Assessment of Vibration-based Damage Identification Techniques using Localized Excitation Source

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Abstract

The Bridge Engineering Laboratory in Kitmai Institute of Technology, Japan has introduced a number of different damage identification techniques to detect structural damage and identify its location utilizing piezoelectric actuators as a localized excitation source. Several spectral functions, such as Cross Spectral Density (CSD), Power Spectral Density (PSD), Phase Angle and Transfer Function Estimate (TFE), were used to estimate the dynamic response of the structure. Each function's magnitude, measured in a specified frequency range, is used in the damage identification methods. The change of the spectral function magnitude between the baseline state and the current state is then used to identify the location of possible damage in the structure. It is then necessary to determine which spectral function is best able to estimate the dynamic response and which algorithm is best able to identify the damage. The first part of this paper compares the performance of different spectral functions when their magnitude is used in one damage identification algorithm using experimental data from a railway steel bridge. The second part of this paper compares the performance of different damage identification algorithms using the same data.

Key Words: damage detection; modal parameters; vibration data; health monitoring

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1. Introduction

Civil engineering structures may experience extreme loading conditions during their design life such as earthquakes, hurricanes or even tsunamis. If a structure manages to withstand these extreme events it becomes crucial to examine its condition to make sure no substantial damages have occurred. On the other hand, even if a structure has not been exposed to extreme loads, it is of vital importance that the condition of an aging structure is monitored to detect damage that could possibly lead to failure of the structure. Thus damage detection and continuous evaluation of structural integrity of important structures is indispensable. Traditional damage detection techniques include visual inspection and localised non-destructive evaluation such as X-ray, radiographic, eddy current and ultrasonic techniques. All these methods require that the vicinity of the damage is known a priori and the portion of the structure being inspected is readily accessible [1]. As a result these methods are not suitable for global monitoring of large civil engineering structures. A more promising and practical approach is based on collecting data about critical structural elements using sensors to provide indicators when some anomalies are detected in a structure. This approach of continuous monitoring of structures integrity is referred to as structural health monitoring (SHM).

In the past two decades, large number of SHM related research has been reported and several Vibration-Based Damage Detection (VBDD) techniques [2-4] have been introduced. Many studies of global damage detection methods applied to structures primarily examined changes in modal properties such as resonant frequencies [5, 6], mode shapes [7-9], and modal damping, determined during measured–input and ambient vibration [1, 10-12]. Several studies found that resonant frequencies and modal damping were insensitive to low levels of damage. Changes in experimentally determined mode shapes were found to be more sensitive indicators of damage. However, only few modes can be measured experimentally which may decrease the accuracy of the obtained results.

The bridge Engineering Laboratory in Kitami Institute of Technology, Japan has introduced a number of VBDD techniques that use the dynamic response at each frequency component rather than using only the modal peaks, which can be called operational mode shapes. The proposed methods are coupled with the implementation of piezoelectric actuators as a localized excitation source. Previous research works by the authors have shown the advantage of using localized excitation in detecting small damage levels. In this study, the performance of the proposed methods will be assessed and compared using experimental data from the same structure with the hope that the relative merits of the various methods as well as their shortcomings may be better identified and understood.

2. Spectral functions

Before the damage identification algorithms are presented, several terms related to spectral analysis are defined. For a continuous time series, $x\,(t)$, defined on the interval from 0 to T, the Fourier Spectrum (Fourier Transform), $X\,(f)$, is defined as [12]

$$X(f) = \int_0^T x(t) e^{-i2\pi ft} dt \tag{1}$$

where $i = \sqrt{-1}$, and f = cyclic frequency (Hz).
The power spectrum P_s is defined as

$$P_s = X(f) X^*(f) \tag{2}$$

where * denotes a complex conjugate. The PSD, $G_{xx}(f)$ is defined as

$$G_{xx}(f) = \frac{2}{T} E\left[X(f) X^*(f)\right] \tag{3}$$

where $E[\ ]$ indicates an ensemble average for a specific f over n samples of $X(f)$.
The CSD, $G_{xy}(f)$, relating two time histories, $x(t)$ and $y(t)$ is defined as

$$G_{xy}(f) = \frac{2}{T} E\left[X^*(f) Y(f)\right]. \tag{4}$$

For a linear system, TFE which relates an input, $X(f)$, to a response, $Y(f)$, is defined as

$$T_{xy}(f) = \frac{Y(f)}{X(f)} = \frac{G_{xy}(f)}{G_{xx}(f)}. \tag{5}$$

The coherence function $H_{xy}(f)$, relating two time histories, $x(t)$ and $y(t)$ is defined as

$$H_{xy}(f) = \frac{\left|G_{xy}(f)\right|^2}{G_{xx}(f) G_{yy}(f)}. \tag{6}$$

The phase angle $A_{xy}(f)$, between two time histories $x(t)$ and $y(t)$ can be computed from the real and imaginary values of G_{xy} as

$$A_{xy}(f) = \tan^{-1}\left[imag(G_{xy}(f)) / real(G_{xy}(f))\right]. \tag{7}$$

3. Vibration-based damage identification techniques: a review

In this section a brief summary of each damage identification method (DIM) will be summarized. For a more detailed summary the reader is referred to the cited references.

3.1 Strain energy method

Strain energy method is developed based on damage index method that was presented by Stubbs et al. (1995) [13]. At each frequency component, f, the magnitude of CSD at channel i can be computed from the real and imaginary values as [14]

$$\psi_{if} = \sqrt{(real(G_{xy}(f)))^2 + (imag(G_{xy}(f)))^2}. \tag{8}$$

$\{\psi_f\}$ is a vector representing CSD magnitudes at all measured points at the same frequency, f,. ψ_f data is normalized with respect to the square root of the sum of squares (SRSS) as follows

$$\{\phi_f\} = \frac{1}{\sqrt{\sum_{i=1}^{n} \psi_{if}^2}} \{\psi_f\} \tag{9}$$

Interpolation using cubic polynomial function is carried out to approximate the normalized CSD magnitudes between sensors. Thus, at each frequency line f, $\phi_f(x)$ represents the normalized magnitude of CSD at distance x after interpolation. The damage index is calculated using the magnitude of $\phi_f(x)$ as follows

$$\alpha_{f,j} = \frac{\left(\int_a^b [\phi_f^{*''}(x)]^2 dx + \int_0^L [\phi_f^{*''}(x)]^2 dx\right) \int_0^L [\phi_f^{''}(x)]^2 dx}{\left(\int_a^b [\phi_f^{''}(x)]^2 dx + \int_0^L [\phi_f^{''}(x)]^2 dx\right) \int_0^L [\phi_f^{*''}(x)]^2 dx} \tag{10}$$

where $\phi_f''(x)$ $\phi_f^{*''}(x)$ are the second derivative of $\phi_f(x)$ corresponding to the undamaged and damaged structure, respectively. L is beam length and a, b are the limits for element j.

A normalized damage indicator can be obtained as

$$Q_{f,j} = \frac{\alpha_{f,j} - \overline{\alpha_f}}{\sigma_f} \tag{11}$$

where $\overline{\alpha_f}$ and σ_f represent the mean and standard deviation of the damage indices, respectively. Values of four standard deviations from the mean are assumed to be associated with damage locations. $Q_{f,j}$ values less than four are discarded and values greater than or equal to four are added over different frequencies on the measurement range, as shown in the following expressions

$$\text{if} \left|Q_{f,j}\right| < 4 \text{ then let } Q_{f,j} = 0 \text{ and } L_{f,j} = 0 \tag{12}$$

$$\text{if} \left|Q_{f,j}\right| \geq 4 \text{ then let } Q_{f,j} = Q_{f,j} \text{ and } L_{f,j} = 1 \tag{13}$$

$L_{f,j}$ is used to count the number of frequency components at which damage is detected. The sum of $Q_{f,j}$ and $L_{f,j}$ at different frequency components in the frequency range of f_1 to f_m is then estimated. The damage localization indicator is defined as

$$DI_j = \sum_{f=f_1}^{f_m} \left|Q_{f,j}\right| \times \sum_{f=f_1}^{f_m} L_{f,j} \tag{14}$$

3.2 PSD curvature 1 method

Denote $G_i(f)$ the PSD magnitude measured at channel number i at frequency value f. PSD data is normalized with respect to the SRSS as shown in the following expression [15]

$$\{P(f)\} = \frac{1}{\sqrt{\sum_{i=1}^{n} G_i^2(f)}} \{G(f)\} \tag{15}$$

where $\{G(f)\}$ is a vector representing PSD magnitudes measured at all channels but at the same frequency, f. A polynomial can be fit to the PSD magnitudes and then subsequently differentiated to obtain curvature values. The damage index is defined as the absolute difference in PSD curvature before and after damage as

$$S_i(f) = \left|P_i''(f) - P_i^{*''}(f)\right| \tag{16}$$

where $P_i''(f)$ and $P_i^{*''}(f)$ are the second derivative of PSD magnitude at frequency f at node i, corresponding respectively to an undamaged and damaged structure. A normalized damage indicator is obtained as

$$Q_i(f) = \frac{S_i(f) - \overline{S}(f)}{\sigma(f)} \tag{17}$$

where $\overline{S}(f)$ and $\sigma(f)$ represent the mean and standard deviation of the damage indices, respectively. The damage indicator is estimated at different measuring channels (i = 1: n) and at different frequency components ($f = f_1 : f_m$) to construct the matrix [Q]. Then 80% of the maximum value of Q is used as a threshold limit. Values less than 80% of Q_{max} are removed and values greater than or equal to 80% of Q_{max} are added over different frequencies on the measurement range from f_1 to f_m . In other words,

$$\text{if} \left|Q_i(f)\right| < (0.80 \times Q_{\max}) \text{ then let } Q_i(f) = 0 \text{ and } D_i(f) = 0 \tag{18}$$

$$\text{if} \left|Q_i(f)\right| \geq (0.80 \times Q_{\max}) \text{ then let } Q_i(f) = Q_i(f) \text{ and } D_i(f) = 1 \tag{19}$$

The damage localization indicator is defined at node i as

$$DI_i = \sum_{f=f_1}^{f_m} \left|Q_i(f)\right| \times \sum_{f=f_1}^{f_m} D_i(f) \tag{20}$$

3.3 PSD method

The absolute difference in PSD magnitude before and after damage is estimated as [16]

$$D_i(f) = \left|G_i(f) - G_i^*(f)\right| \tag{21}$$

where G_i (f) and G_i^* (f) represent PSD magnitude for the undamaged and damaged structures, respectively. When the change in spectral function magnitude is measured at different frequencies on the measurement range from f_1 to f_m, a matrix [**D**] can be formulated as follows

$$\mathbf{D} = \begin{bmatrix} D_1(f_1) & D_2(f_1) & .... & D_n(f_1) \\ D_1(f_2) & D_2(f_2) & .... & D_n(f_2) \\ . & . & .... & . \\ . & . & .... & . \\ . & . & .... & . \\ D_1(f_m) & D_2(f_m) & .... & D_n(f_m) \end{bmatrix} \tag{22}$$

where n represents the number of measuring points. The first damage indicator is calculated from the sum of columns of matrix [**D**] as

$$\mathbf{TC} = \left\{ \sum_f D_1(f) \quad \sum_f D_2(f) \quad . \quad . \quad . \quad \sum_f D_n(f) \right\}. \tag{23}$$

However, the total change in PSD magnitude was found to be a weak indicator of damage localization. The following procedure is employed to determine the location of damage. The first step in this procedure is the selection of the maximum change in PSD magnitude at each frequency value (the maximum value in each row of matrix [**D**]) and discarding all other changes measured at other nodes. For example in matrix [**D**], if D_3 (f_1) is the maximum value in the first row then this value will be used as M_3 (f_1) and other values in this row will be discarded. The same process is applied to the different rows in matrix [**D**] to formulate the matrix of maximum changes of PSD magnitude at different frequencies, [**M**]

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & M_3(f_1) & 0 & .... & 0 \\ 0 & M_2(f_2) & 0 & 0 & .... & 0 \\ 0 & 0 & 0 & M_4(f_3) & .... & 0 \\ . & . & . & . & .... & . \\ . & . & . & . & .... & . \\ 0 & 0 & M_3(f_m) & 0 & .... & 0 \end{bmatrix}. \tag{24}$$

The total of maximum changes in PSD magnitude is calculated from the sum of the columns of matrix [**M**]

$$\mathbf{TM} = \left\{ \sum_f M_1(f) \quad \sum_f M_2(f) \quad . \quad . \quad . \quad \sum_f M_n(f) \right\}. \tag{25}$$

In order to monitor the frequency of damage detection at any node, a new matrix [**C**] is formulated. The matrix consists of 0's at the undamaged locations and 1's at the damaged locations as

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 & 0 & .... & 0 \\ 0 & 1 & 0 & 0 & .... & 0 \\ 0 & 0 & 0 & 1 & .... & 0 \\ . & . & . & . & .... & . \\ . & . & . & . & .... & . \\ 0 & 0 & 1 & 0 & .... & 0 \end{bmatrix}. \tag{26}$$

The total number of times of detecting the damage at different nodes is calculated from the sum of the columns of matrix [**C]** as

$$\mathbf{TC} = \left\{ \sum_f C_1(f) \quad \sum_f C_2(f) \quad . \quad . \quad . \quad \sum_f C_n(f) \right\}. \tag{27}$$

The first damage localization indicator is defined as the scalar product of {**TM**} and {**TC**}

$$\mathbf{DI0} = \{TM\} \cdot \{TC\}. \tag{28}$$

In order to reduce the effect of noise or measurement errors, a value of one standard deviation of the elements in vector {**TM**} will be subtracted from the vector {**TM**}. Any resulting negative values will be discarded. The same procedure will be applied to the vector {**TC**}. The second damage localization indicator is defined as the scalar product of the resulting vectors

$$\mathbf{DI1} = \{TM^*\} \cdot \{TC^*\}. \tag{29}$$

3.4 PSD Curvature 2 method

The same algorithm for PSD method [17] is used here but PSD curvature data is used as the damage sensitive feature (Equation (21)) instead of PSD data.

3.5 Phase angle method

Let $P_{xy}(f)$ denote the phase angle between a response point, $x(t)$ relative to a reference point $y(t)$. The absolute difference in absolute phase angle before and after damage can then be defined as [18]

$$\Delta_{xy}(f) = \left\| P_{xy}(f) \right| - \left| P^*_{xy}(f) \right\| \tag{30}$$

where $P_{xy}(f)$ and $P_{xy}^*(f)$ represent the phase angle of the undamaged and damaged structures, respectively. When the change in phase angle is measured at different frequencies on the measurement range from f_1 to f_m, a matrix $[\Pi_r]$ can be formulated as follows

$$\Pi_r = \begin{bmatrix} \Delta_{1r}(f_1) & \Delta_{2r}(f_1) & .... & \Delta_{nr}(f_1) \\ \Delta_{1r}(f_2) & \Delta_{2r}(f_2) & .... & \Delta_{nr}(f_2) \\ . & . & .... & . \\ . & . & .... & . \\ . & . & .... & . \\ \Delta_{1r}(f_m) & \Delta_{2r}(f_m) & .... & \Delta_{nr}(f_m) \end{bmatrix}_r \tag{31}$$

where n represents the number of measuring points and r represents the number of reference channel. In matrix [Π_r], every row represents the changes in phase angle at different measuring channels but at the same frequency value. Each measuring channel will be used as a reference for the other channels ($r = 1 : n$). Therefore, the matrix [Π_r] will be formulated n different times (3D matrix). The summation of phase angle changes over different frequencies using different references can be used as the indicator of damage occurrence. In other words, the first damage indicator is calculated from the sum of columns of each matrix, [Π_r] and then summing up these changes over different references

$$TC = \sum_r \left\{ \sum_f \Delta_{1r}(f) \quad \sum_f \Delta_{2r}(f) \quad ... \quad \sum_f \Delta_{nr}(f) \right\}_r \tag{32}$$

where $f = f_1 : f_m$ and $r = 1 : n$.

The matrix of maximum changes of phase angle at different frequencies, [B_r] is formulated by selecting the maximum change at each frequency component (each row). It should be noted that [B_r] is a 3D matrix where each value of r ($r = 1 : n$) formulates one matrix

$$B_r = \begin{bmatrix} 0 & 0 & B_{3r}(f_1) & 0 & ... & 0 \\ 0 & B_{2r}(f_2) & 0 & 0 & ... & 0 \\ 0 & 0 & 0 & B_{4r}(f_3) & ... & 0 \\ . & . & . & . & ... & 0 \\ . & . & . & . & ... & \\ 0 & 0 & B_{3r}(f_m) & 0 & ... & 0 \end{bmatrix}_r . \tag{33}$$

The total of maximum changes in phase angle is calculated from the sum of the columns of matrix [B_r] using different references. At each value of r, the sum of columns of matrix [B_r] will result in one vector. Therefore, n different vectors can be obtained. The sum of these vectors is stored in one vector {Z};

$$Z = \sum_r \left\{ \sum_f B_{1r}(f) \quad \sum_f B_{2r}(f) \quad ... \quad \sum_f B_{nr}(f) \right\}_r . \tag{34}$$

In order to monitor the frequency of damage detection at any node, a new matrix [E_r] is formulated. The matrix consists of 0's at the undamaged locations and 1's at the damaged locations, as explained previously. The total number of instances of detecting the damage at different nodes is calculated from matrix [E_r] as

$$K = \sum_r \left\{ \sum_f E_{1r}(f) \quad \sum_f E_{2r}(f) \quad ... \quad \sum_f E_{nr}(f) \right\}_r . \tag{35}$$

In order to reduce the effect of noise or measurement errors, a value of two times standard deviation of the elements in vector {K} will be subtracted from the vector {K}. Any resulting negative values will be removed. The same procedure is applied to the vector {Z} as follows

$$\mathrm{T} = \left\{ \mathrm{Z}_1 - 2\sigma \quad \mathrm{Z}_2 - 2\sigma \quad \ldots \quad \mathrm{Z}_n - 2\sigma \right\} \tag{36}$$

$$\mathrm{I} = \left\{ \mathrm{K}_1 - 2\beta \quad \mathrm{K}_2 - 2\beta \quad \ldots \quad \mathrm{K}_n - 2\beta \right\} \tag{37}$$

where σ and β represent the standard deviation value of the elements in vectors Z and K, respectively. The damage localization indicator is defined as the scalar product of { T } and { I }

$$Dam\_Ind = \left\{ \mathrm{T}_1.\mathrm{I}_1 \quad \mathrm{T}_2.\mathrm{I}_2 \quad \ldots \quad \mathrm{T}_n.\mathrm{I}_n \right\}. \tag{38}$$

3.6 TFE method

Let $T_{xr}(f)$ denote the TFE which relates a response $x(t)$ to a reference response $r(t)$. Every channel will be used once as a reference for other channels. Thus, $T_{rx}(f)$ represents the TFE which relates a response $r(t)$ to a reference response $x(t)$. The relative TFE between x and r can then be defined as [19]

$$R_{xr}(f) = T_{xr}(f) - T_{rx}(f). \tag{39}$$

$R_{xr}(f)$ represents the relative movement (response) between x and r in the frequency domain. The absolute difference in absolute value of $R_{xr}(f)$ before and after damage can then be defined as

$$D_{xr}(f) = \left| \left| R_{xr}(f) \right| - \left| R^*_{xr}(f) \right| \right| \tag{40}$$

where the asterisk denotes the damaged structure. When the change in relative TFE, $D_{xr}(f)$, is measured at different frequencies on the measurement range from f_1 to f_m, a matrix [$\mathbf{D_r}$] can be formulated as

$$\mathbf{D_r} = \begin{bmatrix} D_{1r}(f_1) & D_{2r}(f_1) & \ldots & D_{nr}(f_1) \\ D_{1r}(f_2) & D_{2r}(f_2) & \ldots & D_{nr}(f_2) \\ . & . & \ldots & . \\ . & . & \ldots & . \\ D_{1r}(f_m) & D_{2r}(f_m) & \ldots & D_{nr}(f_m) \end{bmatrix}_r \tag{41}$$

where n represents the number of measuring points and r represents the number of reference channel. In matrix [$\mathbf{D_r}$], every row represents the changes in $D_{xr}(f)$ at different measuring channels but at the same frequency value. Each measuring channel will be used as a reference for the other channels ($r = 1 : n$). Therefore, the matrix [$\mathbf{D_r}$] will be formulated n different times (3D matrix). The total change in the relative TFE in the frequency range of f_1 to f_m can be estimated from the sum of columns of matrix [$\mathbf{D_r}$] as:

$$\mathbf{ST}_r = \left\{ \sum_f D_{1r}(f) \quad \sum_f D_{2r}(f) \quad \ldots \quad \sum_f D_{nr}(f) \right\}_r \tag{42}$$

where $f = f_1 : f_m$ and $r = 1 : n$.

The matrix of maximum changes of relative TFE at different frequencies, [$\mathbf{M_r}$] is formulated by selecting the maximum change in TFE at each frequency component. The total of maximum changes in relative TFE is calculated from the sum of the columns of matrix [$\mathbf{M_r}$] using different references. At each value of *r*, the sum of columns of matrix [$\mathbf{M_r}$] will result in one vector. Therefore, *n* different vectors can be obtained;

$$\mathbf{SM}_r = \left\{ \sum_f M_{1r}(f) \quad \sum_f M_{2r}(f) \quad \ldots \quad \sum_f M_{nr}(f) \right\}_r . \tag{43}$$

Assuming that the collection of the damage index {$\mathbf{SM}_r$} represents a sample population of a normally distributed random variable [1, 12], a normalized damage localization indicator is obtained as follows

$$\mathbf{SMN_r} = \frac{\{SM_r\} - \beta_r}{\sigma_r} \tag{44}$$

where β_r and σ_r represent the mean and standard deviation of the elements in vector {$\mathbf{SM}_r$}. Damage localization indicator is estimated from the sum of $\mathbf{SMN_r}$ over different references

$$\mathbf{DI} = \sum_r \{\mathbf{SMN_r}\} \tag{45}$$

4. Railway steel bridge: description and experimental setup

The structure used to illustrate the application of the proposed methodology was an out-of-service steel railway bridge located in the city of Kitami, Japan, which was tested by a group of researchers from the Kitami Institute of Technology. The bridge was removed from its service location and simply supported on two wooden blocks at its ends. Seen in Figure 1(a), the bridge consisted of two varying-depth steel plate girders, two steel stringers that supported the rails, and ten transverse floor beams located at various intervals. A schematic elevation of the bridge, showing its dimensions and the layout of various components used for the vibration tests, is shown in Figure 1(b).

Two piezoelectric actuators [20], located in the upper part of the web of one of the main girders near midspan (see Figure 1(b)), were used to apply identical dynamic excitation forces to the web in the horizontal out-of-plane direction. Excitation was applied in the form of a sine sweep over a frequency range of 0 to 400 Hz with a duration of 20 seconds. Although the excitation force was not measured, the forces applied for all undamaged and damaged states were equal in amplitude and had the same vibration waveform. The actuator force amplitude was estimated to be approximately 200N.

The lateral out-of-plane acceleration response was measured using eight accelerometers (model: NP-3130, manufacturer: ONO Sokki, Japan, precision: 10 mV/(m/s2)), range: 5-4000 Hz), each mounted at the geometrical centre of gravity of a panel of the main girder, as shown in Figure 6. For this study, 20-second time histories were sampled at a rate of 1600 Hz, producing 32000 time points. Five separate time histories were recorded for the intact case and for each damage state.

As the owner of the bridge did not permit the introduction of any permanent damage, damage was simulated by removing bolts from one stiffener attached to the main girder (Figure 1). The first damage case was introduced by removing the upper bolt from the stiffener located near sensor 5. Three additional damage cases were introduced by removing two, three and four bolts from the top of the same stiffener, respectively.

5. Comparing the performance of various spectral functions

The damage identification algorithm described in section 3.3 will be applied here using the magnitude of different spectral functions to compare the performance of these functions.

5.1 Damage identification using different spectral functions

PSD was calculated at each measuring channel from the acceleration time history data using a Hanning window size of 256 and a sampling rate of 1600. Consequently, PSD was measured at 128 frequency components in the frequency range of 1-800 Hz (frequency step = 800*2/256). PSD in the total measured range of 1-800 Hz will be used in the damage identification algorithm. The first damage case was introduced by removing one bolt from the stiffener located near channel 5, as shown in Figure 1. Damage localization indicators DI0 (Equation (28)) and DI1 (Equation (29)) are used to identify the damage location, as shown in Figures 2 (a) and (b), respectively. Damage localization indicator DI0 determined the damage location at channel 5 accurately with some false positive readings at other channels. These false positive readings are assumed to be due to the presence of noise and measurement errors in the measured data. When a value of one standard deviation was used as a threshold to eliminate the effect of noise, experimental variations and measurement errors, more accurate results were obtained in identifying damage location, as shown in Figure 2 (b). As clearly indicated in this figure, almost all false positive readings were eliminated except the reading at channel 4. This is a considerably high level of accuracy since removing one bolt from a stiffener can be considered a very small damage compared to the size of the test structure. Moreover, no false positive readings were detected when the damage localization indicator 1 was used to detect the damage after removing two bolts or more from the same stiffener. The results of these cases are not shown here for brevity. Figure 2 (c) shows the frequency components at which the damage location is detected at channel 5. This figure is obtained from the fifth column in matrix [**M**] (Equation (24)) and illustrates how the damage is detected at channel 5 using PSD data at each frequency component in the frequency range of 1-800 Hz. For example, the damage location is determined most accurately when PSD is measured in the frequency range of 720-800 Hz. In this range, damage is detected at channel 5 at almost every frequency component and the changes in PSD are higher. The frequency ranges (or components) that provided accurate results are randomly distributed along the total measurement range (from 1 Hz to 800 Hz). It was also observed that the frequency range that provided accurate results in certain damage case changed when the damage level increased at the same location or when the damage location changed. Therefore, it was decided to use PSD (and other spectral functions) data in the total measured frequency range.

The number of bars in Figure 2 (c) represents the number of frequency components at which damage was detected or, in other words, the number of times of detecting the damage at channel 5. In this damage case, damage was detected at 45 frequency components out of 128 frequency components used for measuring the PSD data. This value can be estimated using Equation (27). The estimated value at the damage location will be used to compare the performance of each spectral function when it is used in the proposed algorithm.

CSD was estimated from the acceleration time history measured at each channel relative to the acceleration time history at channel 3. Using channel 3 as a reference yielded the most accurate results when CSD data is used. Therefore, the obtained results, using this channel as a reference, were used for comparison with PSD function. The influence of changing the location of the reference channel on the accuracy of identifying damage location will be discussed later in this section.

Figure 3 (a) shows that more accurate results in identifying damage location were obtained at channel 5 than by using PSD. The difference in accuracy is very small and changing the reference channel sometimes produced less accurate results than that obtained by using PSD function. Figure 3 (b) shows that the frequency ranges (or components) that provided accurate results in this case were not always the same ranges that provided accurate results in case of using PSD. This illustrates one important fact that the frequency range which gives false positive readings when using one spectral function does not always contain less sensitive modes to specific damage since the same range may give accurate results when another spectral function is used. This provides another reason to use the magnitude of the spectral function in the total measured frequency range rather than looking for the best frequency range that may give the most accurate results. Similar results and observations were obtained when TFE, Phase Angle and Coherence data were used in the proposed algorithm, as shown in Figures 4 through 6. All functions yielded accurate results in identifying the damage location.

5.2 The confidence of detecting damage using different spectral functions

Table (1) shows the number of times of detecting the damage (number of frequency components at which damage is detected) at channel 5 for different damage levels using different spectral functions data. All functions yielded similar results when the first bolt was removed. When the second bolt was removed, the number of times of detecting the damage increased significantly using PSD compared to using other spectral functions. Large number of times of detecting the damage at a specific location will result in high indicator value at this location and small indicator values at other locations (false positive readings) which in turn will increase the confidence of detecting damage at this location. After removing the third and the fourth bolts, the use of PSD data yielded the most accurate results. Using Coherence data yielded the least number of times of detecting the damage when 2, 3 and 4 bolts were removed. Only TFE has shown consistent increase in the number of times of detecting the damage with increasing the damage level.

5.3 Monitoring the growth in damage

Damage identification techniques investigated in this paper cannot be used to estimate damage severity or the extent of damage, but it can be used to some extent to monitor the growth in damage. As the amount of damage increased by removing more bolts from the stiffener, the total change in PSD increased at channel 5, as indicated in Figure 7 (a). CSD (Figure 7 (b)) and TFE (Figure 7 (c)) indicated an increase in damage after removing the third bolt. However, they could not show a clear indication of damage increase after removing the second or the fourth bolt. The growth in damage could be monitored using Phase angle (Figure 7 (d)) and Coherence (Figure 7 (e)) data except when the fourth bolt was removed. Thus, it can be concluded that using PSD data yielded the most accurate results in monitoring the growth in damage.

5.4 The influence of changing the location of reference channel

All spectral functions presented previously, except PSD, are estimated from the time history response at each measuring channel relative to the time history response at a reference channel. When one channel is used as a reference, this channel cannot be used to locate the damage. The influence of changing the reference channel location is examined by using each channel once as a reference for other channels. Figure 8 shows the obtained results for the different spectral functions. This figure clearly indicates that the location of the reference channel influences the accuracy of locating damage. For example, in case of using CSD, using channel 3 as a reference yielded the most accurate results while using channel 8 yielded the least accurate results. Another observation from this figure is that the best reference location varied with changing the spectral function. The accuracy of locating the damage did not change significantly when using Phase angle data. The previous observations may change when these functions are used with different structure or in different circumstances. Using the input force as a reference needs further investigations.

6. Comparing the performance of different damage identification algorithms

Chapter 5 compares the performance of different spectral functions when their magnitude is used in one damage identification algorithm whereas this Chapter compares the performance of different damage identification algorithms using the same experimental data measured from the railway steel bridge. Damage indicators of various damage identification methods are normalized such that the maximum relative amplitude for any indicator is 100 (percentage). Thus a direct comparison between different cases is possible. The normalized damage indicator is estimated by dividing the indicator value at each measuring channel (or node number) by the sum of the indicator values at all channels and then multiplying the result by 100. The Strain Energy method, PSD Curvature 1 method, and PSD Curvature 2 method use interpolation schemes to approximate the dynamic response between the measuring points. Ten points were used to divide the readings between each pair of measuring channels. Points between measuring channels are filled with zeros when PSD method, Phase Angle method and TFE method are applied because damage indicators of these methods can be estimated at the measuring channels only. Damage identification methods will be applied to the case of single damage and then to the case of double damage. In all cases, the dynamic response is measured in the frequency range of 1-800 Hz, and the spectral function magnitude (or curvature) in the total measurement range is used in different algorithms. The threshold limit of 3 (Equations (12) and (13)) yielded the most accurate results for the Strain Energy method.

6.1 Single damage

Figures 9 through 12 show the predicted results for the single damage near channel 5; damage cases 1-4 (removing one to four bolts). Strain Energy method identified damage location with low indicator value for all damage levels. It was found that the accuracy of this method was sensitive to the measurement range of CSD data. This is due to the fact that the most sensitive frequency range for identifying specific damage case depends on many factors such as damage location, damage size, geometry of structure and frequency content of the noise. As it is not possible to predict the characteristics of damage before it occurs, the most effective frequency range cannot be determined and hence the total measurement frequency range must be used. Using the total measurement frequency range has reduced the accuracy of Strain Energy method significantly. The PSD Curvature 1 method could not identify the damage location accurately for cases 1-3. This method showed the least accurate results for the case of single damage. Phase angle method predicted damage location with low values of damage indicator for damage cases 1-3. However, this method showed high accuracy in damage case 4. PSD, PSD Curvature 2 and TFE methods identified damage location very accurately and with high indicator value for all damage cases.

6.2 Double damage

Double damage is introduced by removing bolts from two stiffeners shown in Figure 1. Channel 5 is the closest sensor to the first damage location and channel 3 is the closest sensor to the second damage location. Equal number of bolts is removed from the two stiffeners in each damage case. Detecting multiple damage locations becomes more challenging if the locations are close to each other as the influence of one damage location can easily spread to the other location. As shown in Figure 1 (b), the distance between damage locations was about 1.5m in this case, which can be considered small compare to the total length of the main girder (19.61m). It should also be noted that the sensor at channel 5 is closer to the first damage location than channel 3 to the second damage location. The predicted results for damage cases 1 through 4 are plotted in Figures 13 through 16, respectively. Strain Energy method identified the damage location at node 5 in all damage cases and failed to show any indication of damage at node 3. PSD Curvature 1 method identified damage at node 3 only for cases 1-2 without any indication of damage at node 5. In case 3, damage at node 5 is indicated by small readings at nodes 5 and 6. This method performed very well in case 4. PSD method identified damage at node 3 only for case 1 and identified damage at node 5 only for cases 2-4. This method always shows high indicator value and very few false positive readings. This remark was also observed for all single damage cases. PSD Curvature 2 method indicated damage locations at nodes 3, 4 and 5 for damage case 1 and indicated the damage at node 5 only for cases 2-4. Phase angle method showed low accuracy for most of damage cases. TFE method indicated damage location at nodes 3, 4 and 5 for case 1. In case 2, both damage locations are predicted; however, the indicator value at node 5 is much higher than at node 3. In case 3, only damage at node 5 is detected. In case 4, both damage locations are indicated accurately. In general, the accuracy of various algorithms varied with changing the level of damage. The TFE method provided the most accurate results for most of the damage cases.

7. Comparison of the main characteristics of the proposed damage identification methods

7.1 Reference point

All spectral functions presented in this study except PSD are measured from the acceleration response at each measuring channel relative to a reference channel. The accuracy of DIM may be influenced by the location of the reference channel. For example, if the reference channel is located near the damage location, damage will change the response at the reference channel which in turn will alter the response at all other locations including the undamaged locations. Also, in case of low signal to noise ratio at the reference channel or in case of high measurement errors at the reference channel, the accuracy of locating damage is expected to decrease. It is, therefore, assumed that the accuracy of Strain Energy method will be influenced by the choice of the reference channel. Phase angle method uses every channel once as a reference for other channels and then summing up the results. This technique has the advantage of creating large sets of data which can be used to detect damage with more confidence. It should be noted that damage will be identified inaccurately at least once when the reference channel exists near the damage location. Moreover, the accuracy of this method is dependent on the number of channels that will give accurate results when they are used as a reference. In TFE method, this problem is avoided by showing damage identification results at different channels versus the reference channel number instead of adding the obtained results over different references, as illustrated in the cited reference [13]. In this study, it was decided to sum the predicted results at each reference for the purpose of comparison with other methods.

7.2 Data normalization

In Strain Energy method and PSD Curvature 1 method, CSD data or PSD data are normalized (Equations (9) and (15)) before they are used in the damage identification algorithm. The normalization process is useful when the excitation forces used for the undamaged and damaged structures are not exactly equal, as in case of using ambient vibration or in case of low signal to noise ratio. On the other hand, normalization process introduces some approximations or changes to the data at all locations including the undamaged locations, which may decrease the accuracy of these methods.

7.3 Damage sensitive feature

Strain Energy method uses strain energy stored in one element divided by the total strain energy stored in the beam (Equation (10)). The total strain energy will always change after damage because it includes the damaged element(s). The change in the total strain energy will in turn change the damage parameter of all elements including the undamaged elements, which may decrease the accuracy of this method. Other methods avoided such problem by simply estimating the absolute difference in the structural response (spectral function magnitude or curvature) before and after damage (Equations (16), (21), (30) and (39)).

7.4 Curvature data

Strain Energy, PSD Curvature 1 and PSD Curvature 2 use the second derivative of CSD or PSD. The higher derivatives of CSD or PSD data are more sensitive to damage, but the differentiation process enhances the experimental variations inherent in these data.

7.5 Threshold level

The best threshold value for Strain Energy method and PSD Curvature 1 method (Equations (12) and (18)) is dependent on the structure type, damage size and damage locations. The most accurate value for the threshold can be estimated from the finite element model (FEM) of the test structure after investigating several damage scenarios. However, creating an accurate FEM for complex structures is not an easy task. For PSD, PSD Curvature 2, Phase angle and TFE methods, the maximum change in spectral function magnitude (or curvature) at each frequency component is assumed to occur at the damage location (e.g., Equation (24)). One special advantage of this technique is that the setting of the threshold value is based on the measured data with sensors. This feature is desirable when the method is used in an automated structural health monitoring system. However, the accuracy of detecting multiple-damage might be reduced because the maximum change in spectral function magnitude usually occurs at only one damage location and rarely occurs at two or more locations at the same time.

7.6 Measurement range

All methods can be applied using the total measurement range. However, the accuracy of the predicted results is dependent on the used algorithm. The accuracy of Strain energy method decreased significantly when applied using the total measurement range.

8. Concluding remarks

1. The first part of this paper provided a direct comparison of various spectral functions that can be used to estimate the dynamic response. Five different spectral functions were used to estimate the dynamic response of the structure. Damage could be detected and located at the nearest sensor location using all the spectral functions. However, PSD function provided the highest confidence for detecting damage and the most accurate results for monitoring damage growth. Another observation from this study is that when CSD, TFE, and Coherence functions are used, the accuracy of locating the damage is dependent on the location of the reference channel. The best reference location varied with changing the spectral function. It was also observed that the best frequency range in which the magnitude of the spectral function should be used in the proposed algorithm depends not only on the damage location, size or type but also on the type of the spectral function. Therefore, it is recommended to use the spectral function magnitude in the total measurement range because it is extremely difficult to determine the best frequency range in which each spectral function data should be used in the damage identification algorithm.
2. In the second part of this paper, a study was undertaken to compare six vibration based damage identification methods. This comparison was accomplished with experimental data from a railway steel bridge. In general, all methods identified the damage location correctly for the cases of severe single damage. However, the accuracy of the methods was inconsistent when they were applied to the less severe damage cases and double damage cases.
3. Results of this study show that the Strain Energy method showed the least accurate results when the entire set of analyses and experiments are considered. This method requires identifying the frequency range in which CSD data has to be used, as explained in the cited reference [14]. However, for the purpose of comparison, the entire measurement range was used, which decreased the accuracy of this method. PSD, PSD Curvature 2 and TFE methods have shown the most accurate results for all cases of single damage. TFE method performed the best in case of double damage.
4. When the main characteristics of the proposed methods are compared, it was demonstrated that each method may perform better for specific situation but cannot perform well in another different situation. When these methods were applied to different types of structures with various damage levels and types, the accuracy of each method varied with changing the type of structure, damage location, damage type, and damage size, and no method performed the best in all cases. Therefore, applying all the proposed methods together is recommended for the purpose of continuous health monitoring of a structure. Consequently, detecting the damage at a specific location with more methods may increase the confidence of the predicted results.
5. Due to the limited space of the paper, only one damage indicator was selected for each damage identification method to compare the performance of these methods. However, other damage indicators sometimes give more accurate results in some damage cases. Therefore, the performance of these methods can not be completely judged based on this comparative study. The application of all methods using all damage localization indicators is required for better identification of damage locations.

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Table 1: Number of times of detecting the damage

	1 Bolt	2 Bolts	3 Bolts	4 Bolts
PSD	45	80	76	79
CSD	47	45	57	56
TFE	44	48	56	60
PHASE	45	55	55	57
COHERENCE	48	39	43	42

List of figures