Study of weakly bound nuclei with an extended cluster-orbital shell model

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We develop an approach for a unified description of bound and unbound states in the framework of the cluster-orbital shell model (COSM). In order to see the reliability of this approach, we study ^{16}O$+XN$ systems.

1. INTRODUCTION

Recent progress on experiments and theoretical studies for the unstable nuclei have broadened the area of nuclear physics [1]. Many interesting phenomena have been observed and investigated. In order to study such the particular nature of drip line nuclei, in this work, we extended the COSM formalism so as to treat the dynamics of the core. We call our approach as "Neo-COSM" [2].

In the Neo-COSM formalism, the interaction between the core and a valence nucleon is constructed in a microscopic way by using the core density. Hence, the change of the core affects to the energy of the total system through the core-density dependence of the core-N interaction. As the first step calculation, we change the core-size parameter. In this paper, we show the binding energies and r.m.s.radii of ^{16}O$+XN$ systems up to $X = 4$ using the Neo-COSM calculation and discuss the effect of the change of the core-size parameter to the property of the total system.

In the theoretical study of weakly bound systems, the component of unbound states has been considered to be very important. Many approaches have been developed to include the components of unbound states. The Gamow shell model (GSM) has been developed to treat the unbound states in the shell model formalism [3,4]. In order to investigate the importance of the component of the unbound states, we perform a comparison between our Neo-COSM and GSM approaches.

2. EXTENDED CLUSTER-ORBITAL SHELL MODEL APPROACH

2.1. Formalism of the extended cluster-orbital shell model

We construct the Hamiltonian for the valence nucleons in the framework of the cluster orbital shell model [5]. In oder to introduce a degree of freedom to the core, we change the size parameter b in the harmonic oscillator function and explicitly indicate the dependence

of the core-size parameter b as "$[b]$".

The core-N Hamiltonian is obtained by multiplying the core wave function from the left hand side as

$$\left\langle \Phi_C[b] \left| \hat{H} \right| \mathcal{A}' \left\{ |\Phi_C[b]\rangle |\Phi_V\rangle \right\} \right\rangle = \left[\sum_{i \in \mathrm{V}} \hat{h}_i[b] + \sum_{i < j \in \mathrm{V}} \left\{ \hat{T}_{ij} + \hat{v}_{ij} \right\} + \langle H_C[b]\rangle \right] |\Phi_V\rangle . \tag{1}$$

Here $\hat{h}_i[b]$ is one body operator for the i-th valence nucleon defined as $\hat{h}_i[b] \equiv \hat{t}'_i + \hat{V}_i[b]$. $\hat{t}'_i$ is the kinetic energy operator for a valence nucleon, and $\hat{V}_i[b]$ is the microscopic core-N potential constructed by taking into account the anti-symmetrization between the nucleons in the core and the valence ones as

$$\hat{V}_i[b]|\Phi_V\rangle \equiv \sum_{k \in \mathrm{C}} \left\langle \Phi_C[b] \left| \hat{v}_{i\,k} \right| \mathcal{A}' \left\{ |\Phi_C[b]\rangle |\Phi_V\rangle \right\} \right\rangle . \tag{2}$$

The eigen function Ψ_{JM} is obtained as a linear combination of basis sets:

$$\Psi_{JM} = \sum_m c^{(m)} \mathcal{A} \left\{ F^{(m)}(r_1, r_2, \cdots, r_N) |JMTM_T\rangle \right\} . \tag{3}$$

Here, the radial part $F^{(m)}(r_1, r_2, \cdots)$ is the products of the Gaussian functions as

$$F^{(m)}(r_1, r_2, \cdots, r_N) \equiv g_1^{(m)}(r_1) \cdot g_2^{(m)}(r_2) \cdots g_N^{(m)}(r_N) . \tag{4}$$

The linear combination of $F^{(m)}(r_1, r_2, \cdots)$ contains a set of different size parameters $a_i^{(m)}$, which are determined by the diagonalization of the Hamiltonian.

The energy of the core $\langle H_C[b]\rangle$ can be calculated analytically in a certain case [6]. Therefore, the energy of the total system $E(b)$ is obtained as the sum of the core and valence parts, $E(b) = \langle H_C[b]\rangle + \langle H_V[b]\rangle$. The energy for the valence part $\langle H_V[b]\rangle$ depends on b through the core-N interaction $\hat{V}_i[b]$.

2.2. Stochastic variational approach

The variational parameters in this calculation are the size parameters $a_i^{(m)}$ of the Gaussian basis function and angular momenta $j_i^{(m)}$ and $l_i^{(m)}$. Once the eigen values with the M basis sets is obtained, we diagonalize the Hamiltonian by adding the $(M+1)$th base as a candidate and calculate the sum of the difference to the ith eigen values as

$$\mathcal{E} = \sum_{k=1}^{k_{\max}} |E_k^{(M)} - E_k^{(M+1)}| , \tag{5}$$

where the suffix k is the label of the eigen states, for example, $k = 0$ stands for the ground state. If, $\mathcal{E}$, the sum of the differences between the eigen values $E_k^{(M)}$ and $E_k^{(M+1)}$ becomes larger than a parameter $\mathcal{E}_0$, i.e. $\mathcal{E} > \mathcal{E}_0$, the new candidate is included to the total basis functions of the calculation as the $(M+1)$th basis set.

2.3. The Core-N interactions

We use the Volkov potential [7] as the effective NN-interaction. The Majorana parameter is chosen as $M_k = 0.58$, and we fix this value in all calculations.

The Hamiltonian for ith valence particle $\hat{h}_i[b]$ becomes as follows:

$$\hat{h}_i[b] = \hat{t}_i + \hat{V}_i^d[b] + \hat{V}_i^{ex}[b] + \hat{V}_{ls}[b] + \lambda\hat{\Lambda}_i[b] \; . \tag{6}$$

Here, $\hat{V}_i^d$ and $\hat{V}_i^{ex}$ are the direct and exchange parts, respectively, and $\hat{\Lambda}$ is the projection operator to the Pauli forbidden states. For the exchange part $\hat{V}_i^{ex}$, we apply the "knock on" exchange kernel [8]. Further, we introduce an effective LS-potential to reproduce the energies of the positive parity states, $5/2^+$ and $3/2^+$ of ^{17}O.

3. RESULTS

3.1. Neo-COSM calculations

We calculate binding energies and r.m.s. radii of oxygen isotopes and $N = 8$ isotones. Fixed-b calculations are shown in Fig. 1. As a first step to take account the dynamics of the core, we introduce a degree of freedom for the core-size parameter b. The effect of the dynamics of the core to the total system can be discussed with a microscopic treatment through the change of the core-size parameter [3]. We call this approach as "Neo-COSM".
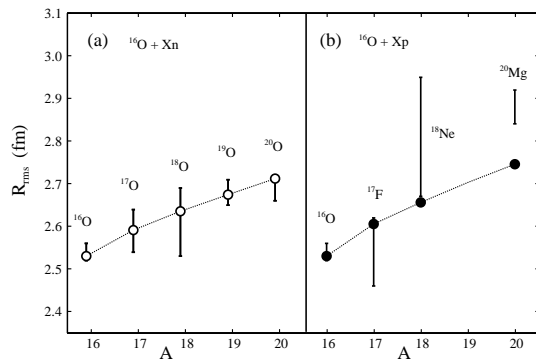

Figure 1. Calculated r.m.s.radii for ^{16}O$+XN$ systems with a fixed b-parameter. Open and solid circles are our calculation, and lines with error bars are the experiments [1]. Dotted lines is for the eye-guide.
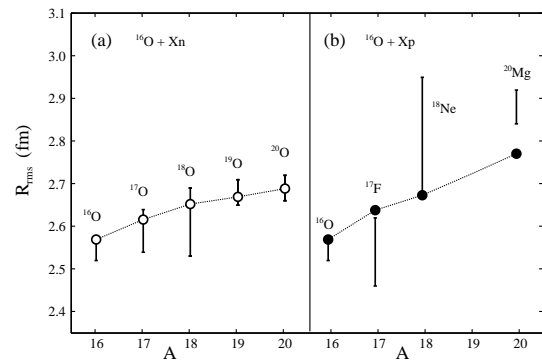
Figure 2. Calculated r.m.s.radii for ^{16}O$+XN$ systems with Neo-COSM, Open and solid circles are our calculation, and lines with error bars are the experiments [1]. Dotted lines is for the eye-guide.

We calculate the energies of of ^{17}O and ^{17}F by changing b. We consider that the difference for the energy of $1/2^+$ in the change of b relates the Thomas-Ehrman shift and will be a key mechanism to create the isotope and isotone dependence in the Neo-COSM calculation.

Calculated results for ^{16}O$+XN$ systems from $X = 2$ to 4 and comparison to the experiment [1] are shown in Fig. 2(a) for the oxygen isotopes (^{16}O$+Xn$ systems) and in 2(b) for the $N = 8$ isotones (^{16}O$+Xp$ systems). We find that the change of the r.m.s. radii with increase of the number of valence neutrons becomes smaller than those obtained by

Table 1
R_{rms} and adopted b values obtained by the Neo-COSM calculations. All units are fm.

	^{16}O	^{17}O	^{18}O	^{19}O	^{20}O	^{17}F	^{18}Ne	^{20}Mg
R_{rms}	2.57	2.62	2.65	2.67	2.69	2.64	2.68	2.77
b	1.75	1.76	1.74	1.72	1.71	1.76	1.74	1.74

the fixed b calculation. Results are shown in Table 1. This is because the adopted b values of the Neo-COSM calculation decrease as the neutron number increases. Therefore, the r.m.s. radius of ^{20}Mg becomes much larger than that of ^{20}O, and the A-dependence of the r.m.s.radii is qualitatively improved to reproduce the experiment.

3.2. Comparison with the GSM approach

The essential difference between COSM and GSM [3,4] is the treatment of the unbound components of the single-particle states. We expand the wave function obtained by COSM and compare the weights of the single-particle components to those obtained by GSM. As an example of weakly bound systems, we calculate the helium isotopes. Energies and radii are almost reproduced. The main component $p_{3/2}$-wave of the single-particle state does not show any difference between COSM and GSM. We found the difference appears in the second main component in the wave function, for example, $(0p1/2)^2$ in ^{6}He.

4. SUMMARY

We studied the ^{16}O$+XN$ systems in an extended cluster-orbital shell model, that we call Neo-COSM. Neo-COSM has two important features. First one is that the correlations among the valence nucleons and the asymptotic behavior of the wave functions can be correctly described in a unified formalism. Second one is that the calculation is performed by taking into account the dynamics of the total systems. Further, we studied pole and continuum contributions to the wave function obtained by COSM. As examples of normal nuclei and weakly bound nuclei, we study oxygen and helium isotopes.

We showed the Neo-COSM is a useful approach to discuss the many-body weakly bound systems. Discussions and investigations for the effect of non-central forces, e.g. tensor force, and the core-polarization are important future works,

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