

Decay of Vorticity in Separated Shear Layer*

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Abstract

In this study, process of vorticity decay caused by viscous interaction in a separated shear layer was investigated based on velocity measurements and flow visualization in the immediate vicinity of the separation point of a test body. The experiment was carried out at a fixed Reynolds number of 19,000. The research results innovate the important aspects of clarification on vorticity decay including i) part of the vorticity in the separated shear layer behind a body are caused by viscous interaction associated with transition of velocity distribution from boundary layer like shape to free shear layer like shape in the immediate vicinity of the separation point; ii) Reynolds shear stress associated with turbulent diffusion is not concerned in the vorticity decay process in the immediate vicinity of the separation point; iii) since the Reynolds shear stress around an intermediate size vortex produced by pairing of smaller vortices is considerably large, the flow field becomes turbulent and three dimensional.

Key words: Shear Flow, Separation, Viscous Flow, Flow Measurements, Flow Visualization, Vorticity Decay, Transition

1. Introduction

When a body is placed in a flow, vortex streets are formed behind the body and fluctuating fluid forces act on it, creating with many problems. For instance, the fluid forces and the vortex formation cause vibration and sound noise of bridges and buildings or cause destruction of the body by resonance such as the accident of the fast breeder nuclear reactor (1995, Japan). Circulation of the vortex is considered as one of the primary factors to cause disastrous fluid forces, and entrainment and decay of vorticity discharged into separated shear layer are intensely concerned in determining strength of the circulation. The previous researches in this regard are reported by Gerrard⁽¹⁾ and Sarpkaya & Shoaff⁽²⁾ but poorly detail. And Mair & Maull⁽³⁾ reported that about 15% of circulation in the separated shear layer on one side was decayed by cancelation due to the entrainment of counter signed circulation from the separated shear layer on the other side. However, the separated shear layer behind the body is thin and time variation of velocity vectors around this region is strong, and quantitative investigations are very scant.

Lou et al.⁽⁴⁾ and Tong⁽⁵⁾ quantitatively investigated variations of circulation, vorticity and Strouhal number of wake generated behind a rectangular prism by varying Reynolds number using a Laser Doppler Velocimetry (LDV). Wang et al.⁽⁶⁾ studied velocity distribution, Strouhal number and Reynolds shear stress as well as vorticity and power

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spectral distribution of velocity fluctuation in wakes of a circular cylinder placed near a wall for varying clearance between them. Downes et al.⁽⁷⁾ also placed a cantilever circular cylinder in a flow, and investigated the trajectory of its displacement and vorticity at each trajectory. However, no previous report is found in literature for the investigation in the immediate vicinity of the test body. Haniu et al.^(8,9) conducted quantitative measurements by means of LDV, and reported that viscous force arising from shear is concerned in rapid vorticity decay near the separation point from a body. This fact implies the presence of another primary factor to cause vorticity decay in addition to that remarked by Mair & Maul⁽³⁾. Namely, in a separated shear layer near the separation point from a body, viscous interaction while the velocity profile is evolving from boundary layer like shape to free shear layer like shape will be implicated in the vorticity decay. In particular, the major dimension of the separated shear layer is as small as the petite vortices generated by Kelvin-Helmholtz instability, and hence the Reynolds number in the separated shear layer is fairly small. Therefore, the viscous shear strongly acts on the fluid deformation caused by the pairing of small vortices. On the other hand, separated shear layer from a bluff body, such as triangular prism or circular cylinder, possesses stream line curvature to form a centrifugally stable flow field. This aspect makes the separated shear layer has different dynamic characteristics from an ordinary straight lined mixing layer. Explicitly, the centrifugally stable flow prevents the two small vortices apart from their original streamline pass toward opposite directions due to their vortex induced velocity, and enables them start to pairing⁽⁸⁾.

In this study, in order to avoid cancelation of vorticities due to their mixing in opposite sign, a triangular prism was attached at the bottom of an open channel, and a separated shear layer was issued in downstream from the top edge of the prism. Besides, to investigate the behavior of the thin separated shear layer more detail than that of studies carried out by Haniu et al.⁽⁸⁾, dimension of the test body was enlarged by about 4 times. Moreover, size of the measuring region of time mean velocity with LDV was set to about 4% of the major dimension of the test body at the immediate downstream of the separation point. In contrast, it was about 25% of the major dimension in the previous investigation⁽⁸⁾. In addition, flow visualization was also conducted in this measuring region, and involvements of the vorticity decay in the immediate vicinity of the separation point with the evolving process in the separated shear layer were investigated.

2. Experimental Apparatus and Methods

In this experiment, a water circulating open channel with 400mm width and 890mm depth measuring cross-section was used. The experimental arrangement and definition of symbols are presented in Fig.1. A right isosceles triangular prism with a fixed separation point was used as a test body, as shown in Fig.1(b). Its major dimension h was 20cm. The major dimension, measured from bottom apex to top apex of a triangular prism, which was denoted by D in the previous report⁽⁸⁾ corresponds to $2h$. A dye injection hole of 1mm diameter for flow visualization was provided near the separation point at the mid span of the prism. During the experiment, main stream velocity was kept 10cm/s, and the Reynolds number at that condition was 1.9×10^4 . Velocity measurements were carried out by means of LDV, for which a minute amount of latex particles were added in the open channel. Since the forming frequency of the instability originated small vortices in the separated shear layer was about 1.1Hz, analogue low pass filter was set to 3Hz and sampling frequency for data acquisition was set to 6Hz for prevention of aliasing. Also, in order to obtain statistically stable results of time mean velocities and Reynolds shear stress, 8,000 data (for about 2 minutes) were acquired at each measuring point.

As shown in Fig.1(c), the measuring region was in the ranges of $0.075 < x/h < 0.295$ and $0 < y/h < 0.225$, and the data were obtained at intervals of $0.02h$ in x direction and $0.015h$ in y direction. Prior to the velocity measurements, the region where characteristics phenomena were occurring was located by flow visualization. The velocity measurements were carried out only at the located region for experimental efficiency. Namely, velocity measurements were not conducted in the regions near the upper left and lower right corners as of Fig.1(c).

In the presented study, vorticity, shear strain rate and vorticity decay rate distributions as well as Reynolds shear stress distribution were calculated from measured time averaged u (in x direction) and v (in y direction) component velocity distributions. And in order to investigate the vorticity decay phenomenon, contour line charts were generated for each of the above mentioned quantities and compared with flow visualization. The quantities have been obtained with the equations (1), (2) and (3).

Vorticity:

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \dots\dots\dots(1)$$

Shear strain rate:

$$e = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \dots\dots\dots(2)$$

Vorticity production rate:

$$\varepsilon = u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} \dots\dots\dots(3)$$

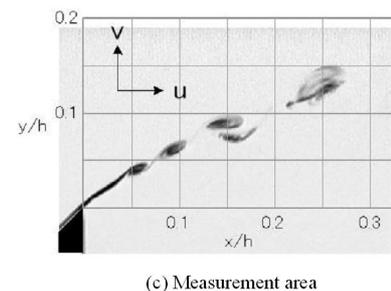
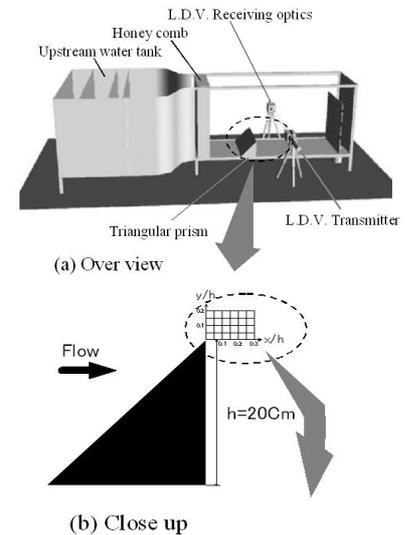


Fig.1 Experimental arrangement and definitions of symbols

Where the vorticity production rate obviously is the producing rate of circulation per unit area in two dimensional flow and was obtained from balance of circulation on the basis of vorticity data and velocity path through an infinitesimal fluid element.

Equation (4) is the vorticity equation in two dimensional laminar flow, and it can be identified that the vorticity production rate in Eq.(3) is the same as the convection term on left hand side of the Eq.(4). Therefore, this fact suggests that either vorticity production or decay would occur due to viscous interaction as indicated on the right hand side of the Eq.(4). On the other hand, Reynolds shear stress was obtained by Eq.(5), where velocities were measured both at $+30^\circ$ and -30° clockwise inclination to x axis, and rms values of velocity fluctuation for each inclined component were defined as rms_+ and rms_- respectively.

Vorticity equation:

$$u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \nu \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \dots\dots\dots(4)$$

Reynolds shear stress:

$$\overline{u'v'} = \frac{rms_-^2 - rms_+^2}{4 \cos 30^\circ \sin 30^\circ} \dots\dots\dots(5)$$

For the flow visualization, as mentioned before, fluorescent dye was introduced into the flow through 1mm diameter dye injection hole at the mid span of the test body. Video recording was made at 1/30s intervals, and the recorded images were compared with the distributions of quantities obtained from velocity data.

3. Results and Discussion

3.1 Flow visualization behind the test body

Figure 2 shows visualization photos of the separated shear layer at 1/30s intervals. In the figure, grid lines are plotted 0.05 times of major dimension h of the body for both x and y directions. In the presented figures, readers can see some lumps of dye concentration of several sizes. Since a vortex entrains fluids from surroundings, each lump can be considered as an individual vortex, and hence a smaller size vortex is pronounced as “small vortex” and larger size vortex produced by pairing of two smaller vortices is pronounced as “intermediate vortex”. The readers can also see from the time series photos of 1/30s to 6/30s in Fig.2 that “small vortices” are formed successively in the separated shear layer due to Kelvin-Helmholtz instability. The time series photos show that a “small vortex”

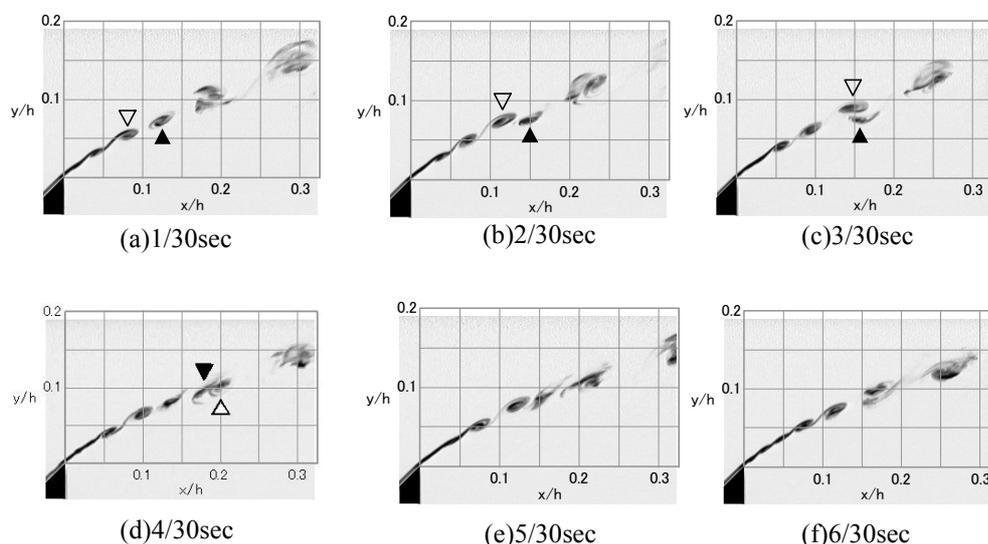


Fig.2 Time series visualization photos

generated at about $x/h=0.1$ in Fig.2(a) which is indicated by symbol ∇ started to pairing with neighboring “small vortex” at about $x/h=0.15$ in Fig.2(c) which is indicated by symbol \blacktriangle , and completed pairing at about $x/h=0.2$ in Fig.2(d). Therefore, in this separated shear layer in the immediate vicinity of the separation point, the characteristics of flow fields in upstream and downstream of vortices pairing region will be fairly different.

In upstream of the separation point, flow field is laminar boundary layer flow because of the presence of solid surface, and vorticity is produced by viscous interaction. In contrast, in separated shear layer sufficiently downstream of the separation point, flow field is free shear layer because of the absence of solid surface. Therefore, in the measuring region of presented study, velocity distribution in the separated shear layer would be shifting from boundary layer like shape to free shear layer like shape, and rest of this report describes viscous interaction and behavior of vortices in the transition process.

3.2 Velocity profiles

Figure 3 shows the moment when two “small vortices” start their pairing at about $x/h=0.15$. Visualization photo is the same as that of Fig.2(c) at 3/30s, and time mean velocity profiles of u component in y direction are superimposed. In a flow field where “small vortex” is generated near the separation point as shown in Fig.3, there is a rapid increase of velocity from zero, and the velocity profile is similar to a laminar boundary layer. In contrast, in the flow field at about $x/h=0.255$, where “intermediate vortex” is formed after pairing, velocity profile is similar to a free shear layer. Therefore, present measuring region is considered to be in transition of velocity distribution from boundary layer like shape to free shear layer like shape.

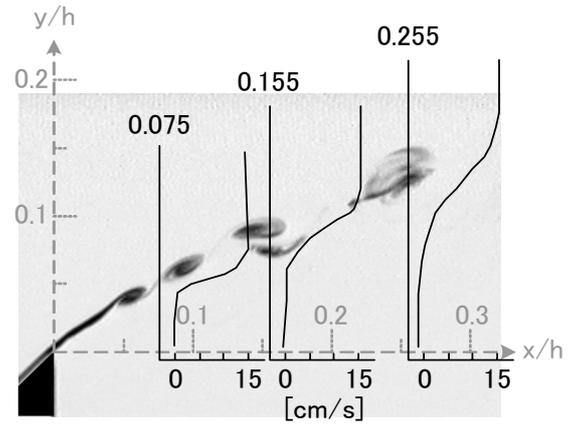


Fig.3 Visualization photo and velocity distribution at each x/h position

3.3 Contour maps of measured quantities

Figure 4 shows contour maps of vorticity, shear strain rate and vorticity decay rate as well as Reynolds shear stress that are calculated from time mean velocity distributions.

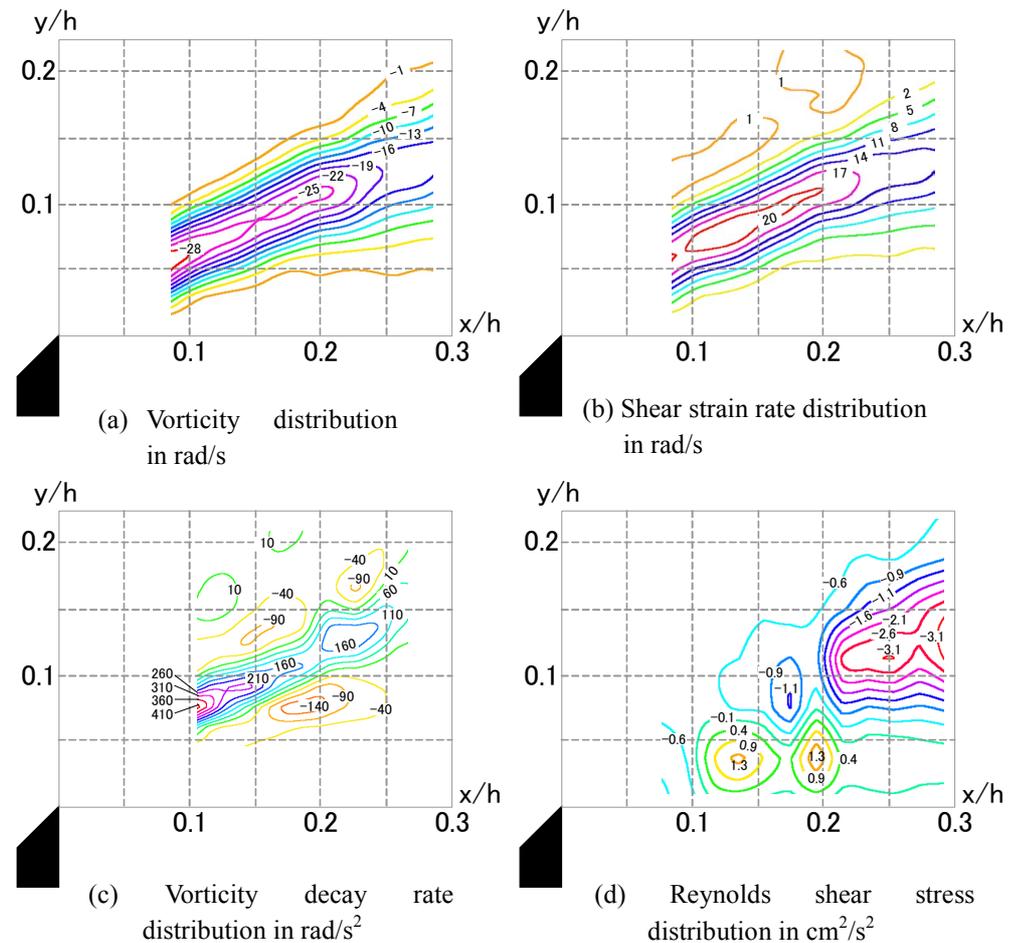


Fig.4 Various contour lines

3.3.1 Vorticity and shear strain rate contour maps

Figure 5 is constructed by superimposing outlines of vortices in visualization photo at 3/30s shown in Fig.2(c) on the vorticity contour map shown in Fig.4(a). Figure 6 is also constructed by superimposing major contour lines of shear strain rate shown in Fig.4(b) with broken lines on the vorticity contour map shown in Fig.4(a).

From Fig.5, one can depict the maximum vorticity at about $x/h=0.1$, the vorticity decreases moderately toward downstream up to about $x/h=0.2$, and then decreases rapidly beyond $x/h=0.2$. Similar trend is found in shear strain rate contour lines that are presented by broken lines in Fig.6. These decreasing trends may be due to the fact that the term $\partial u / \partial y$ both in Eq.(1) of vorticity and in Eq.(2) of shear strain rate is significantly larger than the term $\partial v / \partial x$ in the shear layer, and hence the decrease of u velocity gradient in y direction towards the downstream dominates the decreasing trends of the vorticity and shear strain rate distributions.

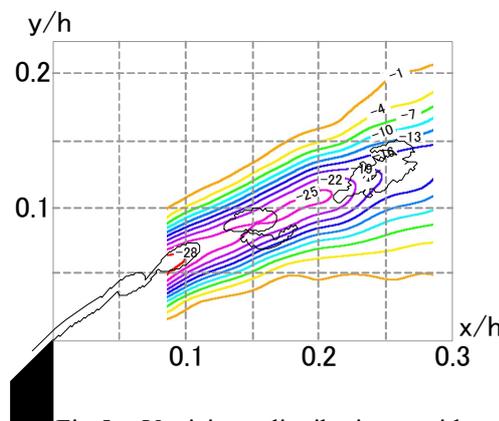


Fig.5 Vorticity distribution with outlines of visualization photo

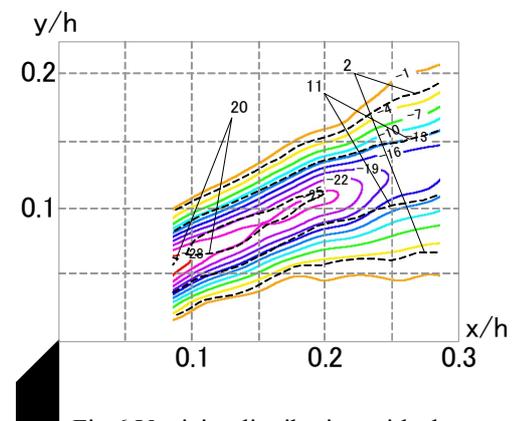


Fig.6 Vorticity distribution with shear strain rate (black dotted lines)

3.3.2 Vorticity decay rate contour map

Figures 7 and 8 are constructed by superimposing vorticity distribution of Fig.4(a) and shear strain rate distribution of Fig.4(b), respectively on the vorticity decay rate distribution of Fig.4(c). Normally, positive vorticity production rate indicates growth and negative vorticity production rate indicates decay. As the coordinate system has been employed in this experiment, sign of the vorticity issued into the separated shear layer is considered negative (clock wise), and hence positive vorticity production rate means decay of negative vorticity or negative vorticity production rate means growth of negative vorticity.

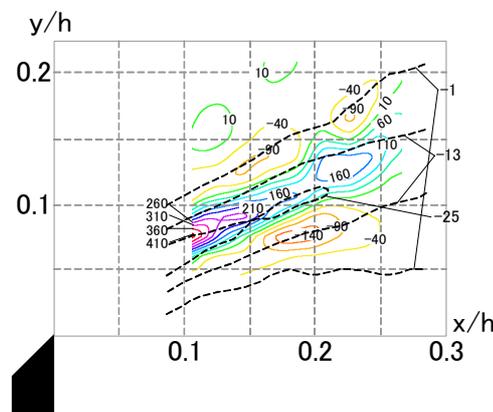


Fig.7 Vorticity decay rate distribution with vorticity (black dotted lines)

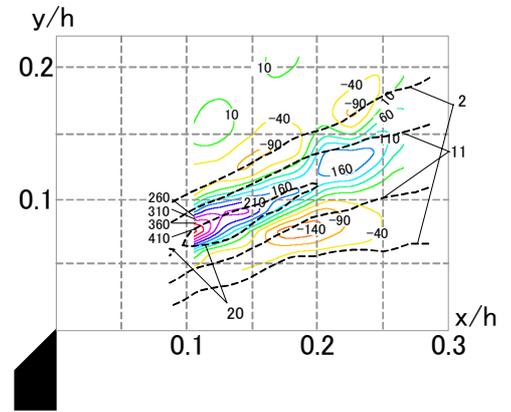


Fig.8 Vorticity decay rate distribution with shear strain rate (black dotted lines)

Therefore, in the presented study, with paying attention to the growth and decay of the negative vorticity, the vorticity production rate is re-pronounced as vorticity decay rate.

From Figures 7 and 8, it can be seen that region of large vorticity decay rate coincides with the region of large vorticity and shear strain rate. As mentioned in Fig.3, viscous interaction associated with transition of velocity distribution from boundary layer like shape to free shear layer like shape cause shear in the flow and it would resulted in vorticity decay. It is remarkable that the velocity profile is almost similar to that of free shear layer at $x/h=0.255$ in Fig.3, and vorticity decay is sufficiently subsided around this region. Mair & Maull⁽³⁾ reported that the large vorticity decay in a Kármán vortex issued from a bluff body was caused by entrainment of counter signed circulation from the separated shear layer on the other side, and it reduced the circulation of the vortex in the far down stream. However, from presented study, it becomes clear that the vorticity decay is also caused by viscous interaction associated with transition of velocity profile in the immediate vicinity of the body. On the other hand, at about $x/h=0.2$ in Figs.7 and 8, there is a small region of vorticity production underneath of the ridges of vorticity and shear strain rate. In order to investigate the small vorticity producing region, superimposition of vorticity decay rate on the outlines of visualized photo at 3/30s is presented in Fig.9. The figure shows that “small vortices” are pairing around this small region, and the “small vortex” on downstream side is stretched as it goes through the small vorticity producing region. Therefore, the small vorticity producing region is considered to be caused by viscous interaction associated with stretching of the “small vortex” on downstream side.

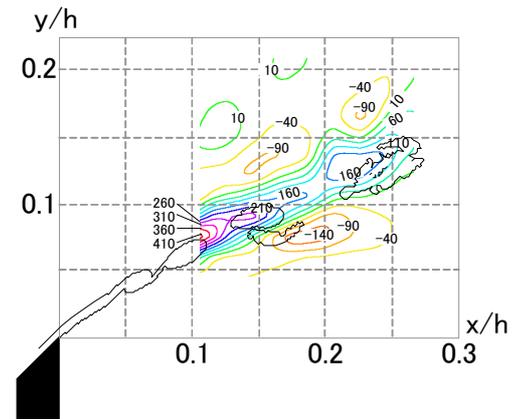


Fig.9 Vorticity decay rate distribution with visualization photo

3.3.3 Reynolds shear stress and flow two-dimensionality

In the issue of Kármán vortex from a circular cylinder, which is the most popular two-dimensional body, presence of flow three-dimensionality such as vortex dislocation is clearly observed at low Reynolds number⁽¹⁰⁾. At a little higher Reynolds number, it is also known that the flow separation line on the surface of circular cylinder meanders along the span and the flow just behind the cylinder becomes three-dimensional⁽¹¹⁾. Moreover,

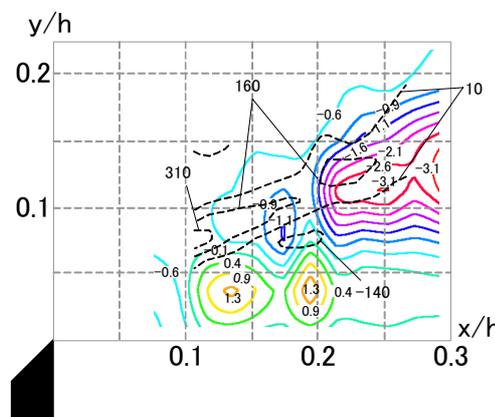


Fig.10 Reynolds shear stress distribution and vorticity decay rate (black dotted lines)

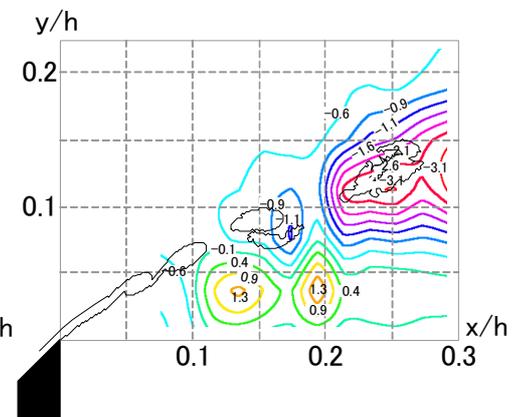


Fig.11 Reynolds shear stress distribution and visualization photo

even in the case of a body with fixed flow separation point, such as triangular prism or square prism with its diagonal line of the cross section is placed parallel to the main stream direction, flow behind the body is known to become three-dimensional⁽¹²⁾. So far in this report, investigations were carried out on the basis of two-dimensional flow measurements. However, it is not certain whether the observed vorticity decay phenomenon is purely originated from the two-dimensional viscous interaction or originated from the appeared weakening of vorticity due to inclination of rotating axis of vortex from the span-wise direction caused by the presence of flow three-dimensionality. On the other hand, one of the characteristics in the process of flow becomes turbulent is the flow field being three-dimensional. Hence the vorticity obtained by two-dimensional measurement would appear to be weakened due to presence of turbulence. In this study, measurements to investigate directly the flow three-dimensionality were not conducted. However, assuming the Reynolds shear stress, which is one of the indicators for flow being turbulent, implies the flow field is becoming three-dimensional, the regions of sufficiently small Reynolds shear stress were dealt as two-dimensional flow fields. In Fig.10, distribution of vorticity decay rate is superimposed on the distribution of Reynolds shear stress. And in Fig.11, outlines of the visualization photo at 3/30s are superimposed on the distribution of Reynolds shear stress. From Fig.10, it can be seen that Reynolds shear stress is fairly small in the region between $x/h=0.1$ and 0.2 , where the vorticity decay is the most significant, and hence the flow field in this region is not turbulent yet. On the other hand, from Fig.11, one can see that the Reynolds shear stress is very large at about $x/h=0.25$, where the pairing of two "small vortices" is completed. Thus, an "intermediate vortex" formed by pairing of two "small vortices" would have three-dimensional flow structure along with flow disturbance.

4. Conclusions

From above experimental investigation, the following conclusions can be drawn on vorticity decay in a separated shear layer.

- (1) Vorticity decay in a separated shear layer behind a body is not only due to the cancelation of vorticity with the counter signed vorticity but also due to viscous interaction associated with transition of the velocity distribution in the separated shear layer from boundary layer like shape in the immediate vicinity of the separation point to free shear layer like shape in the downstream.
- (2) A small region of vorticity production was found around the starting region of pairing of two "small vortices" due to stretching in the process of pairing.
- (3) Reynolds shear stress associated with turbulent diffusion is not directly concerned in the vorticity decay in the immediate vicinity of the separation point from the body.
- (4) Significant increase of Reynolds shear stress, implying the flow field is becoming turbulent and three dimensional, is observed around an intermediate vortex produced by pairing of "small vortices".

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Nomenclature

e : Shear strain rate

h :	Major dimension of the test body
rms :	Root mean square of velocity fluctuation
rms_+ :	rms of velocity fluctuation in $+30^\circ$
rms_- :	rms of velocity fluctuation in -30°
u :	Time mean velocity component in x direction
v :	Time mean velocity component in y direction
$u'v'$:	Reynolds shear stress
x :	Main stream wise (longitudinal) coordinate
y :	Transverse coordinate
ε :	Vorticity production rate or decay rate
ν :	Kinematic viscosity
ω :	Vorticity

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