LETTER *Special Section on Analog Circuit Techniques and Related Topics*

Design Considerations for RC Polyphase Filters with Simultaneously Equal Ripple Both in Stopband and Passband

Hiroaki TANABE[†], ***Student Member and*** **Hiroshi TANIMOTO**[†a)], ***Member***

SUMMARY This paper describes a numerical design procedure of element values of RC polyphase filters with equal minima in stopband and equal ripple in passband. Determination of element values of RC polyphase filters with equal-ripple characteristic have not been solved to the best knowledge of the authors. There found a paper tackling with the problem; however, it can only give sub-optimal solutions via numerical calculation [3]. We propose a numerical element value design procedure for RC polyphase filters with equi-ripple gain in both stopband and passband by using the coefficient matching method. Some design examples are given.

key words: *RC polyphase filter, equal ripple, element value design, image rejection, analog complex coefficient filters*

1. Introduction

The RC polyphase filter is a kind of analog complex coefficient filters, and is becoming very popular for its capability of image rejection in wireless communication systems [1].

The RC polyphase filters are often used in cascaded sections in order to obtain wider stopband and/or passband widths. In such cascaded RC polyphase filters, to have simultaneously equal ripple in both stopband and passband may be a major concern. Such equal ripple RC polyphase filter has been introduced and analyzed by Gingell [2]; however, he treats the equal ripple transfer function only, and it seems that the procedure determining its exact element values has not completely been solved to date, to the best knowledge of the authors.

Recently, Wada et al tackled with this problem and presented an element value design procedure for chebyshev gain in stopband and very flat gain in passband [3]. They make use of symmetry of the zero locations for the equal ripple transfer function of the RC polyphase filter, which were obtained in [2], and introduced a single design parameter in order to determine the element values.

The parameter is determined in such a way that the passband becomes equal ripple. This seemingly yields an "equal ripple" response in the passband and stopband. However, the resulting number of ripples in passband is always less than a number of cascaded stages, n, for $n > 3$. This passband behavior could be attributed that they introduced only one design parameters for any number of cascaded stages. So, the resulted design is sub-optimal in the sense of chebyshev approximation and could be improved to have much smaller ripple value.

We propose a design procedure that yields exact element values to realize the optimal equal ripple transfer function of the RC polyphase filter given by Gingell, by using the coefficient matching method instead of introducing new design parameters.

Some design examples and considerations for cascading order of the each RC polyphase filter section are presented.

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[†]The authors are with the Department of Electrical and Electronic Engineering, Kitami Institute of Technology, Kitami-shi, 090-8507 Japan.
a) E-mail: tanimoto@elec.kitami-it.ac.jp

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2. Transfer Function of Equal Ripple RC Polyphase Filter [2]

An equal ripple RC polyphase filter can be characterized by three parameters n, x, and ε. Here, n is a number of RC polyphase filter sections, x is a square root of a reciprocal bandwidth defined by $x \equiv \sqrt{\omega_L/\omega_H}$ (< 1), where ω_L and ω_H are angular frequencies of lower edge and higher edge of passband and/or stopband, respectively. ε is a ripple parameter.

These three parameters are not independent but must satisfy the following constraint, to have the equal ripple characteristic [2]:

$$4n \times K(x^2)\Big/K(\sqrt{1-x^4}) = K(\sqrt{1-\varepsilon^4})\Big/K(\varepsilon^2), \tag{1}$$

where, $K(m)$ is a complete elliptic integral of the first kind with a modulus m. Using the parameter ε, passband ripple a_p and stopband ripple a_s can be calculated in dB:

$$a_p = 10\log(1+\varepsilon^2),\ a_s = 10\log(1+1/\varepsilon^2). \tag{2}$$

The voltage transfer function of the RC polyphase filter with simultaneously equal ripple in both stopband and passband is given by [2]:

$$H(s) = \prod_{r=1}^{n} \frac{sx + \mathrm{j}\,\mathrm{dn}\left[\left(\frac{2r-1}{2n}\right)K(\sqrt{1-x^4}),\ \sqrt{1-x^4}\right]}{sx + \mathrm{cs}\left[\left(\frac{2r-1}{2n}\right)K(\sqrt{1-x^4}),\ \sqrt{1-x^4}\right]}, \tag{3}$$

where, dn(·) and cs(·) are Jacobian elliptic functions [4].

Equation (3) can be factorized in the following form:

$$H(s) = \frac{(1-\mathrm{j}s\tau_{z1})(1-\mathrm{j}s\tau_{z2})\cdots(1-\mathrm{j}s\tau_{zn})}{(1+s\tau_{p1})(1+s\tau_{p2})\cdots(1+s\tau_{pn})} = \frac{(1-\mathrm{j}s\tau_{z1})(1-\mathrm{j}s\tau_{z2})\cdots(1-\mathrm{j}s\tau_{zn})}{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}, \quad (4)$$

where, time constants of zeros and poles are given by

$$\tau_{zr} = \left\{\frac{1}{x}\mathrm{dn}\left[\left(\frac{2r-1}{2n}\right)K,\ \sqrt{1-x^4}\right]\right\}^{-1}, \quad (5)$$

$$\tau_{pr} = \left\{\frac{1}{x}\mathrm{cs}\left[\left(\frac{2r-1}{2n}\right)K,\ \sqrt{1-x^4}\right]\right\}^{-1}. \quad (6)$$

3. Circuit Analysis by Using F Matrix

Circuit topology of a single stage RC-polyphase-filter is shown in Fig. 1. If the four input voltage sources constitute a single perfect four-phase voltage source (voltages with equal magnitude and 90° phase shift each other), then the output voltages and currents in the network are also symmetric four phase signals, because the network is also a symmetric four phase circuit. That is, if input voltages and currents of network are given by

$$V_{in2} = jV_{in1},\ V_{in3} = -V_{in1},\ V_{in4} = -jV_{in1}, \quad (7)$$

$$I_{in2} = jI_{in1},\ I_{in3} = -I_{in1},\ I_{in4} = -jI_{in1}, \quad (8)$$

then, output voltages and currents are given by

$$V_{out2} = jV_{out1},\ V_{out3} = -V_{out1},\ V_{out4} = -jV_{out1}, \quad (9)$$

$$I_{out2} = jI_{out1},\ I_{out3} = -I_{out1},\ I_{out4} = -jI_{out1}. \quad (10)$$

Thus, the circuit can be described by an F matrix for a single phase circuit, owing to the circular symmetry of the circuit [3]. Now, the single stage RC polyphase filter is described by

$$\boldsymbol{F} = \frac{1}{1-jsCR}\begin{pmatrix} 1+sCR & R \\ 2sC & 1+sCR \end{pmatrix}. \quad (11)$$

When n RC polyphase filter sections are connected in cascade, the total F matrix is given by

$$\begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix} = \boldsymbol{F}_1\boldsymbol{F}_2\cdots\boldsymbol{F}_n\begin{pmatrix} V_{out} \\ I_{out} \end{pmatrix} \quad (12)$$

$$= \frac{1}{\prod_{i=1}^{n}(1-jsC_iR_i)}\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} V_{out} \\ I_{out} \end{pmatrix}, \quad (13)$$

where, $\boldsymbol{F}_i$ is an F matrix of the i-th stage, and R_i, C_i are resistor and capacitor values for the stage. Thus, a voltage transfer function of the RC polyphase filter, $H(s)$, is given by the following equation:

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{(1-jsC_1R_1)\cdots(1-jsC_nR_n)}{A}. \quad (14)$$

Because the matrix entry A is an n-th order polynomial of s, we can write it down with real coefficients a_i ($i = 1, 2, \ldots, n$). We have

$$H(s) = \frac{(1-jsC_1R_1)(1-jsC_2R_2)\cdots(1-jsC_nR_n)}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}. \quad (15)$$

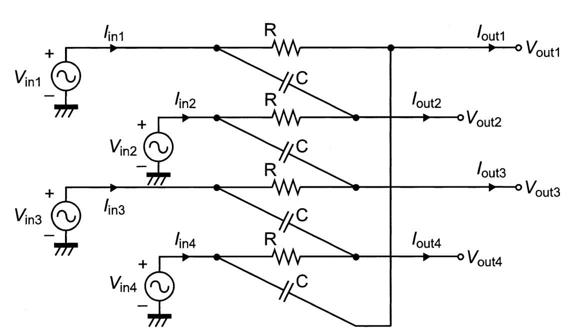

Fig. 1 Basic single stage RC polyphase filter circuit.

4. Element Value Design by Coefficient Matching Method

We already have the transfer function Eq. (4), which gives equal ripple stopband and passband *in terms of poles and zeros*, and now we have the transfer function Eq. (15) of the n-stage RC polyphase filter *in terms of its element values*. Thus, we are ready to obtain element values that give the equal ripple characteristics by using these two transfer functions, in principle.

However, we observe the fact that while only the time constant, R_iC_i, for each stage is determined directly from the zeros of $H(s)$, we cannot solve for individual R_i and C_i values in terms of pole locations, because the equations become nonlinear. Thus, we have to adopt some numerical method to find the poles for general RC polyphase filter circuits. We employ the coefficient matching method which has been widely used in the field of filter synthesis [5].

The coefficient matching method solves the problem by equating the same order coefficients of denominator polynomials and numerator polynomials, respectively, from Eqs. (4) and (15), i.e, solving the nonlinear simultaneous equations for $i = 1, 2, \ldots, n$:

$$\begin{cases} a_i(R_1, C_1, \ldots, R_n, C_n) = b_i(\tau_{p1}, \tau_{p2}, \ldots, \tau_{pn}) \\ R_iC_i = \tau_{zi} \end{cases} \quad (16)$$

One important observation from this equation is that the i-th zero inevitably sticks to the i-th section. So, if we change assigning the order of zeros, different RC values may appear. This point will be revisited later.

We arbitrarily set "nominal impedance" of $R_1 = 1\,[\Omega]$ to obtain normalized solution, because always the relation $a_0 = b_0 = a_n = b_n = 1$ holds. Other impedance solutions may simply be obtained through impedance scaling as usual.

For numerical solution of Eq. (16), a mathematical software *Mathematica*™ was employed.

5. Design Example

Designed element values by the coefficient matching method for ω_H/ω_L = 10, 30, 100 and n = 3, 4, 5 are shown in Table 1. This table exactly corresponds to the Table 1 of reference [3], and hence, can be compared directly.

These results are obtained by placing the time constants of zeros in descending order; that is, the first stage has the largest zero time constant and the second stage has second largest one, ..., and the last section has the smallest one. The descending order is chosen to conform with the realization presented in reference [3]. Changing the order of zeros will later be discussed in Sect. 6.

Figure 2(a) shows a comparison of passband response by this work's design and the design given in reference [3]. The previous design for n = 5 has only three peaks in the passband while the present design has just five peaks and much smaller ripple, which exactly meets the theoretically expected value. The ripple values are very small for these two designs, however, and may not cause significant differ-

Table 1 Element value of RC polyphase filters ($\sqrt{\omega_L \omega_H}$ = 1 [rad/s]).

n	$\frac{\omega_H}{\omega_L}$	ripple ε ($\omega_L \leq \omega \leq \omega_H$)
Time constant of zero : $\tau_{z1}, \tau_{z2}, \ldots, \tau_{zn}$ [s]		
Time constant of pole : $\tau_{p1}, \tau_{p2}, \ldots, \tau_{pn}$[s]		
Resistor : $R_1(=1), R_2, \ldots, R_n[\Omega]$		
Capacitor : $C_1(=\tau_{z1}), C_2, \ldots, C_n$[F]		
3	3	0.0048188
1.604593, 1, 0.6232110		
3.976315, 1, 0.2514891		
1, 1.94382, 3.77844		
1.60459, 0.51445, 0.164939		
3	10	0.0360503
2.649642, 1, 0.3774095		
4.819564, 1, 0.2074877		
1, 1.61159, 2.59723		
2.64964, 0.620504, 0.145312		
3	30	0.0907769
4.102008, 1, 0.2437830		
6.184086, 1, 0.1617054		
1, 1.41595, 2.00492		
4.10201, 0.706239, 0.121592		
3	100	0.168996
6.487809, 1, 0.1541352		
8.524637, 1, 0.1173070		
1, 1.27502, 1.62568		
6.48781, 0.784299, 0.0948125		
4	10	0.009452267
2.855521, 1.506315, 0.6638717, 0.3501988		
6.605261, 1.674982, 0.5970215, 0.1513945		
1, 1.68378, 3.23279, 5.44332		
2.85552, 0.894601, 0.205355, 0.0643354		
4	30	0.0323814
4.624798, 1.774832, 0.5634337, 0.2162256		
8.625454, 1.866104, 0.5358758, 0.1159359		
1, 1.59840, 2.46547, 3.94080		
4.62478, 1.11038, 0.22853, 0.0548684		
4	100	0.0741678
7.729276, 2.092523, 0.4778921, 0.1293782		
12.180770, 2.137451, 0.4678469, 0.08209662		
1, 1.45137, 2.01845, 2.92953		
7.72928, 1.44175, 0.236761, 0.0441635		
5	10	0.00247835
2.959996, 1.894867, 1, 0.5277415, 0.3378383		
8.363023, 2.333639, 1, 0.4285152, 0.1195740		
1, 1.550276, 3.450017, 7.677742, 11.902616		
2.959996, 1.22228, 0.289854, 0.0687365, 0.0283835		
5	30	0.0115508
4.904764, 2.468595, 1, 0.4050887, 0.2038834		
11.011944, 2.757818, 1, 0.3626055, 0.09081049		
1, 1.705158, 2.862169, 4.804255, 8.192012		
4.90476, 1.44772, 0.349385, 0.0843187, 0.0248881		
5	100	0.0325472
8.439469, 3.226576, 1, 0.3099260, 0.1184909		
15.731648, 3.4072611, 1, 0.2934909, 0.06356613		
1, 1.597025, 2.410326, 3.637808, 5.809671		
8.43947, 2.02037, 0.414882, 0.0851958, 0.0203955		

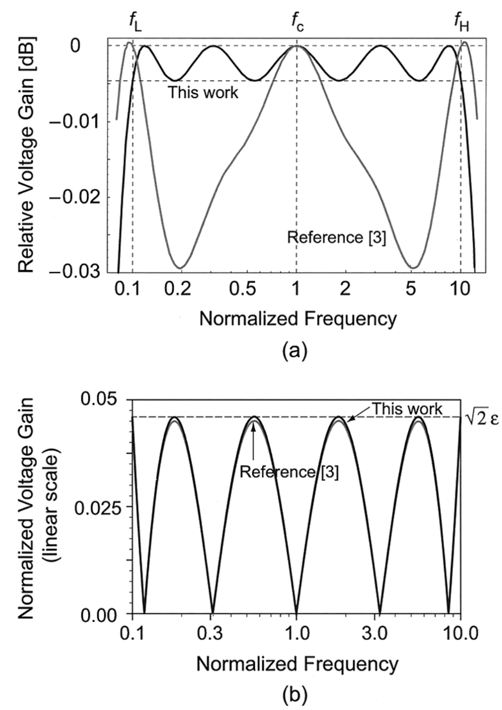

Fig. 2 Design example for n = 5, ω_H/ω_L = 100, ε = 0.0325472 (a_p = 0.004598 dB). (a) Passband detail. The 0 dB level actually is 3.0103 dB. (b) Stopband detail. Gain has been raised by 3 dB in accordance with (a). These plots are drawn from spice simulation results.

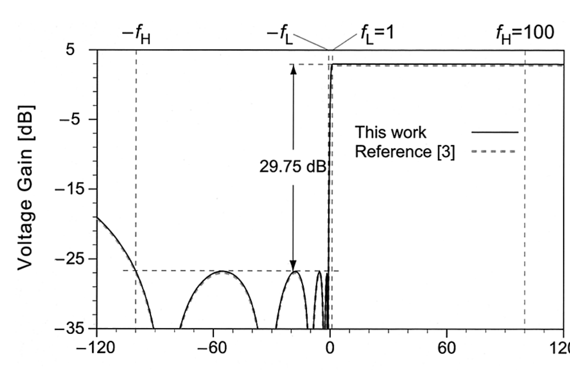

Fig. 3 Total frequency response of design example.

ences in practice, for this example. The stopband details are almost the same (see Fig. 2(b) and Fig. 3). Actual element values of this example are listed in the bottom of Table 1 of the present paper and the Table 1 of reference [3].

6. Cascading Order of Sections (Sequence of Zeros)

Wada et al. designed the element values with descending or ascending order only [3]. In contrast, our method does not use any symmetric nature of zero locations, so that we can freely assign the zero order when cascading the sections, in principle. This can be done just interchanging the order of zeros in the second equation of Eq. (16).

Table 2 Element value dependence on cascading order.

sequence	R_1	R_2	R_3	R_4
	C_1	C_2	C_3	C_4
1234	1.000E+00	1.598E+00	2.465E+00	3.941E+00
	4.625E+00	1.111E+00	2.286E-01	5.487E-02
1243	1.000E+00	-	-	-
1324	1.000E+00	-	-	-
1342	1.000E+00	3.942E-01	7.107E-01	2.903E+01
	4.625E+00	1.429E+00	3.043E-01	6.113E-02
1423	1.000E+00	4.468E-01	1.963E+00	1.011E+01
	4.625E+00	4.839E-01	9.041E-01	5.576E-02
1432	1.000E+00	1.109E-01	1.176E+06	6.549E+07
	4.625E+00	1.950E+00	4.791E-07	2.710E-08
2134	1.000E+00	-	-	-
2143	1.000E+00	-	-	-
2314	1.000E+00	5.147E+00	2.261E+01	1.011E+01
	1.775E+00	1.095E-01	2.045E-01	2.140E-02
2341	1.000E+00	1.769E+01	5.045E+07	1.196E+08
	1.775E+00	1.597E-02	4.286E-09	3.866E-08
2413	1.000E+00	1.049E+00	8.538E+00	8.954E+00
	1.775E+00	2.061E-01	5.417E-01	6.293E-02
2431	1.000E+00	4.977E+00	2.338E+01	7.565E+01
	1.775E+00	4.344E-02	2.410E-02	6.113E-02
3124	1.000E+00	4.085E+01	7.364E+01	2.903E+01
	5.634E-01	1.132E-01	2.410E-02	7.448E-03
3142	1.000E+00	8.607E+00	3.277E+00	2.820E+01
	5.634E-01	5.373E-01	6.598E-02	6.293E-02
3214	1.000E+00	5.569E+01	7.072E+05	7.843E+04
	5.634E-01	3.187E-02	1.041E-06	2.757E-06
3241	1.000E+00	1.621E+01	8.678E+00	8.294E+01
	5.634E-01	1.095E-01	2.492E-02	5.576E-02
3412	1.000E+00	-	-	-
3421	1.000E+00	-	-	-
4123	1.000E+00	2.372E+00	9.535E+04	1.686E+06
	2.162E-01	3.103E-01	2.962E-06	5.319E-08
4132	1.000E+00	9.557E+00	5.116E+00	8.294E+01
	2.162E-01	7.702E-02	1.753E-02	3.406E-03
4213	1.000E+00	3.236E+00	1.520E+01	7.565E+01
	2.162E-01	8.729E-02	4.842E-02	1.185E-03
4231	1.000E+00	-	-	-
4312	1.000E+00	-	-	-
4321	1.000E+00	4.165E+00	2.024E+01	8.429E+01
	2.162E-01	2.153E-02	1.396E-02	8.733E-03

· Note1: "Sequence" is given in the indices of zero's time constants; i.e, $\tau_{z1} > \tau_{z2} > \tau_{z3} > \tau_{z4}$, and the sequence "1324" implies the zeros are allocated to each single stage RC-polyphase-filter section in the order of τ_{z1}-τ_{z3}-τ_{z2}-τ_{z4}, from input side to output side.
· Note2: "–" indicates no convergence.

To explore this advantage, we designed RC polyphase filters of $n = 4$ with different zero orders. There exists $n!$ permutations of sections for an RC polyphase filter with n sections, so that an attempt to check all the permutations may soon fail except for very small n.

For the RC polyphase filter of which specifications are $n = 4$, $\omega_H/\omega_L = 30$, and $\varepsilon = 0.0323814$, Table 2 shows the result of design by the proposed method for all the possible zeros sequences. The table shows only the element values for R's and C's, for each zeros sequence. Some data are lacking due to numerical non-convergence.

The numerical solutions are very sensitive to initial guesses, and in some cases, we could not obtain the solution. Beside the numerical methods, we need some theoretical analysis to explore further this degree of freedom.

However, it is very interesting that the minimum element value spread, which is defined by the largest ratio of element values of the like kind, of 28.2 was obtained for 2-4-1-3 and 3-1-4-2 sequences, while it is 84.3 for the descending order. Some sequences like 1-4-3-2 and 2-3-4-1 result in extremely large element spread as large as 5.9×10^8. This preliminary result clearly indicates the importance of choice for zeros' sequence in the design.

7. Conclusion

Element value design technique based on coefficient matching method has been proposed for RC polyphase filters with simultaneously equal ripple in both stopband and passband.

The proposed method has a capability of arranging the zeros sequence in any order. We found that there exists a zeros sequence that gives minimal element value spread.

No convergence in numerical calculations and analytical approach to find the best zeros sequence are future problems.

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