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Arrangement Analysis of Leaves Optimized on Photon Flux Density or Photosynthetic Rate*

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Abstract

By clarifying a plant evolutive process, useful information may be obtained on engineering. Consequently, an analysis algorithm that investigates the optimal arrangement of plant leaves was developed. In the developed algorithm, the Monte Carlo method is introduced and sunlight is simulated. Moreover, the arrangement optimization of leaves is analyzed using a Genetic Algorithm (GA). The number of light quanta (photon flux density) that reaches leaves, or the average photosynthetic rate of the same was set as the objective function, and leaf models of a dogwood and a ginkgo tree were analyzed. The number of leaf models was set between two to four, and the position of the leaf was expressed in terms of the angle of direction, elevation angle, rotation angle, and the representative length of the branch of a leaf. The chromosome model introduced into GA consists of information concerning the position of the leaf. Based on the analysis results, the characteristics of the leaf of an actual plant could be simulated by ensuring the algorithm had multiple constrained conditions. The optimal arrangement of leaves differs in maximization of the photon flux density, and that of the average value of a photosynthetic rate. Furthermore, the leaf form affecting the optimal arrangement of leave and also having a significant influence also on a photosynthetic rate was shown.

Key words: Leaf Arrangement, Simulation, Photosynthesis, Genetic Algorithm, Monte Carlo Simulation, Dogwood, Ginkgo Biloba

1. Introduction

It is thought that many kinds of plants have evolved so that the photosynthetic rate may be maximized. If the relation between the evolution of a plant and configuration (Here, the figure of a leaf and arrangement) is clarified, information useful for industry may be obtained. For example, the form of a leaf and the distribution state of branches may be applicable to a distributed energy system. The goal of this research involves obtaining information on the light received surface form with the high efficiency of the light received. Therefore, in this paper, the relation between the form of the leaf with high diversity and the amount of sunlight received is investigated. In this paper, the algorithm for analyzing the relation between the arrangement and the amount of light received (photon flux density) of the leaf, and the average photosynthetic rate is developed. When a plant grows higher, it can absorb considerably more sunlight than its peers. However, plant-resources (biomass) are

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restricted and it is necessary to ensure they are distributed effectively in leaves, stalks, a trunk, and roots. It is thought that this distribution percentage is decided based on the operations for leaving a descendant, competition for light received and dynamic strength, etc. Because the biomass is produced from photosynthesis, top priority is given to the configuration evolution of the leaf, which influences the amount of light a plant receives. Generally, the light that reaches a plant leaf is separated into a reflective component, an absorption component to the chloroplast, and a transmission component [(1) (2)]. A plant will not see its branches and leaves increase when in a low position, virtually inaccessible to sunlight. This configuration is remarkable as the individual in a stock [(3)]. Furthermore, the photosynthetic ability and respiration rate of leaves reached easily by sunlight increase [(4)]. The analysis algorithm developed in this paper calculates the number of light quanta that reach a leaf using Monte Carlo simulation. Furthermore, the optimal arrangement of the leaves to maximize the amount of light receives or the average photosynthetic rate is investigated by introducing a Genetic Algorithm (GA). The Monte Carlo simulation has been applied to many such problems to date. In particular, in radiation heat transmission analysis, the radiation view factor of the complex form, including multiplex integration, is easily calculable [(5)]. Therefore, in this paper, the amount of light a leaf with complex form receives is obtained using the Monte Carlo method. On the other hand, GA is applicable to a nonlinear problem with many variables [(6)]. The relation between the amount of light received and the photosynthetic rate of a leaf is nonlinear [(7)]. The former value changes with the angle of direction of a leaf, namely the elevation angle, rotation angle, light source position, and length of the leaf branch. Consequently, these variables are expressed with a chromosome model and the optimal solution using GA is sought. When the photosynthetic rate increases, a plant sees its leaves benefit from a considerable volume of light received. However, when the light becomes superfluous, "damage to the photochemical system" and "moisture loss due to evaporation on leaves" will occur, adversely affecting the photosynthetic ability [(8)]. It is thought that the plant is controlling the amount of light received by adjusting the angle of the leaves, which allows sunlight to reach the rear leaves. Consequently, the photosynthetic rate for the whole plant in question is expected to increase. In this paper, the relation between the configuration of a plant leaf and the photosynthetic rate is investigated using the developed analysis algorithm.

2. Analysis Model

2.1 Leaf model

A plant produces organic matter (biomass) by photosynthesis. Plants have evolved to maximize the amount of sunlight taken in by optimizing the form of their leaves, or changing the length of the branches of leaves. In this paper, the amount of light received (photon flux density) and photosynthetic rate are investigated using the model of the leaf of a dogwood (Fig. 1) and a ginkgo tree. Details of the element dividing of each leaf (Figs. 2 and 3) are described in Section 2.2.3, while Figure 4 shows a model showing the relation between the amount of light received, and the photosynthetic rate (It was prepared based on the reference [(8)]). In regions with small amounts of light received, the photosynthetic rate increases based on an increase in the amount of light received. However, when light is superfluous, damage to the photochemical system and moisture loss due to evaporation on the leaf occur, which adversely affects the photosynthetic rate. In such cases, the actual plant is thought to be adjusting the amount of light it receives by changing the direction of its leaves, and folding the same. The leaf of a dogwood can also adjust the amount of light received via these methods. In the analysis of this paper, the amount of light received by the leaves of a dogwood and ginkgo tree is adjusted by on the direction of the leaf.

2.2 Coordinate systems and a shading check

2.2.1 Coordinates of the leaf position

The coordinate system used for analysis, the form of a leaf, and the symbol showing its arrangement is shown in Fig. 5. The subscripts m and n in Fig. 5 show the surface element numbers of Leaves 1 and 2, respectively. The position of a leaf is expressed in terms of the angle of direction θ, the elevation angle φ, the rotation angle β, and the representation length r of the branch of the leaf. The reference planes of θ and φ are the x-z and x-y planes, respectively, while β is based on the axis perpendicular to the x-y plane. The end of a leaf is connected with a branch of the leaf, and the angle of the central line of the leaf and the leaf surface is right-angled. Depending on the photosynthetic ability of a plant leaf, the environmental condition of light may be known (4). Consequently, the content of the photosynthesis enzyme ε is given for every surface element of a leaf model.

2.2.2 Model of light source

Figure 6 shows the model of the light source introduced into the analysis, and emits a light quanta towards leaves from the radiation surface P_{v1} - P_{v2} - P_{v3} - P_{v4} . In this paper, sunlight is simulated with many light quanta and the number of light quanta reaching each leaf is calculated using the Monte Carlo method, while the photon flux density is taken further. Using the analysis calculation described later, many light quanta are emitted to a leaf from the random point $P_{e,q}$ on the virtual radiation surface shown in Fig. 6. The point of light quantum having reached Leaves 1 and 2 is expressed with $P_{t,a}$ and $P_{u,b}$, respectively. Here, subscripts a and b represent the numbers of the light quantum reaching Leaves 1 and 2 directly. Figure 7 shows the leaf model of two sheets that overlap optically. When a light quantum flux reaches the surface element m on Leaf 1, they will be distributed with reflectivity $\gamma_{t,m}$, absorptivity $\alpha_{t,m}$, and transmissivity $\tau_{t,m}$. The relations among reflectivity, absorptivity, and transmissivity are expressed in Eq. (1). When the transmission light quantum flux of Leaf 1 reaches the surface element n on Leaf 2, they will be further distributed by the probability of reflectivity $\gamma_{u,n}$, absorptivity $\alpha_{u,n}$, and transmissivity $\tau_{u,n}$. The relation of Eq. (1) is also realized in this case. However, in the analysis of this paper, the light quantum reflected by a certain leaf is not treated. Many plants change the transmissivity of a leaf by the photon flux density. By the analysis calculation described later, the transmissivity of all the surface elements of leaves is rendered the same (1) (2). In addition, with an actual plant, there is a light received amount by

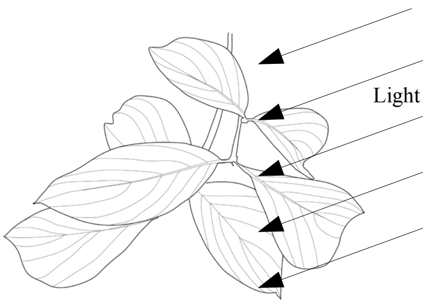


Fig. 1 Example of the leaf form of a dogwood

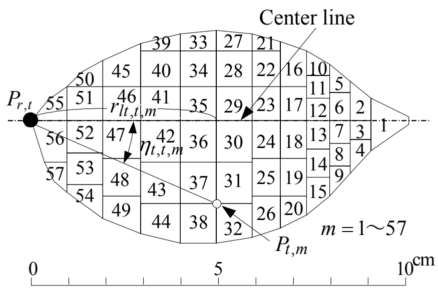


Fig. 2 Dogwood leaf model

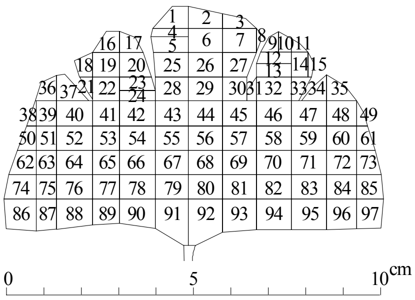


Fig. 3 Ginkgo biloba model

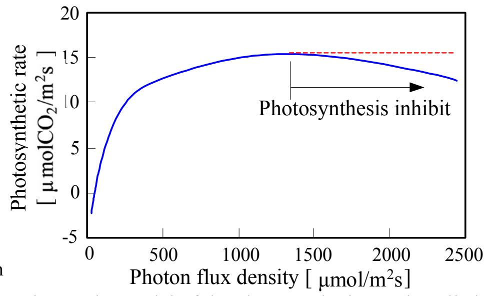


Fig. 4 The model of the photosynthetic rate installed into analysis

diffraction or dispersion. Although these are not taken into consideration in this paper, the light received amount of a leaf may be influenced greatly.

$$\alpha_{t,m} + \gamma_{t,m} + \tau_{t,m} = 1 \tag{1}$$

2.2.3 Surface element model of leaf

The shade area caused by an overlap of leaves differs based on the form of a leaf, the inclination, area, and the length of a branch of the leaf. For example, the length of a branch of a ginkgo biloba differs greatly in the case of each leaf. The length of the branch of a leaf varies and it is thought that the shade area due to the overlap of leaves is reduced. On the other hand, the branch length of a dogwood leaf is usually short. In the analysis in this paper, since the form of a leaf is characteristic, leaf models of a dogwood and ginkgo biloba are used. Figure 2 shows the leaf model of a dogwood, which consists of quadrangle surface elements of 57. $P_{r,t}$ is the corner point of the central line that simulates the costa of the leaf. Each of the apex coordinates on the surface element of a leaf are given in angle $\eta_{lt,t,m}$ and distance $r_{lt,t,m}$, as shown in Fig. 2. On the other hand, the model (Fig. 3) of the ginkgo biloba consists of surface elements of 97 and the area ratio of the leaf model of a ginkgo biloba to a dogwood is 0.81.

2.2.4 Light quantum interrupts

Figure 8 shows the model of the shadow due to an overlap of a leaf. In the model of Fig. 8, the light quantum emitted from a virtual radiation surface passes the surface element m on Leaf 1, before reaching the surface element n on Leaf 2. The existence of the leaf that interrupts the light quantum is checked in the following procedures. $\overrightarrow{V_{e,q}}$ defines the normal line vector in the arbitrary point $P_{e,q}$ on the virtual radiation surface that emits a light quantum. Below, '$\overrightarrow{\ \ }$' expresses a vector. Moreover, the light quantum emitted from $P_{e,q}$ passes the point $P_{t,m}$ on the surface element m of Leaf 1, and reaches the point $P_{u,n}$ on the surface element n of the leaf 2. When the distance of $P_{e,q}$ and $P_{t,m}$ is expressed with R, the intersection coordinates $P_{t,m}$ on Leaf 1 will be calculated by Eq. (2). However, '·' in Eq. (2) expresses an inner product. Moreover, '×' in Eq. (3) expresses the outer product of a vector. When each side of Leaf 1 is expressed with vectors, the normal line vector on the apex $P_{s1,m}$ of a surface element is calculable by Eq. (3). When $P_{t,m}$, seen from the $P_{e,q}$ side, is located on the right side of all sides, intersection $P_{t,m}$ exists on the surface element m of Leaf 1. Equations (4) are conditions in case $P_{t,m}$ is located on the right side of the side 1-2 of the surface element m. Likewise, it is confirmed whether $P_{t,m}$ is located on right side about all the sides.

$$P_{t,m} = P_{e,q} + R \cdot \overrightarrow{V_{e,q}} \tag{2}$$

$$\overrightarrow{N_{t,m}} = (P_{s2,m} - P_{s1,m}) \times (P_{t,m} - P_{s1,m}) \tag{3}$$

$$\{(P_{s2,m} - P_{s1,m}) \times (P_{t,m} - P_{s1,m})\} \cdot \overrightarrow{N_{t,m}} \geq 0 \tag{4}$$

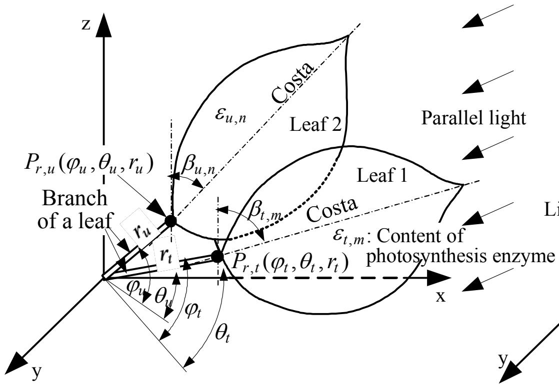


Fig. 5 System of light received analysis of leaves

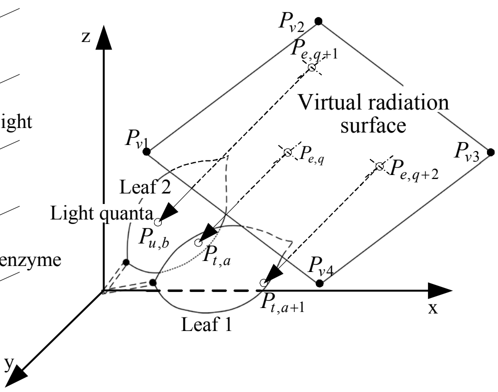


Fig. 6 Virtual radiation surface and light received of leaves

2.3 Objective function

The light quanta flux emitted from the virtual radiation surface calculates the number that reaches the surface element k on leaf w using Eq. (5). The first term of the right-hand side of Eq. (5) is a number that arrives at k directly among the emitted light quanta. Moreover, the 2nd term of the right-hand side is a number that transmits leaves except for w and reaches k among the light quanta emitted from the virtual radiation surface. Equation (6) is a formula of $q'_{h,i,k}$ in Eq. (5) on the right-hand side. Here, $\tau_{j,k}$ is the transmissivity in the surface element k of Leaf j. The number of light quanta that transmits the surface element k of Leaf j is a product of the light quanta flux and transmissivity. However, in the Monte Carlo method, transmission of a light quantum is judged by making transmissivity into a threshold value using a random number between 0-1.

$$E_{w,k} = q_{d,w,k} + \sum_{i=1}^{J_l} q'_{h,i,k} \quad (i \neq w) \tag{5}$$

$$q'_{h,i,k} = \prod_{m=1}^{J_{e,i}} \left(\tau_{j,m} \cdot q_{d,m,k}\right) \quad (j \neq w) \tag{6}$$

The objective functions include (a) Maximization of the number of light quanta reaching each leaf from the virtual radiation surface, and (b) Maximization of the average value of the photosynthetic rate of all the leaves. Concerning the objective function (a), the number of light quanta reaching all the leaves using Eq. (5) is calculated. The arrangement of leaves (θ, φ, β, r) in case this number of light quanta is the maximum is decided as the optimal solution. On the other hand, with reference to the objective function (b), the relation between the photon flux density and the photosynthetic rate, which was shown in Fig. 4, is used, while the photon flux density, as calculated by Eq. (5), is introduced into Fig. 4, and the photosynthetic rate is taken. Arrangement with the largest average value of the photosynthetic rate of each leaf represents the optimal solution.

3. Analysis Method

3.1 Calculation of the amount of light received due to the Monte Carlo Method

On virtual radiation surface shown in Fig. 6, the radiant point $P_{e,q}$ of a light quantum is

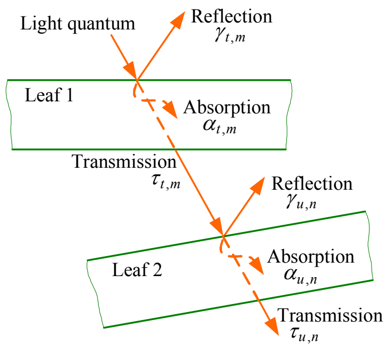


Fig. 7 Optical system model

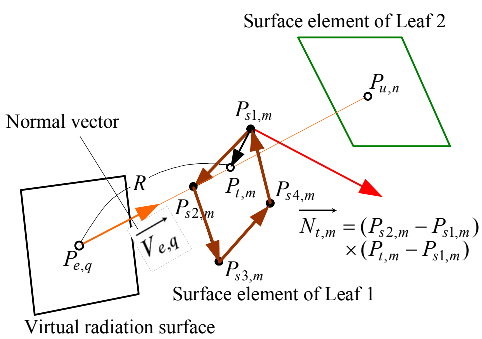


Fig. 8 Interrupt check of light quanta

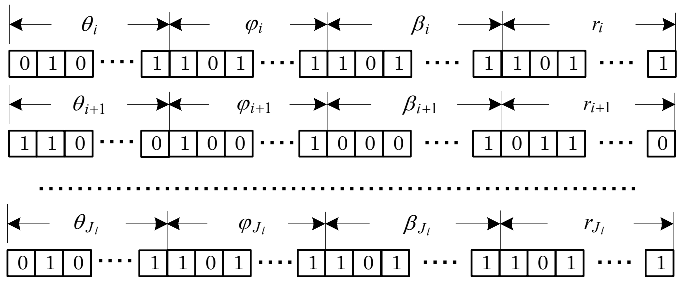


Fig. 9 Chromosome model

determined using a random number. Since the perpendicular vector of $P_{e,q}$ and virtual radiation surface is calculable, this is decided to be the path of a light quantum. The number of leaves of the analysis system is set to J_l. The surface element data of each leaf given to the program is the point $P_{r,i}$ (i is a leaf number) shown in Fig. 2, and its relative position ($\eta_{t,i,m}$ and $\eta_{t,i,m}$, m is the surface element number of the leaf). Furthermore, the initial value of the position (θ, φ, β and r) of all the leaves is determined using random numbers. If θ_i, φ_i, β_i, r_i, $\eta_{t,i,m}$ and $\eta_{t,i,m}$ are decided, apex coordinates ($P_{sx,i,m}, P_{sy,i,m}, P_{sz,i,m}$) of the surface element m of Leaf i will be taken by calculating Eqs. (7) to (9).

$$P_{sx,i,m} = P_{rx,i} - \eta_{t,i} \cdot \frac{\cos(\pi/2 - \eta_{t,i,m} - \beta_i)}{\cos \eta_{t,i,m}} \cdot \sin\theta_i - \eta_{t,i} \cdot \sin\varphi_i \tag{7}$$

$$P_{sy,i,m} = P_{ry,i} + \eta_{t,i} \cdot \frac{\cos(\pi/2 - \eta_{t,i,m} - \beta_i)}{\cos \eta_{t,i,m}} \cdot \cos\theta_i - \eta_{t,i} \cdot \sin\varphi_i \tag{8}$$

$$P_{sz,i,m} = P_{rz,i} + \eta_{t,i} \cdot \frac{\cos\varphi_i}{\cos \eta_{t,i,m}} \cdot \sin(\pi/2 - \eta_{t,i,m} - \beta_i) \tag{9}$$

When the apex coordinates of all the surface elements of each leaf are decided, the surface element on the path of the light quantum can be determined from the method shown in Fig. 8. Moreover, Eqs. (5) and (6) can be calculated by counting the number of light quanta that reach the surface element of each leaf directly, and those that reach after being transmitted to leaves in the vicinity multiple times. Accordingly, the number and rate having reached each leaf of the light quanta flux emitted from virtual radiation surface can be determined.

3.2 Optimization of the leaf arrangement due to GA

3.2.1 Chromosome model

The chromosome model introduced into the analysis of GA is shown in Fig. 9. The chromosome model expresses θ_i, φ_i, β_i and r_i with a 10-bit code of 0 and 1, respectively. Here, subscript i expresses a leaf number and J_l is the quantity of the leaf. The analysis error having occurred by expressing the real number with 10-bit binary numbers is 1% or less. The initial generation's chromosome models are determined using random numbers, and it searches for solutions with an adaptive value repeated high and more for a generation, adding the gene manipulation described later.

3.2.2 Gene manipulation

The low group of adaptive value is selected from the chromosome models, while the high group of adaptive value is made to survive at a constant rate. However, in order to keep up the diversity of the chromosome models with a high adaptive value, gene manipulation is added, based on the aforementioned probability. In this paper, mutation is added to each gene group showing θ_i, φ_i, β_i and r_i in the chromosome model shown in Fig. 9. During the operation of mutation, the gene of the position determined at random is reversed compulsorily.

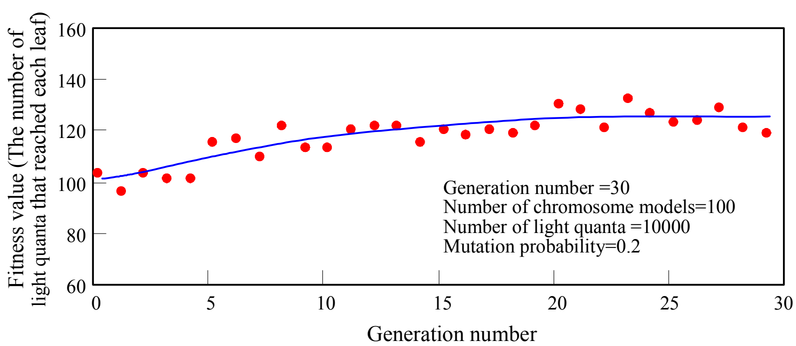
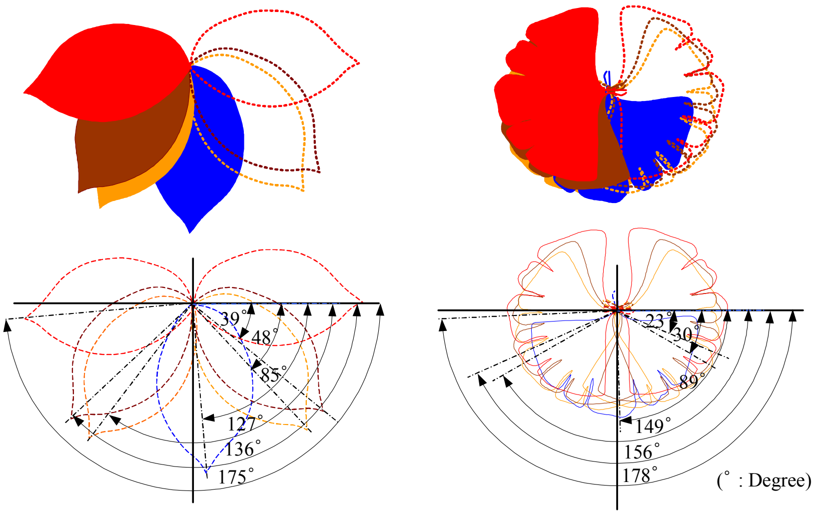


Fig. 10 Convergence result

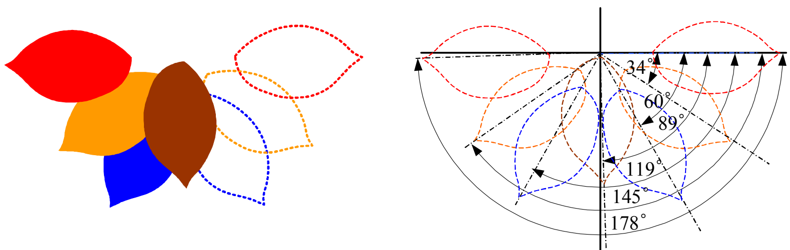
3.3 Analysis procedure

The form data (Figs. 2 and 3) of each leaf model are initially input, and the initial generation's chromosomes model (Fig. 9) is further generated at random. The point P_r of each leaf is then decided by decoding the gene in a chromosome mode. Next, Eqs. (7), (8), and (9) are calculated, and the apex coordinates of the surface element of each leaf are obtained. The emission position $P_{e,q}$ in a virtual radiation surface is determined based on a Monte Carlo method calculation, and the path of a light quantum is decided by calculating a normal line vector further (Fig. 6). Moreover, the surface element number of leaves reached by a light quantum can be determined by calculating Eqs. (2) to (4). If the number of light quanta reaching all the leaf models is determined, the adaptive value of each chromosome model is calculable using Eqs. (5) and (6). The gene manipulation described in Section 3.2.2 is added to the chromosome models of high adaptive value, while the new chromosomes determined at random are also added, and next-generation chromosomes are generated. The calculation described at the top is repeated based on the set-up frequency (the last generation). For the last generation's chromosome models, an individual with the highest adaptive value is decided as the optimal solution. By decoding the optimal solution, the

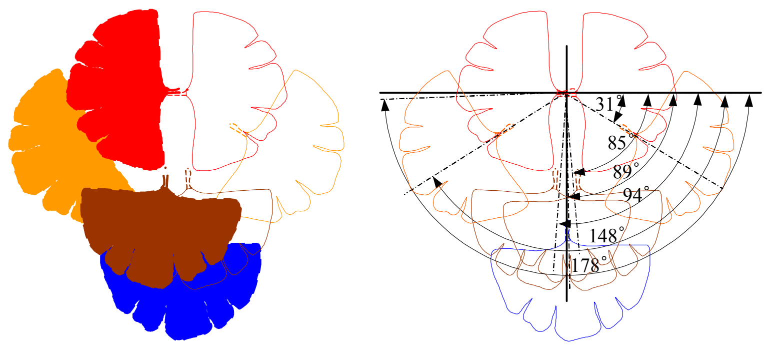


(a) Leaf of dogwood (b) Ginkgo biloba

Fig. 11 Analysis result in case the total of the number of light quanta that leaves take is the maximum. In the case of $\theta = \varphi = r = 0$.



(a) Leaf of dogwood



(b) Ginkgo biloba

Fig. 12 Analysis result in case the total of the number of light quanta that leaves take is the maximum. In the case of $\theta = \varphi = 0$.

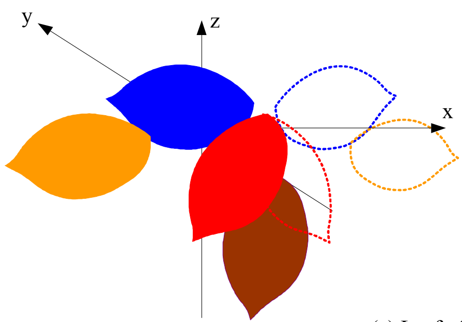
optimal arrangement of each leaf (θ , φ , β and r) can be taken.

4. Analysis Result

4.1 Analysis conditions

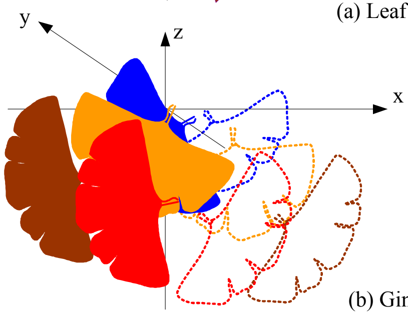
4.1.1 Analysis parameters

The analysis time of the developed algorithm changes significantly based on the number of light quanta operated by the Monte Carlo calculation, and the parameters given to GA. Therefore, in this paper, the following analysis parameters were set up by trial and error. The number of light quanta emitted from a virtual radiation surface is 10000 pieces, and was decided to be equivalent to the photon flux density of 13 μmol/m2s . There are 100 chromosome models to operate (Fig. 9). Gene manipulation is only mutation and the mutation probability is 0.2. Less than 10% is chosen at random from each gene group showing θ , φ , β and r in the chromosome model, and mutation is added. Because the convergence condition of the analysis calculation changes according to the analysis problems, in this analysis, the final generation number is adjusted and determined to be between 30 and 200. Figure 10 shows the relationship of the generation number and



	θ	φ	r	β
Leaf	120.4	-4.2	26.6	149.9
Leaf	24.0	-37.0	46.7	175.6
Leaf	380.5	-42.2	35.8	101.7
Leaf	422.4	-0.2	36.8	91.3

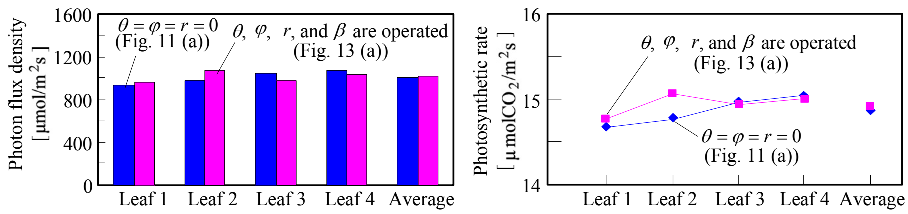
(a) Leaf of dogwood



	θ	φ	r	β
Leaf	134.7	-1.4	54.9	91.8
Leaf	287.7	-45.3	56.9	102.7
Leaf	326.8	-14.9	12.9	174.6
Leaf	40.0	-36.8	13.5	176.8

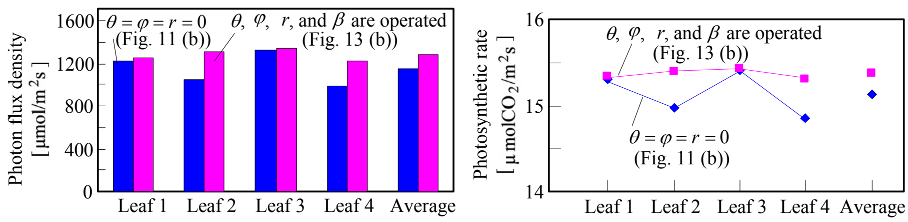
(b) Ginkgo biloba

Fig. 13 Analysis result in case the total of the number of light quanta that leaves take is the maximum



(a) Photon flux density (b) Photosynthetic rate

Fig. 14 Analysis result in case the number of total light quanta that the leaves of dogwood take is the maximum



(a) Photon flux density (b) Photosynthetic rate

Fig. 15 Analysis result in case the number of total light quanta that the ginkgo bilobas take is the maximum adaptive value when analyzing using the parameters shown in this figure. The adaptive value has the tendency to increase, however, meaning the generation number also increases. However, the solution to the maximum adaptive value is not necessarily taken in the last generation. This proximate cause is a calculation error of Monte Carlo calculation and GA analysis.

4.1.2 Coordinate system and virtual radiation surface

In analysis, the coordinate system of Fig. 5 or Fig. 6 is used. Moreover, the virtual radiation surface is set as the region whereby the light quanta can reach all the leaf models. In the analysis of this paper, the virtual radiation surface was decided to be a plane of $P_{V,1}=$ (500,500,500), $P_{V,2}=$ (500,-500,500), $P_{V,3}=$ (500,-500,-500) and $P_{V,4}=$ (500,500,-500). Moreover, the reflection and transmission on the leaf of a light quantum are ignored because of the negligible influence of the analysis result.

4.1.3 Constrained conditions

The total J_l of a leaf may be two to four sheets. Moreover, the constrained condition is set up as follows and the result of the rotation symmetry is obtained.

$$0 \text{ degree} \le \theta \le 90 \text{ degree} \quad (10)$$

$$-90 \text{ degree} \le \varphi \le 0 \text{ degree} \quad (11)$$

$$90 \text{ degree} \le \beta \le 270 \text{ degree} \quad (12)$$

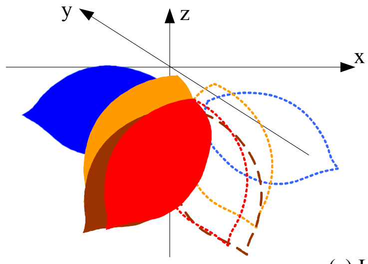
4.2 Analysis result in case the total number of light quanta that leaves take is the maximum

Figure 11 shows the analysis result of the arrangement of leaves of a dogwood and ginkgo biloba, where the total number of light quanta taken by leaves is maximized. However, the angle of direction θ of the leaf, the elevation angle φ, and the representation length r of the branch of the leaf were set as zero, and only the optimal solution of the rotation angle β was investigated. These results are an arrangement of leaves, as seen from the virtual radiation surface. Although the overlap area of Leaves 2 and 3 of the dogwood is considerable, the arrangement of Leaves 1, 4, and 2 or Leaf 3 is distributed relatively widely. On the other hand, the leaf of a ginkgo biloba is in sector form and the overlap area of each leaf is very considerable compared with a dogwood leaf. Therefore, in order for the ginkgo bilobas to increase the amount of light received, the size of each leaf or the representation length r of the branch of the leaf must be adjusted. Consequently, β and r are set as variables and the next step involves exploring the optimal solution. However, because the characteristics of the length of branch of a leaf differ between an actual dogwood and an actual ginkgo tree, the constrained condition of Eqs. (13) and (14) are set to r. These conditions were decided based on the actual plant shoot. Figure 12 shows the result of the optimal leaf arrangement, as seen from the virtual radiation surface. A dogwood and ginkgo tree will distribute each leaf more in the distance, if flexibility is given regarding the length of a branch of a leaf. Consequently, each leaf can increase the amount of light received. In particular, the ginkgo biloba reduces the overlap area of leaves by changing the length of the leaf branch and considerably increasing the amount of light received.

$$\text{Leaf of dogwood}: \quad 0 \le r \le 50 \text{mm} \quad (13)$$

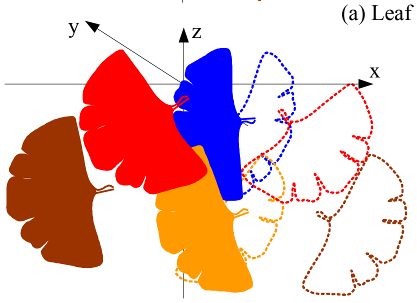
$$\text{Ginkgo biloba}: \quad 0 \le r \le 100 \text{mm} \quad (14)$$

Figure 13 shows the result of analyzing the optimal arrangement as a variable for θ, φ, r and β. Moreover, Figs. 14 and 15 show the analysis result of the photon flux density and the photosynthetic rate of a dogwood and ginkgo tree, respectively. The distribution of



	θ	φ	r	β
Leaf 1	19.9	-41.3	25.5	177.3
Leaf 2	12.6	-25.0	36.3	174.9
Leaf 3	15.2	-9.0	8.3	143.5
Leaf 4	32.3	-2.9	23.1	95.1

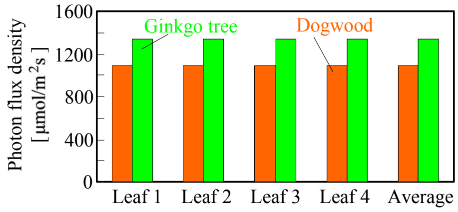
(a) Leaf of dogwood



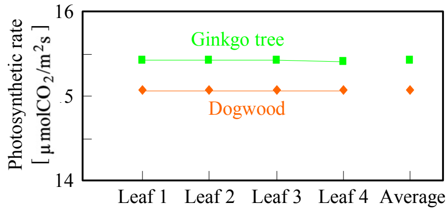
	θ	φ	r	β
Leaf 1	46.8	-3.5	2.0	123.0
Leaf 2	88.2	-43.2	98.7	104.5
Leaf 3	22.8	-25.1	86.5	128.7
Leaf 4	0.9	-27.3	80.6	102.3

(b) Ginkgo biloba

Fig. 16 Analysis result in case the total photosynthetic rate of leaves is the maximum

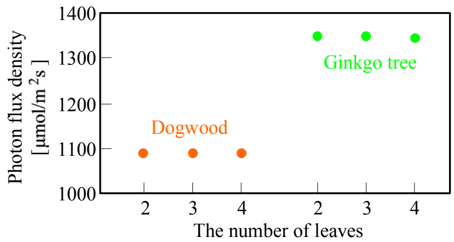


(a) Photon flux density

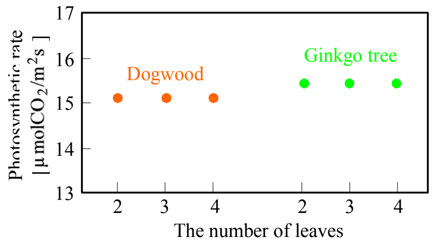


(b) Photosynthetic rate

Fig. 17 Analysis result in case the total photosynthetic rate of leaves is the maximum



(a) Average photon flux density



(b) Average photosynthetic rate

Fig. 18 The number of leaves, and the relationship of the optimal arrangement

the leaf of the ginkgo tree, as shown in Fig. 13 (b), has a wide range of θ, φ, r compared with a dogwood. However, compared with Fig. 12 (b), Fig. 13 (b) of the range of the length of r is narrower. Even if r is restrained by the narrow range, the amount of light received can be made to increase, when the flexibility of a ginkgo biloba of θ and φ is considerable. On the other hand, even if a dogwood changes θ, φ, r significantly, as shown in Fig. 14 (a), there are few increases in the amount of light received compared with $\theta = \varphi = r = 0$. Therefore, the relation between the arrangement of leaves and the amount of light received is considered to have a significant influence on the form of a leaf.

4.3 Analysis result in case the average value of the photosynthetic rate of leaves is maximum

Figure 16 shows the analysis result of the arrangement of leaves in case the average of the total photosynthetic rate of leaves is the maximum. Moreover, the calculation result of the photon flux density and the photosynthetic rate of each leaf are shown in Fig. 17. The ginkgo biloba distributes r widely within constrained conditions, and is increasing the photosynthetic rate. On the other hand, the leaf of a dogwood has a narrow distributed range of θ and r, and the photosynthetic rate is mainly raised by altering φ and β. It is thought that the characteristic arrangements of the actual leaves have been simulated by applying simple restricted conditions for the developed algorithm, based on the analysis result shown in Figs. 16 and 17. When the leaf of a dogwood is viewed from a light source, leaves are distributed in a rotative direction centering on a branch. On the other hand, the ginkgo bilobas are distributed in the direction of an axis line of the branch. Moreover, in order to increase the amount of light received, it is necessary to change the length of the branch of a leaf significantly. In order to perform high-precision analysis, analysis capable of significantly changing the form and size of leaves is required.

4.4 Relation between the leaf form, leaves arrangement and photosynthetic rate

When the analysis result, where leaves take in a maximum of light quanta, is compared with the analysis result, in case the average value of the photosynthetic rate of leaves peaks, the characteristics of the leaf arrangements differ. As shown in Fig. 17, when the average value of the photosynthetic rate of leaves peaks, the analysis result of photon flux density and photosynthetic rate is almost the same for all leaves. The reason is based on the relation between the photon flux density and photosynthetic rate, which were shown in Fig. 4. It is considered since the analysis program arranges leaves in order that the rapid increase range of a photosynthetic rate may be avoided.

The relationship among the number of leaves, the photon flux density, and the photosynthetic rate, in the event that the average value of the photosynthetic rate of leaves is maximized, is shown in Fig. 18. When leaves are placed in an optimal situation, within the limits analyzed in this paper, the photon flux density and the photosynthetic rate will hardly influence the number of leaves. Moreover, despite the considerable photon flux density of a ginkgo tree at 24% when compared with a dogwood, the photosynthetic rate stops at a rise of about 2%, because the increase in the photosynthetic rate following the rise of the photon flux density becomes loose (Fig. 4).

In the leaf model used for these analyses, the area of the leaf of a dogwood is larger than that of the ginkgo biloba, meaning the volume of light received for a dogwood in terms of light quanta is advantageous compared with a ginkgo tree. However, when the result of the optimal arrangement of leaves is compared, the photosynthetic rate of a ginkgo tree is larger. The form of a leaf and the flexibility of length of the branch of a leaf of a ginkgo tree are considered to be advantageous in increasing the photosynthetic rate. It is thought that the form of a leaf and the arrangement of leaves are influenced by a dynamic factor and the survival of the next generation, etc. in addition. It is necessary to add these factors to an improvement in analytical accuracy.

5. Conclusions

An analysis algorithm that introduced the Monte Carlo method and Genetic Algorithm (GA) was developed, and the configuration of a plant leaf and the relationship of the photosynthetic rate were investigated. Using the Monte Carlo method, sunlight was simulated by light quanta flux. Moreover, the optimal arrangement of leaves was determined in the analysis using GA. Consequently, the characteristics of the leaf of an actual plant could be simulated by applying multiple constrained conditions to the developed algorithm. The leaf model of a dogwood and ginkgo tree were analyzed as a case study. The difference in the form of a leaf has a significant influence on the optimal arrangement and the photosynthetic rate. Moreover, the optimal arrangement of leaves differs in terms of the optimization of the photosynthetic rate, and that of the light volume received. Moreover, in order to realize highly precise analysis, it is necessary to take into consideration the influence of difference in the size of a leaf, dynamic factors and survival of the next generation, etc.

Nomenclature

E : The number of leaves

J_e : The number of the leave surface elements
J_l : The number of the leaves
$\overrightarrow{N}$: Normal line vector of the leaf model
P : The point on coordinates
P_e : Coordinates of the radiation position on the virtual radiation surface
P_r : End point of the central line on a leaf model (Fig. 2)
P_s : Apex of the surface element of the leaf
P_t , P_u : Coordinates of the light quantum achieved on the leaf
$\overrightarrow{P}$: Vector of P
P_v : Apex coordinates of a virtual radiation surface
q_d : Coordinates of the light quantum that reached directly from the virtual radiation
q'_h : Coordinates of the light quantum that reaches after transmitting other leaves
R : Distance (Fig. 8) (mm)
r : Branch of a leaf (mm)
r_{lt} : Distance showing the apex of the leaf model (Fig. 2) (mm)
$\overrightarrow{V_e}$: Normal line vector of P_e

Roman character

α : Absorptivity
β : Angle of rotation (degree)
ε : Content of photosynthesis enzyme
φ : Angle of elevation (degree)
γ : Reflectivity
$\eta_{t,m,s1}$: Angle of the central line on the leaf, and the apex $s1$ of element m (degree)
θ : Angle of direction (degree)
τ : Transmissivity

Subscript

a , b : Light-quantum number that reached the leaf directly from the virtual radiation surface
i , j : Number of the leaf model
k , m , : Surface element number of the leaf
q : Simulated light-quantum number
t , u , w : Number of the leaf model

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