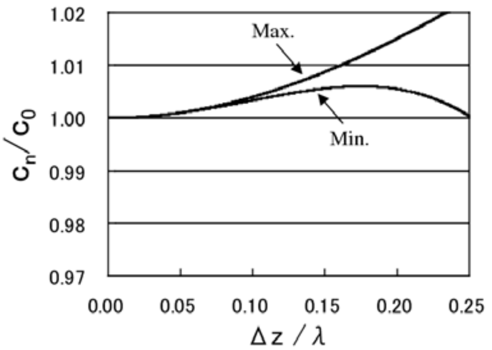

Fig. 8. Numerical phase velocity of the FDTD(2,4) method in the case of cubic grids(18)

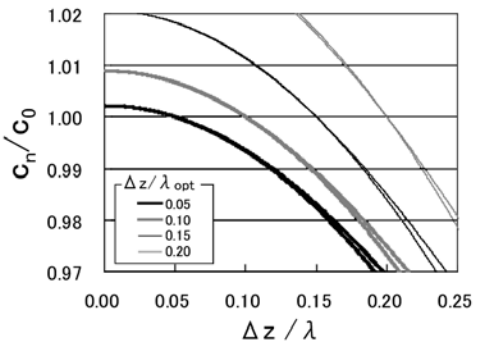

Fig. 9. Numerical phase velocity of the NS-FDTD method in the case of cubic grids(18)

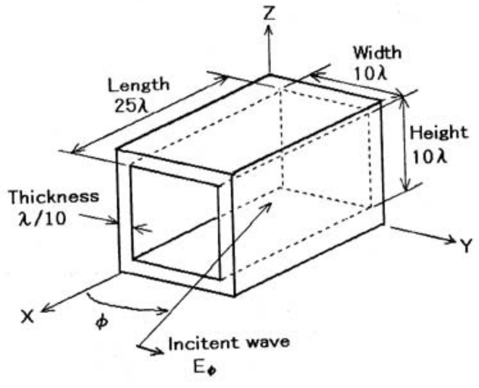

Fig. 10. Large-scale rectangular cavity(19)

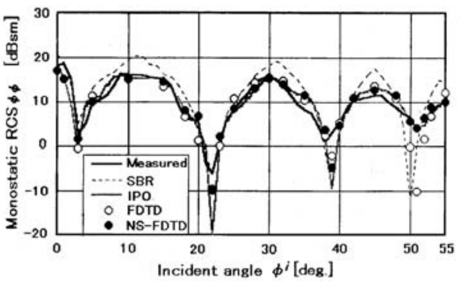

Fig. 11. Monostatic radar cross section of a rectangular cavity for θ^i=90 [deg.] at 10 GHz(19)

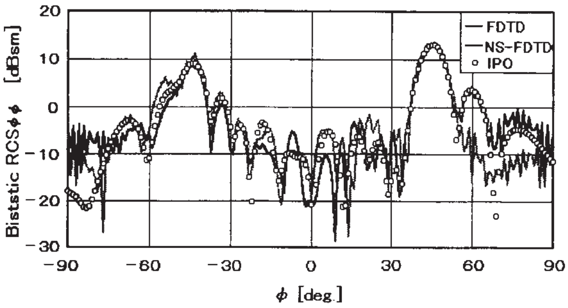

Fig. 12. Bistatic radar cross section of a rectangular cavity for θ^i=90 [deg.] and ϕ^i=45 [deg.] at 10 GHz(19)

obtained by the FDTD method with large phase errors in the region with strong geometrical optics components appear to coincide with the monostatic radar cross section. Fig. 12 shows the obtained bistatic radar cross sections. The results of the nonstandard FDTD method agree extremely well with those obtained by the IPO method near the main lobes. On the other hand, the results of the FDTD method exhibit discrepancies near the main lobes.

3. FDTD Parallel Computation(22)~(24)

3.1 Parallel Computer Parallel computers can be classified into shared-memory types and distributed-memory types. In a shared-memory computer, multiple processors share one main memory through buses and cross-bar switches. On the other hand, in a distributed-memory computer, each processor is connected to a corresponding main memory, where the set of a processor and main memory is generally called the node. A parallel supercomputer generally has an architecture consisting of both shared-memory and distributed-memory types to boost speed as shown in Fig. 13.

An automatically parallelizing compiler performs well in a shared-memory computer, but not very well in a distributed-memory computer. Consequently, parallel programming is necessary in order to gain the maximum performance from a parallel supercomputer.

3.2 Parallel Programming Using Fortran90 and the MPI Library In the parallel programming, we can use either a parallel programming language or a message passing library. When using a parallel programming language such as high-performance Fortran (HPF), we only have to add some commands to the head of a conventional code. This approach is thus comparatively easy to implement, but the performance is not so good. On the other hand, when a message passing library is used, we have to drastically modify the conventional code, but the performance is quite high. As a message passing library, MPI and PVM libraries are well known, and the MPI library is becoming a de facto standard in parallel computation. In our study, the MPI library is used.

In our study, Fortran90 was used as a programming language. In our system, dynamic memory allocation was used. Therefore, each subregion can have a memory of different size. Furthermore, we can easily handle the global index because the local index of a subregion can correspond to the global index of a total region as shown Fig. 14.

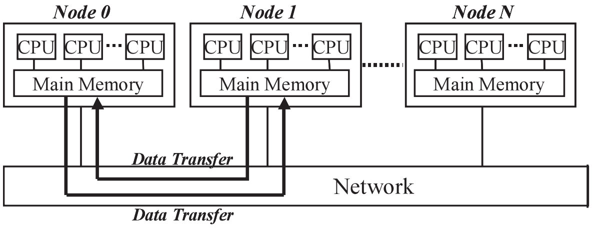

Fig. 13. Shared - distributed memory computer(24)

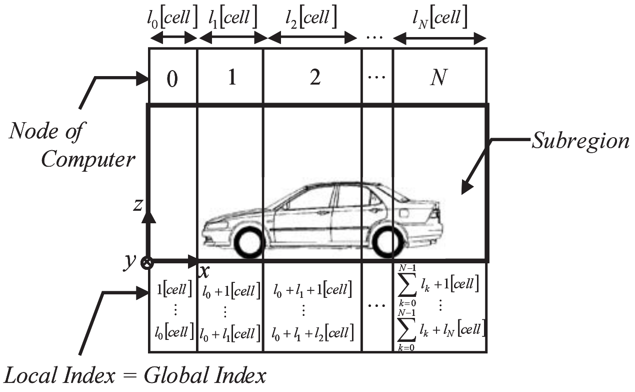

Fig. 14. FDTD parallel computation using dynamic memory allocation(24)

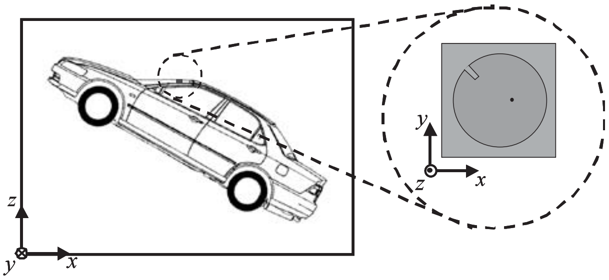

Fig. 15. Vehicle and position of the antenna used in the simulation(24)

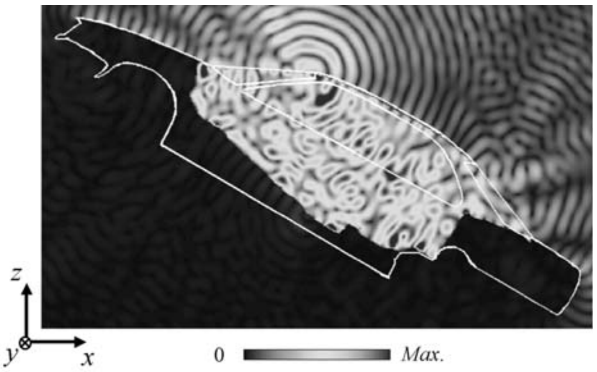

Fig. 16. Snapshot of the instantaneous amplitude distribution of electric field inside and outside the vehicle(24)

3.3 FDTD Large-Scale Parallel Supercomputing to Obtain the Radiation Characteristics of an Antenna Mounted on a Vehicle We applied the FDTD parallel computation system to the analysis of the radiation characteristics of an antenna mounted on a vehicle. Fig. 15 shows a vehicle and position of the antenna used in the simulation. The patch antenna is mounted inside the windshield of the vehicle. The vehicle is inclined to fit the patch antenna plane to an FDTD lattice. In this model, the ground, tires and so on are not considered. The simulation was carried out by

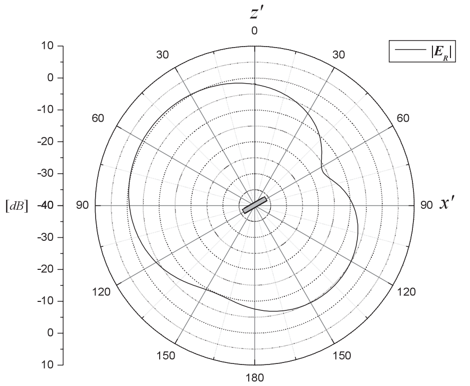

Fig. 17. Radiation pattern of the patch antenna for a right-hand circularly polarized wave at 1.520 GHz(24)

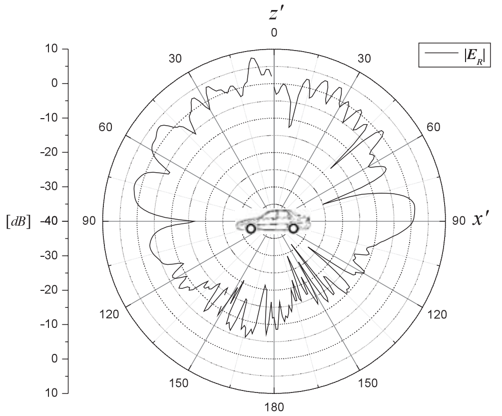

Fig. 18. Radiation pattern of the patch antenna mounted on the vehicle for a right-hand circularly polarized wave at 1.520 GHz(24)

using an SR8000 parallel supercomputer (Hitachi, Ltd). Fig. 16 shows a snapshot of the instantaneous amplitude distribution of electric field inside and outside the vehicle in the zx plane. The radiated waves through the windows propagate mainly in the upper direction. Fig. 17 shows the radiation pattern of the patch antenna for a right-hand circularly polarized wave at 1.520 GHz. Fig. 18 shows the radiation pattern of the patch antenna mounted on the vehicle. The radiation pattern of the antenna mounted on a vehicle shown in Fig. 18 is very different from those of an antenna itself shown in Fig. 17. In Fig. 18, a lot of null points can be observed, because the vehicle is very large compared with the wavelength of radiated waves.

4. Others

The FDTD method is applicable to not only computational electromagnetics but also other computational physics. The method has been applied to the analysis of sound fields such as active noise control system(25)~(27). Furthermore, the method has been applied to the elastic wave ploblem(28).

The visualization of electromagnetic fields is important for the education of electromagnetics. The simulated results of electromagnetic waves by using the FDTD method are visualized in our laboratory and used for the tuition(29).

5. Conclusion

In this paper, recent our studies toward the large-scale FDTD computation were described. First, the characteristics of high accuracy FDTD methods are discussed, next, the FDTD parallel computation system we have developed and its application to the mobile antenna analysis are described. In the future, high accuracy FDTD methods will be incorporated into our CAD system. In the large-scale FDTD computation, various problems we have not encountered in the small-scale computation arise. We are going to clear up these problems.

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