PAPER *Special Section on Analog Circuit Techniques and Related Topics*

Transfer Function Preserving Transformations on Equal-Ripple RC Polyphase Filters for Reducing Design Efforts

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SUMMARY Element value spread of an equal-ripple RC polyphase filter depends heavily on the order of zero assignment. To find the optimum design, we must conduct exhaustive design for all the possible zero assignments. This paper describes two circuit transformations on equal-ripple RC polyphase filters, which preserve their transfer functions, for reducing circuit design efforts. Proposed Method I exchanges (R, C) values to $(1/C, 1/R)$ for each stage. This gives a new circuit with different zero assignment for each stage of its original circuit. Method II flips over the original circuit and exchanges the resulting (R_i, C_i) values for (C_{n-i+1}, R_{n-i+1}) for each i-th stage. Those circuit transformations can reduce a number of circuit designs down to 1/4 of the straight-forward method. This considerably reduces a burden for exhaustive design for searching the minimum element value spread condition. Some design examples are given to illustrate the proposed methods.

key words: *RC polyphase filter, equal ripple, transfer function preserving transformation, element value spread*

1. Introduction

The RC polyphase filter is a kind of analog complex coefficient filters, and is becoming very popular for its capability of image rejection in wireless communication systems [1].

The RC polyphase filters are often used in cascaded sections for obtaining wider stopband and/or passband widths. In such cascaded RC polyphase filters, to have simultaneously equal ripple in both stopband and passband may be a major concern. Such equal ripple RC polyphase filter has been introduced and analyzed by Gingell in transfer function level [2], and its element value design methods have been reported recently [3], [4].

In a multi-stage RC polyphase filter, each stage has its own transmission zero, and we can freely assign transmission zeros among the stages in any sequence. Traditionally, the zero sequence has been designed in such a way that time constants of the zeros are placed in descending or ascending order [1]. However, the authors have shown that despite changes in sequence of the transmission zero assignment, those circuits may have exactly the same transfer function [4]. The authors also found that order of cascading sections in an equal-ripple multi-stage RC polyphase filter may cause drastic changes in element value spread [4].

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Thus, it is highly desirable to check all the cascading orders of the RC polyphase filter and find out the optimum circuit design with minimal element-value spread. Because an n-stage RC polyphase filter has $n!$ different cascading orders, however, it is very difficult and cumbersome to design and check all the different circuits for larger n. Also, the design technique introduced in [4] involves a numerical method, which will not converge in some cases. It may be expected that less number of designs cause fewer non-convergence issues to be encountered.

This paper presents two circuit transformations which preserve a transfer function of an equal-ripple RC polyphase filter while keeping its element value spread unchanged. Those transformations allow systematic ways to reduce the number of different circuits to be designed to about 1/4, by using symmetry natures of equal ripple RC polyphase filter circuits.

Some design examples and considerations are presented for cascading order of each RC polyphase filter section.

2. Design Procedure of Equal-Ripple RC Polyphase Filter

This section gives a brief summary of a design procedure proposed in reference [4], for later use in the present paper.

2.1 Transfer Function and Time Constants for Equal Ripple RC Polyphase Filter

Gingell derived an analytic expression for the voltage transfer function of equal-ripple RC polyphase filters and proved that its poles and zeros must lay on the negative real axis and imaginary axis, respectively, so that they can be expressed by time constants [2].

An equal-ripple RC polyphase filter can be characterized by three parameters n, x, and ε. Here, n is a number of RC polyphase filter sections, x is a square root of a reciprocal bandwidth defined by $x \equiv \sqrt{\omega_L/\omega_H}$ (< 1), where ω_L and ω_H are angular frequencies of lower edge and higher edge of passband and/or stopband, respectively. ε is a ripple parameter.

The voltage transfer function of the RC polyphase filter with simultaneously equal ripple in both stopband and passband is given by [2]:

$$H(s) = \prod_{r=1}^{n} \frac{s + \mathrm{j}\,\dfrac{1}{x}\,\mathrm{dn}\left[\left(\dfrac{2r-1}{2n}\right) K\left(\sqrt{1-x^4}\right),\ \sqrt{1-x^4}\right]}{s + \dfrac{1}{x}\,\mathrm{cs}\left[\left(\dfrac{2r-1}{2n}\right) K\left(\sqrt{1-x^4}\right),\ \sqrt{1-x^4}\right]}, \tag{1}$$

where, $K(\cdot)$ is the complete elliptic integral of the first kind, and $\mathrm{dn}(\cdot)$ and $\mathrm{cs}(\cdot)$ are Jacobiban elliptic functions [5].

Equation (1) can be factorized in the following form:

$$\begin{aligned} H(s) &= \frac{(1-\mathrm{j}s\tau_1^z)(1-\mathrm{j}s\tau_2^z)\cdots(1-\mathrm{j}s\tau_n^z)}{(1+s\tau_1^p)(1+s\tau_2^p)\cdots(1+s\tau_n^p)} \\ &= \frac{(1-\mathrm{j}s\tau_1^z)(1-\mathrm{j}s\tau_2^z)\cdots(1-\mathrm{j}s\tau_n^z)}{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}, \end{aligned} \tag{2}$$

where, time constants of zeros and poles are given by

$$\tau_r^z = \left\{\frac{1}{x}\mathrm{dn}\left[\left(\frac{2r-1}{2n}\right) K\left(\sqrt{1-x^4}\right),\ \sqrt{1-x^4}\right]\right\}^{-1}, \tag{3}$$

$$\tau_r^p = \left\{\frac{1}{x}\mathrm{cs}\left[\left(\frac{2r-1}{2n}\right) K\left(\sqrt{1-x^4}\right),\ \sqrt{1-x^4}\right]\right\}^{-1}. \tag{4}$$

2.2 Circuit Analysis by Using $\boldsymbol{F}$ Matrix

Circuit topology of a single stage RC polyphase filter is shown in Fig. 1. The single section RC polyphase filter is described by [2], [3]

$$\boldsymbol{F} = \frac{1}{1-\mathrm{j}sCR}\begin{pmatrix} 1+sCR & R \\ 2sC & 1+sCR \end{pmatrix}. \tag{5}$$

If n sections are connected in cascade, the total F matrix is given by

$$\begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix} = \boldsymbol{F}_1\boldsymbol{F}_2\cdots\boldsymbol{F}_n\begin{pmatrix} V_{out} \\ I_{out} \end{pmatrix} \tag{6}$$

$$= \frac{1}{\prod_{i=1}^{n}(1-\mathrm{j}sC_iR_i)}\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} V_{out} \\ I_{out} \end{pmatrix}. \tag{7}$$

Where, $\boldsymbol{F}_i$ is an F matrix of the i-th section, and R_i, C_i are resistor and capacitor values for the section. Thus, a voltage transfer function of the RC polyphase filter, $H(s)$, is given

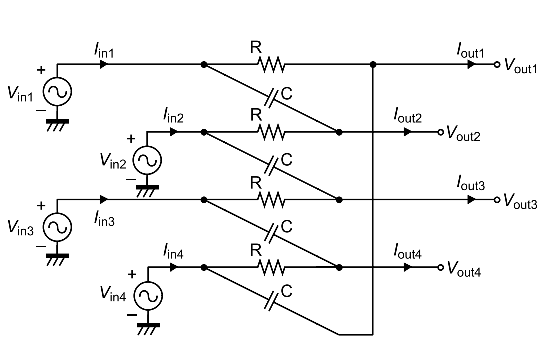

Fig. 1 Basic single stage RC polyphase filter circuit.

by the following equation:

$$H(s) = \frac{(1-\mathrm{j}sC_1R_1)(1-\mathrm{j}sC_2R_2)\cdots(1-\mathrm{j}sC_nR_n)}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, \tag{8}$$

because, the matrix entry A is an n-th order polynomial of s, we can write it down with real coefficients a_i ($i = 1, 2, \ldots, n$).

2.3 Element Value Design by Coefficient Matching Method

We can determine the element values R_i and C_i, by equating Eq. (2) and Eq. (8). The coefficient matching method [6] solves the problem by equating the same order coefficients of denominator polynomials and numerator polynomials, respectively, from Eqs. (2) and (8), i.e., solving the nonlinear simultaneous equations for $i = 1, 2, \ldots, n$:

$$\begin{cases} a_i(R_1, C_1, \ldots, R_n, C_n) = b_i(\tau_1^p, \tau_2^p, \ldots, \tau_n^p) \\ R_iC_i = \tau_r^z \end{cases}. \tag{9}$$

Here, the index $r\,(= 1, 2, \ldots, n)$ in τ_r^z may not be equal to the index i in the second equation above. It is important to note that the order of sections, the index i, is not necessarily arranged in ascending or descending order of zeros, the index r, rather we may freely assign a particular zero to any section. This leads to $n!$ different assignments at most; i.e., we have $n!$ different element value sets which realize the same transfer function. As mentioned earlier, we can take advantage of selecting a set of element values with the minimum element value spread among them.

We arbitrarily set "nominal impedance" of $R_1 = 1\,\Omega$ to obtain normalized solution, because always the relation $a_0 = b_0 = a_n = b_n = 1$ holds. Other impedance solutions may simply be obtained through impedance scaling as usual.

3. Symmetric Zero-Sequences

Due to the periodic nature of Jacobian elliptic functions, the time constants given by Eqs. (3), (4) for zeros and poles exhibit symmetry property in their values. If we assign the subscript r for τ_r^z and τ_r^p in *descending* order of their magnitude, we can show the following properties among zeros and poles, here super script z stands for zeros and p stands for poles:

$$\tau_r^z\tau_{n-r+1}^z = 1, \tag{10}$$

$$\tau_r^p\tau_{n-r+1}^p = 1, \tag{11}$$

where $r = 1, 2, \ldots, n$. For their proof, see reference [3] for Eq. (10), and reference [7] for Eq. (11).

Next, we define a *symmetric* zero-sequence. We assume that indices of zero, r, are arranged in descending order. We say a zero sequence is *symmetric* if the sequence is arranged in such a way that τ_r^z is placed in k-th section from left end of the RC polyphase filter and its reciprocal $1/\tau_r^z = \tau_{n-r+1}^z$ is placed in k-th section from right end, i.e., $(n-k+1)$-th section from the left end.

Table 1 Examples of symmetric zero-sequences.

All symmetric sequences for $n = 4$			
1-2-3-4	1-3-2-4	2-1-4-3	2-4-1-3
3-1-4-2	3-4-1-2	4-2-3-1	4-3-2-1
All symmetric sequences for $n = 5$			
1-2-3-4-5	1-4-3-2-5	2-1-3-5-4	2-5-3-1-4
4-1-3-5-2	4-5-3-1-2	5-2-3-4-1	5-4-3-2-1

For example, zero sequences 1-2-3-4, 1-3-2-4, and 4-3-2-1 are symmetric sequences, while 1-2-4-3, 1-3-4-2, and 4-2-1-3 are not. If the number of total sections, n, is an odd number, the center section must always be assigned to $\tau^z_{(n+1)/2}$. Table 1 shows all the possible symmetric zero sequences for $n = 4$ and $n = 5$ cases.

4. Reduction in Number of Zero Sequences

We introduce two circuit transformations on the equal-ripple RC polyphase filter, which preserves its transfer function. By using those transformations, a number of different circuits to be designed and checked reduces to about 1/4.

In this section, the index i is exclusively used for denote i-th section from the input side of an RC polyphase filter; i.e., C_i and R_i belong to the i-th section. On the other hand, the index r is used for indexing the zero time constants, τ^z_r, which are arranged in descending order of their values. This implies a one-to-one mapping from i to r can be defined, depending on an actual assignment of zero time constants to a particular order of sections.

4.1 Method I (Element Value Exchange)

This method exchanges resistor values for capacitor values of an equal-ripple RC polyphase filter.

Transfer function of an n-section RC polyphase filter is given by the following expression:

$$\frac{1}{H(s)} = \left[\prod_{i=1}^{n} \frac{1}{1 - \mathrm{j}sC_iR_i} \begin{pmatrix} 1 + sC_iR_i & R_i \\ 2sC_i & 1 + sC_iR_i \end{pmatrix} \right]_{11} \tag{12}$$

where, $[\boldsymbol{X}]_{11}$ denotes the (1,1) element of the matrix $\boldsymbol{X}$. Eliminating C_i by using the relation $\tau^z_r = R_iC_i$ and extracting the common factor, the time constant, from the matrix, we have

$$\frac{1}{H(s)} = \left[\prod_{i=1}^{n} \frac{1}{1 - \mathrm{j}s\tau^z_r} \begin{pmatrix} 1 + s\tau^z_r & R_i \\ 2s\tau^z_r/R_i & 1 + s\tau^z_r \end{pmatrix} \right]_{11} \tag{13}$$

$$= \left[\prod_{i=1}^{n} \frac{\tau^z_r}{1 - \mathrm{j}s\tau^z_r} \begin{pmatrix} 1/\tau^z_r + s & R_i/\tau^z_r \\ 2s/R_i & 1/\tau^z_r + s \end{pmatrix} \right]_{11} \tag{14}$$

$$= \left[\prod_{i=1}^{n} \frac{1}{1 - \mathrm{j}s\tau^z_r} \begin{pmatrix} 1/\tau^z_r + s & 1/C_i \\ 2s/R_i & 1/\tau^z_r + s \end{pmatrix} \right]_{11}. \tag{15}$$

In the above modification from Eq. (14) to (15), the identity for equal-ripple design, $\prod_{i=1}^{n} \tau^z_i = 1$ [3], has been used.

Thus, the following exchanges in element values

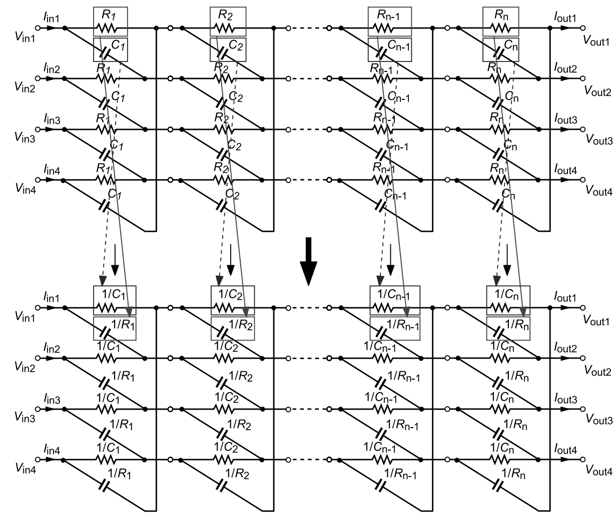

Fig. 2 Illustration of Method I.

Table 2 Time constants of zeros for $n = 5$, $\omega_H/\omega_L = 100$ design.

τ^z_1	τ^z_2	τ^z_3	τ^z_4	τ^z_5
8.439469	3.226576	1	0.309926	0.118491

$$R_i \to \frac{1}{C_i}, \quad C_i \to \frac{1}{R_i} \tag{16}$$

on Eq. (12) preserve the time constant τ^z_r and do not change the transfer function. Figure 2 illustrates those exchanges.

We have assumed that τ^z_r $(r = 1, 2, \ldots, n)$ are arranged in descending order in the above calculation; however, this is not indeed a restriction, because we assumed only the symmetry relation of Eq. (10). Hence, the index r of τ^z_r may run through from 1 to n in any order. It should be noted that the resulting element-value set will correspond to a new zero sequence of which zeros are replaced by their reciprocals, so that we need to design only a half of $n!$ different circuits and the rest can be obtained by using the Method I. Also, the element value spread is preserved in the process of Method I, because an element value spread is equal to an element value spread for its reciprocals.

For illustrative example, we designed two element-value sets for zero sequences 1-3-4-5-2 and 5-3-2-1-4 by the coefficient matching method. Table 2 shows the time constants of zeros used in this example. The result is shown in Table 3. We can obtain an element value set for 5-3-2-1-4, from that of the sequence 1-3-4-5-2 by the proposed Method I.

Table 4 shows how the Method I can be applied to an actual case. The second column of Table 4, which is labeled "Value," is an exact copy of the element-value set for the zero sequence 1-3-4-5-2 given in the Table 3. The third column is a list of inverted and swapped element values for the second column, and the last column is an impedance scaled version of the third column. The impedance scaling is done in such a way that R_1 value becomes 1 Ω as in the original design. We see that the resulting element-value set exactly matches with the directly designed element-value set for the

Table 3 Normalized element values designed by coefficient matching method ($n = 5$, $\omega_H/\omega_L = 100$ case).

Element	Zero sequence			
	1-3-4-5-2	2-5-4-3-1	4-1-2-3-5	5-3-2-1-4
R_1	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00
C_1	8.4395E+00	3.2266E+00	3.0993E−01	1.1849E−01
R_2	3.7370E−01	2.2649E+00	6.1674E+01	3.1538E+00
C_2	2.6760E+00	5.2316E−02	1.3684E−01	3.1708E−01
R_3	6.5879E−01	9.5699E+00	9.9630E+01	1.7939E+01
C_3	4.7045E−01	3.2386E−02	3.2386E−02	1.7986E−01
R_4	1.0642E+00	5.4435E+01	1.7564E+02	7.5799E+01
C_4	1.1134E−01	1.8371E−02	5.6935E−03	1.1134E−01
R_5	6.5635E+01	1.7168E+02	6.5635E+01	1.7168E+02
C_5	4.9159E−02	4.9159E−02	1.8053E−03	1.8053E−03

Table 4 Element values obtained by Method I (inverted and exchanged).

Element		Value [Ω],[F]	Inverted and exchanged	Scaled
τ_1^z	R_1	1.0000E+00	1.1849E−01	1.0000E+00
	C_1	8.4395E+00	1.0000E+00	1.1849E−01
τ_3^z	R_2	3.7370E−01	3.7370E−01	3.1538E+00
	C_2	2.6760E+00	2.6760E+00	3.1708E−01
τ_4^z	R_3	6.5879E−01	2.1256E+00	1.7939E+01
	C_3	4.7045E−01	1.5179E+00	1.7986E−01
τ_5^z	R_4	1.0642E+00	8.9815E+00	7.5799E+01
	C_4	1.1134E−01	9.3966E−01	1.1134E−01
τ_2^z	R_5	6.5635E+01	2.0342E+01	1.7168E+02
	C_5	4.9159E−02	1.5236E−02	1.8053E−03

zero sequence 5-3-2-1-4, which is given in Table 3.

4.2 Method II (Circuit Flipping)

Method II consists of two stages: Transposition of the original circuit followed by exchanges of resistor values for capacitor values.

As $(\boldsymbol{F}_1 \cdots \boldsymbol{F}_n)^T = \boldsymbol{F}_n^T \cdots \boldsymbol{F}_1^T$ holds for rectangular matrices $\boldsymbol{F}_i$ ($i = 1, 2, \ldots, n$), cascaded matrices of $\boldsymbol{F}_i$ satisfy a relation for its $(1, 1)$ element:

$$[\boldsymbol{F}_1 \boldsymbol{F}_2 \cdots \boldsymbol{F}_n]_{11} = [\boldsymbol{F}_n^T \boldsymbol{F}_{n-1}^T \cdots \boldsymbol{F}_1^T]_{11} \tag{17}$$

Here, superscript "T" denotes a matrix transposition. This suggests how to derive a flipped over RC polyphase filter which has the same voltage transfer function of an original RC polyphase filter with a given zero-sequence.

Now, we start from Eq. (12) with its matrix transposed and use the relation Eq. (17). The total voltage transfer function $H(s)$ can be described by the following expressions.

$$\frac{1}{H(s)} = \left[\prod_{i=1}^{n} \frac{1}{1 - \mathrm{j}s\tau_r^z} \begin{pmatrix} 1 + s\tau_r^z & R_i \\ 2sC_i & 1 + s\tau_r^z \end{pmatrix}^T\right]_{11} \tag{18}$$

$$= \left[\prod_{i=n}^{1} \frac{1}{1 - \mathrm{j}s\tau_r^z} \begin{pmatrix} 1 + s\tau_r^z & 2sC_i \\ R_i & 1 + s\tau_r^z \end{pmatrix}\right]_{11} \tag{19}$$

Note that the index i runs inversely, from n to 1, in Eq. (19). Because the diagonal entries of the i-th section have a time constant τ_r^z, the off-diagonal entries in each section of Eqs. (18) and (19) must satisfy a relation $C_i R_i = \tau_r^z$.

Next, we apply an impedance scaling on Eq. (19) by a scaling factor of $1/(2s)$. It should be noted that while diagonal elements of a chain matrix have no physical dimensions, off-diagonal elements have physical dimensions; i.e., $(1, 2)$ element has an impedance dimension and $(2, 1)$ element has an admittance dimension. Then, only the off-diagonal elements are affected by the impedance scaling. We have an impedance scaled version of Eq. (19):

$$\frac{1}{H(s)} = \left[\prod_{i=n}^{1} \frac{1}{1 - \mathrm{j}s\tau_r^z} \begin{pmatrix} 1 + s\tau_r^z & C_i \\ 2sR_i & 1 + s\tau_r^z \end{pmatrix}\right]_{11}. \tag{20}$$

Equation (20) and Eq. (12) have very similar expressions, and have exactly the same transfer function. Comparing those expressions, we observe the following facts:

1. The index i of products inversely runs in Eq. (20). This means that a cascading order of sections in Eq. (20) is a reversed order that of the original Eq. (12) case; that is, Eq. (12) represents the cascading sections in an *ascending* order of i, or, in a corresponding order of r, while Eq. (20) represents the cascading sections in a *descending* order of i, or, in a corresponding order of $(n - r + 1)$.
2. In Eq. (20), an element value exchange $R_i \leftrightarrow C_i$ takes place for each section; however, the zero location of each section remains unchanged because the symmetry property $R_i C_i = \tau_r^z$ must hold in an equal-ripple RC polyphase filter.
3. In the transformation process of the Method II, element value spread remains unchanged because R_i and C_i *values* do not change.

In conclusion, a transformed RC polyphase filter by the Method II has exactly the same transfer function as its original RC polyphase filter. Figure 3 shows an illustration of the procedure for the Method II.

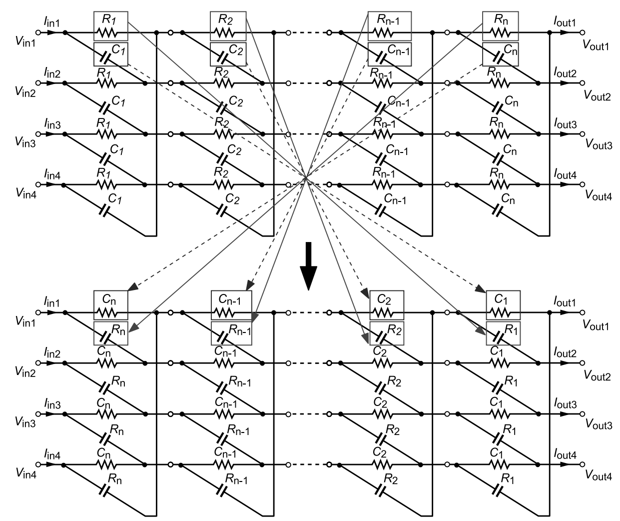

Fig. 3 Illustrative explanation of Method II.

Table 5 shows an example of the Method II. In this example, the original zero sequence is 1-3-4-5-2 and the resulting zero sequence is 2-5-4-3-1. The resulting element

Table 5 Example of Method II. Element values after transposed and exchanged.

τ_r^z		Original values	Transposed	Exchanged	Scaled
τ_1^z	R_1	1	6.5635E+01	4.9159E-02	1.0000E+00
	C_1	8.4395E+00	4.9159E−02	6.5635E+01	3.2266E+00
τ_3^z	R_2	3.7370E−01	1.0642E+00	1.1134E-01	2.2649E+00
	C_2	2.6760E+00	1.1134E−01	1.0642E+00	5.2316E-02
τ_4^z	R_3	6.5879E−01	6.5879E−01	4.7045E−01	9.5699E+00
	C_3	4.7045E-01	4.7045E−01	6.5879E−01	3.2386E−02
τ_5^z	R_4	1.0642E+00	3.7370E−01	2.6760E+00	5.4435E+01
	C_4	1.1134E−01	2.6760E+00	3.7370E−01	1.8371E−02
τ_2^z	R_5	6.5635E+01	1	8.4395E+00	1.7168E+02
	C_5	4.9159E−02	8.4395E+00	1	4.9159E−02

values are re-normalized in such a way that R_1 becomes 1 Ω. The values of the last column exactly match with the values obtained via coefficient matching method (see Table 3).

5. Discussion

5.1 Number of Possible Circuit Design Reduction

We have introduced two reduction methods in number of designs for possible zero sequence combinations. As is known from the above discussion, the number of reduction depends on each zero sequence's symmetry. Figure 4 illustrates how the Methods I and II translate original zero sequences.

If an original zero sequence is *symmetric*, Methods I and II give the same circuits so that the reduction rate will be 1/2, because two consecutive transformations give the original sequence as shown in Fig. 4(a).

On the other hand, a *non-symmetric* zero sequence will give different circuits for Methods I and II, and the order of two consecutive transformations result in different zero sequences. Four non-symmetric zero-sequences can be transformed each other by applying Methods I and II. Hence, the reduction rate will be 1/4 as shown in Fig. 4(b) in this case.

5.2 Number of Stages vs. Possible Circuit Design Reduction

A number of symmetric zero-sequences depends on a number of stages in an RC polyphase filter, n, and whether n is an even number or an odd number.

If n is an even number, it suffices to specify first $n/2$ sequences due to the symmetry constraint of Eq. (10). The latter half sequence can be determined automatically by Eq. (10). There is two possibilities to assign a particular zero for the first half or last half. Thus, all the possible combinations are $2^{n/2}$. In addition, the $n/2$ first half zeros have a total permutations of $(n/2)!$. Then, the total number of possible symmetric zero-sequences becomes $2^{n/2} \times (n/2)!$ for an even n. The number of the rest of possible sequences, $n! - 2^{n/2} \times (n/2)!$, must be non-symmetric zero-sequences.

For an odd n, the symmetric zero-sequence constraint forces the $\tau^z_{(n+1)/2}(= 1)$ to be fixed at the center of the sequence. Then, the number of possible symmetric zero-sequences is given by $2^{(n-1)/2} \times \{(n-1)/2\}!$ for an odd n.

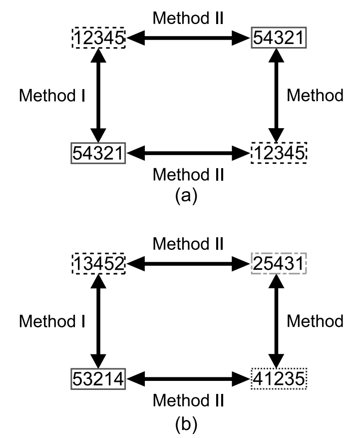

Fig. 4 Possible number of reductions for symmetric sequences (a), and non-symmetric sequences (b).

Table 6 Minimum required number of different designs for n = 1–7.

n	Total (a)	Symmetric (b)	Non-symmetric (c)	Required (d)	Reduction (d)÷(a)
2	2!=2	1	0	1	0.5
3	3!=6	2	4	3	0.5
4	4!=24	8	16	8	0.33
5	5!=120	8	112	32	0.27
6	6!=720	48	672	192	0.27
7	7!=5040	48	4992	1272	0.25

Also, the rest of possible sequences must be non-symmetric sequences.

In summary, we have the following formulae for total required number of zero-sequences. For even n:

$$\frac{1}{2}\left\{2^{\frac{n}{2}} \times \left(\frac{n}{2}\right)!\right\} + \frac{1}{4}\left\{n! - 2^{\frac{n}{2}} \times \left(\frac{n}{2}\right)!\right\}, \tag{21}$$

for odd n:

$$\frac{1}{2}\left\{2^{\frac{n-1}{2}} \times \left(\frac{n-1}{2}\right)!\right\} + \frac{1}{4}\left\{n! - 2^{\frac{n-1}{2}} \times \left(\frac{n-1}{2}\right)!\right\}. \tag{22}$$

Table 6 summarizes the relation of number of stages, n, total number of zero sequences (a), number of symmetric zero sequences (b), number of non-symmetric zero sequences (c), minimum number of required zero sequences (d), and reduction rate for $n = 1$ to 7. The minimum required number of circuits to be designed is calculated by (d) = (b) ÷ 2 + (c) ÷ 4, and the reduction rate is given by (d) ÷ (a).

For example, the minimum number of zero sequences to be designed is 32 for 5 stage case. The rest of 88 circuits can be obtained from the 32 designs by using Methods I and II. Hence, the reduction rate is 0.27 in this case. For large n, the reduction rate approaches to 1/4.

5.3 Merits of Proposed Methods

As mentioned in Sect. 4, the proposed Methods I and II preserve an element value spread of its original circuit. This implies that the proposed methods can be utilized for classifying all the zero sequences which have the same element value spread *before* we go to actual design. This is a big economy and is an advantage of the method, because we only have to design a representative sequence for each class, and the number of different classes are reduced to about 1/4 of the total number of sequences.

This reduction of design effort may also be beneficial for actual element value design process. The element design process is assumed to use the element matching method which numerically solves a nonlinear simultaneous algebraic equations. Unfortunately, existing numerical methods sometimes fail to converge. The proposed methods may reduce the chance of non-convergence issue by reducing the number of to be solved problems, and if a representative sequence fail to converge, we may try other sequences of the same class which must yield the same solution.

6. Conclusion

Two circuit transformations have been proposed, which systematically change the order of zero assignment from the existing designs while preserving voltage transfer function and element value spread for equal-ripple RC polyphase filters. By using the proposed transformations, a required number for circuit designs can be reduced to 1/4 of the previous carpet bombing method.

For example, a five stage RC polyphase filter having 5! = 120 different zero placements can be reduced to only 32 different zero placements. The rest of 88 circuits can be obtained without actual design procedure, but can be obtained from simple calculations on 32 circuits.

These circuit transformations considerably reduce design efforts which are required for searching the minimum element value spread design.

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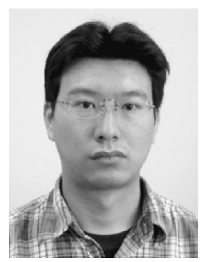

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