Simplified modeling of cluster-shell competition in carbon isotopes

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We study the cluster-shell competition in carbon isotopes. Two parameters are introduced for the ^{12}C core to characterize the transition from a cluster state to a shell-model state as a measure of the cluster-shell competition: one expresses the relative distance between α clusters and the other does the dissolution of one of the α clusters. We show the energy curves as functions of these parameters. The calculation is performed by combining the simplified modeling of cluster-shell competition (SMSO) for the ^{12}C core and AMD triple-S. As the neutron number increases, the α-α distance becomes smaller, on the contrary, the dissolution of α clusters is enhanced. The origins of the anomalously small $B(E2)$ value of ^{16}C are discussed from cluster-shell competition point of view, and the appearance of pure cluster state around the 10 MeV region is suggested.

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I. INTRODUCTION

Nuclear systems show different facets in changes of the proton-neutron number and the excitation energy. One of the standard picture for the nuclear systems is the shell-model view-point that a self-consistent mean-field is created by the nucleons themselves. Here, nucleons are treated as quasi-free particles moving in a potential, despite the original nucleon-nucleon interaction is rather short range, and there is no potential source at the center of the nucleus. Here, the spin-orbit interaction is very strong contrary to the atomic systems so that it makes possible to explain the observed magic numbers [1, 2]. Therefore, basically, the jj-coupling type wave function is adopted in the shell-model.

One of the other important aspects of the nuclear systems is the cluster-model. The main feature of the cluster model is that strongly correlated subsystems are spatially localized as constituents. Here, the most plausible constituent is the α-particle, namely two protons and two neutrons occupy the $(0s)^4$ configuration corresponding to the lowest closed shell. These four nucleons strongly interact with each other, however, on the contrary, the α-α interaction is weak. Hence, the α-particles can be considered as subunits in the nucleus. This molecular view-point of the alpha particles has been introduced even before the shell-model [3], and such cluster structure has been extensively studied for more than forty years [4–6]. Recently, for some light neutron-rich nuclei, theoretical and experimental investigations have been carried out based on the cluster models, which became one of the important topics of light exotic nuclei [7, 8].

Although the shell-model and the cluster-model have quite different feature, the real systems may have both components. To take into account the shell-model component in the cluster model, the effect of dissolution of the α-cluster(s) due to the spin-orbit interaction, which does not act for the $N\alpha$ systems, should be treated. In our previous work [9], we have demonstrated "cluster-shell competition" in light nuclei based on AMD triple-S [10]. In the calculation, the wave functions of antisymmetrized molecular dynamics (AMD) [11] are generated as basis states and superposed by means of an effective way to choose the basis set. In order to take into account the spin-orbit contribution efficiently, the cooling equation is applied only for the imaginary parts of the Gaussian center parameters of the nucleons, whereas their real parts are randomly generated. After performing the angular momentum projection, important basis states are chosen in the same way as stochastic variational method (SVM) [12, 13].

As the second step of the study on the cluster-shell competition, we have proposed a simplified method and discussed that the imaginary parts of the Gaussian center parameters can be a measure of the transition from a cluster state to a shell-model state [14]. When the parameter for the imaginary part, Λ, changes from 0 to 1, the four nucleons in one of the α-clusters gradually shift into the jj-coupling type wave function. This introduction of the parameter Λ is called simplified modeling of spin-orbit interaction (SMSO) [14]. We have also studied how the tensor correlation is taken account in the cluster model in a similar manner [15].

In this work, we study the cluster-shell competition in neutron-rich carbon isotopes by combining SMSO and AMD triple-S. We introduce two parameters for the ^{12}C core part; (1) R_1 as the α-α distance parameter and (2) Λ as a measure of dissolution of one of the α clusters. The change of optimum R_1 and Λ values with respect to the number of neutrons is studied. Also, we study the energy levels of ^{16}C and search for the pure cluster state in the level scheme. Finally, we calculate the $B(E2)$ val-

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ues and show the relation between the $B(E2)$ values and two parameters R_1 and Λ. The origins of anomalously small $B(E2)$ value in ^{16}C are discussed.

II. FRAMEWORK

We briefly discuss our framework of SMSO [14]. The total Hamiltonian has the following form:

$$\hat{H} = \sum_{i}^{A} \hat{t}_i - \hat{T}_{c.m.} + \sum_{i<j}^{A} \hat{v}_{ij} \ . \tag{1}$$

Here, A is the mass number and $\hat{t}_i$ is the kinetic energy operator for each nucleon. The kinetic energy of the center of mass motion ($\hat{T}_{c.m.}$) is exactly subtracted from the total Hamiltonian. $\hat{v}_{ij}$ is the effective nucleon-nucleon interaction, which includes central, spin-orbit and Coulomb terms. In this study, other non-central forces (e.g. tensor force) are not included explicitly. We use the Volkov No.2 interaction [16] for the central part of $\hat{v}_{ij}$,

$$\begin{aligned} V(r) \ &= \ (W - MP^{\sigma}P^{\tau} + BP^{\sigma} - HP^{\tau}) \\ &\times [V_1 e^{-\rho_1 r^2} + V_2 e^{-\rho_2 r^2}] \ , \end{aligned} \tag{2}$$

where the exchange parameters are taken as $W = 1 - M$, $M = 0.61$, and $B = H = 0.125$. For the spin-orbit term, we use the G3RS interaction [17],

$$V_{ls}(r) = V_0^{ls}[e^{-d_1 r^2} + e^{-d_2 r^2}]P(^3O)\, \boldsymbol{L} \cdot \boldsymbol{S} \ , \tag{3}$$

where $d_1 = 5.0$ fm^{-2}, $d_2 = 2.778$ fm^{-2}, $V_0 = 2000$ MeV, and $P(^3O)$ is a projection operator onto a triplet odd state. All of the parameters of the interaction are determined from $\alpha + n$ and $\alpha + \alpha$ scattering phase shifts and the binding energy of the deuteron [18]. However, M is slightly modified from the original value (0.6) to 0.61 for the analysis of the carbon isotopes, and furthermore, the bound state for the $n + n$ system is eliminated by introducing B and H parameters in Eq. (2).

In actual calculations, we introduce SMSO for the ^{12}C core in order to describe the cluster-shell competition by a simple parameter. The total wave function is fully antisymmetrized and the spatial part of the i-th nucleon is described by a Gaussian wave packet centered at $\vec{z}_i/\sqrt{\nu}$. The ^{12}C core is described as three α clusters with an equilateral triangle shape using the length parameter R_1 on the x-z plane. Hence, the parameter R_1 stands for the distance between the Gaussian center parameters of the α-clusters. The limit of $R_1 \to 0$ corresponds to the wave function of the lowest state of the (SU(3)) shell-model. It has been shown in Ref. [19] that the ground state of ^{12}C given by a microscopic three α model has large overlap, e.g., more than 50%, with the lowest shell-model configuration.

Furthermore, in order to describe the dissolution of the α particle, we transform one of the α clusters into four independent (jj-coupling-like) particles by giving the imaginary parts for the Gaussian center parameters. In addition to the real part ($\vec{R}$), we introduce a parameter Λ for the imaginary part as follows:

$$\vec{z}_i/\sqrt{\nu} = \vec{R} + i\Lambda(\vec{e}^{\,i}_{spin} \times \vec{R}) \ , \tag{4}$$

where $\vec{e}^{\,i}_{spin}$ is the unit vector along the spin-direction. The $\Lambda = 0$ case corresponds to the $(0s)^4$ configuration for the each α particle. In the extreme case of $\Lambda = 1$, on the other hand, four nucleons in one of the α-cluster have the wave functions of the $p_{3/2}$-orbit, and hence the spin-orbit interaction acts attractively [14]. Although only one of the α-clusters is dissolved, this condition has been known to be almost sufficient to take into account the spin-orbit contribution in ^{12}C [9].

In order to solve the motion of the valence neutrons around the ^{12}C core, we combine SMSO and AMD triple-S [10] for ^{14}C (^{12}C+2n) and ^{16}C (^{12}C+4n). We superpose the AMD wave functions. In each AMD wave function, the real parts of the Gaussian center parameters for the valence neutrons outside the ^{12}C core are randomly generated. The cooling equation is applied only for their imaginary parts in order to take into account the spin-orbit contribution, efficiently. In the cooling procedure, only project the parity of the system. After performing the angular momentum projection onto 0^+, important basis state are selected in the same way as the stochastic variational method (SVM) [12, 13]. Although the same set of the selected basis states is used for other J^{π} states, the coefficients for the linear combination of the basis states are obtained by diagonalizing the Hamiltonian for each J^{π} state independently.

III. RESULTS

A. Energies of the carbon isotopes

First, we show the calculated energies of the carbon isotopes. For ^{14}C and ^{16}C, there are two and four neutrons outside the ^{12}C core, respectively. The real part of the Gaussian center parameters of these neutrons are randomly generated under the weight function of $w(r) = \exp(-r/R_2)$. We choose the parameter $R_2 = 1.5$ fm in order to achieve the fast convergence of the energy. Here, due to the difference of the number of neutrons, the energy convergence of ^{16}C is slower than that of ^{14}C as shown in Fig. 1. Typical number of basis states introduced for ^{16}C is 250.

In the present framework, the spin orientation of each nucleon is quantized along the z-axis, and the ^{12}C core is placed on the x-z plane based on SMSO. For the closed-shell nucleus, *i.e.* ^{14}C ($N = 8$), the result is insensitive to the spin axis with respect to the ^{12}C core. However in ^{16}C, which is an open-shell nucleus of the *sd*-shell, we introduce the basis states with $S_z = 1$ in addition to $S_z = 0$ so that the valence neutrons can have the optimal

spin direction:

$$\{|\Phi\rangle\} = \{|\Phi(S_z = 0)\rangle\} \oplus \{|\Phi(S_z = 1)\rangle\} \ . \qquad (5)$$

In practice, a basis state with $S_z = 1$ is generated once in every four times, and states with $S_z = 0$ are generated in other three times. The efficiency of inclusion of $S_z = 1$ in ^{16}C is shown in the inserted panel on the right-upper position of Fig. 1, where "Normal" and "$S_z = 1$ mixed" show the calculations using $S_z = 0$ only and $S_z = 1$ mixed basis set, respectively. Superposing the basis states with different spin-orientation is found to increase the speed of the energy convergence.

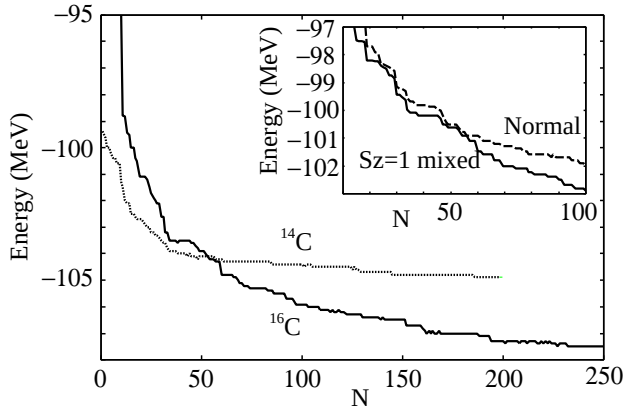

FIG. 1: Energy convergence of ^{14}C and ^{16}C calculated with SMSO and AMD triple-S. The horizontal axis is the number of introduced basis states (N), where distribution of the valence neutrons is randomly generated. In the right-upper panel, the "Normal" and "$S_z = 1$ mixed" show the calculations with $S_z = 0$ only and $S_z = 1$ mixed basis set, respectively.

The energy curves of ^{12}C, ^{14}C, and ^{16}C as functions of R_1 and Λ are shown in Fig. 2. The optimum values of R_1 and Λ are different each other in these nuclei. In ^{12}C (upper panel), the energy of $R_1 = 2.5$ fm gives the lowest than that energy at $\Lambda = 0$, where the spin-orbit interaction have no effect on the system. However, as Λ increases, the energy curve of $R_1 = 1.5$ fm, which lies higher position than that of $R_1 = 2.5$ fm at $\Lambda = 0$, goes down, and the minimum point appears at around $\Lambda = 0.4$. This shows that the contribution of the spin-orbit interaction makes the R_1 value smaller, and both the cluster and shell-model components are mixed in the ground state. In ^{14}C (middle panel) and ^{16}C (lower panel), a smaller R_1 value gives the lowest energy ($R_1 = 0.5$ fm). Due to the presence of valence neutrons, the nucleus becomes a strongly bound system, and the wave function approaches to the shell-model picture. Thus the optimal Λ value increases to 0.8 in these nuclei. The optimum values of R_1 and Λ for $^{12-16}$C and the energies are summarized in Table I.

When the valence neutrons are added to ^{12}C, the optimum value of R_1 becomes smaller, and simultaneously the optimum Λ becomes larger. Therefore, it can be concluded that the 3α-structure of ^{12}C is dissolved as the increase of the number of the valence neutrons. In ^{16}C, the ^{12}C core has completely different structure from the "free" ^{12}C by considering from the small R_1 and large Λ values.

TABLE I: The optimum values of the parameters R_1 and Λ for the carbon isotopes.

	^{12}C	^{14}C	^{16}C
R_1 (fm)	1.5	0.5	0.5
Λ	0.4	0.8	0.8
E (MeV)	−88.6	−106.4	−108.5

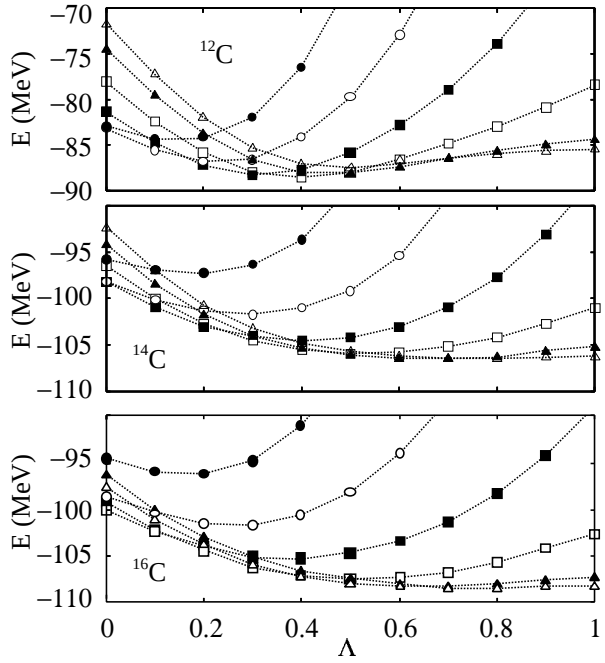

FIG. 2: Calculated energies of ^{12}C (upper panel), ^{14}C (middle panel), and ^{16}C (lower panel) as functions of Λ. Solid and open circles, solid and open squares, solid and open triangles correspond to $R_1 = 3.0$, 2.5, 2.0, 1.5, 1.0 and 0.5 fm, respectively.

Next, we calculate the energy levels of ^{16}C using the wave function with the optimum values of $R_1 = 0.5$ (fm) and $\Lambda = 0.8$. Results are shown in Fig. 3. The energy levels of the 0_1^+, 0_2^+, 2_1^+ and 2_2^+ states are in agreement with the experimental data. The 4_1^+ state lies lower position than the experiment, and the 3_1^+ state is, on the contrary, higher. This tendency of the appearance at low (high) in energy of the 4_1^+ (3_1^+) state is similar to the AMD calculation with variation before (after) the J^π projection as VBP (VAP) [20]. This is because our framework has a sort of mixed procedure of VBP and VAP. The cooling equation is performed before the angular momentum projection, which corresponds to VBP, and the basis set is selected after the angular momentum projection onto $J^\pi = 0^+$, which is VAP. However, the set of the basis states are not optimized for J^π states which has different

character from that of the 0^+ state. Nevertheless, in the recent approach [21], the energy levels are well described by the core+n+n model space.

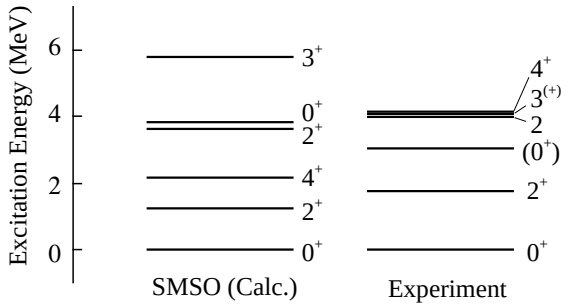

FIG. 3: Energy levels of ^{16}C.

Although the shell-model-like component is dominant in the ground state, cluster state may appear in the excited states. In order to search for such state in ^{16}C, we mix the wave functions with different R_1 and Λ values. In Fig. 4, "Base 1" stands for the wave functions with the optimum values ($R_1 = 0.5$ fm and $\Lambda = 0.8$), and the ^{12}C core is close to the shell-model-like structure. In addition to these basis states, a different base set of "Base 2" is introduced, which is cluster-like ($R_1 = 2.5$ fm and $\Lambda = 0.0$). We found that the energy of the 0^+_3 state goes down significantly, whereas the energies of the ground and second 0^+ states do not change drastically. Therefore, it is considered that 3α cluster structure with a geometric shape appears in $E_x = 10$ MeV region of ^{16}C, in analogy with the crystallization of the clusters in ^{14}C [22].

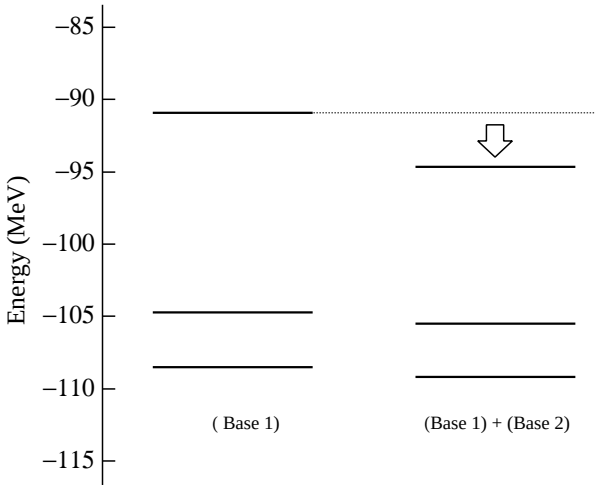

FIG. 4: The 0^+ energy levels of ^{16}C. Here "Base 1" shows the basis states with $R_1 = 0.5$ (fm) and $\Lambda = 0.8$, and "Base 2" shows $R_1 = 2.5$ (fm) and $\Lambda = 0.0$.

B. $B(E2)$ values of the carbon isotopes

In ^{16}C, anomalously small $B(E2)$ value is reported from an experiment [23]. The observed $\mathrm{B}(E2, 2^+_1 \to 0^+_1)$ value is 0.63 e^2fm^4, which is almost 13 times smaller than that of ^{12}C, although both nuclei have the same proton number. Here, we calculate $B(E2; 2^+_1 \to 0^+_1)$ with the optimum values of R_1 and Λ. The obtained $B(E2)$ value is very small, 0.01 e^2fm^4, which is even smaller than the experimental value as shown in Table II. We also calculate the "neutron $B(E2)$" value ($B(E2)_n$ in Table II) by inverting the charges of neutrons and protons as $e_p = 0$ and $e_n = e$. The obtained value is relatively large; $B(E2; 2^+_1 \to 0^+_1)_n = 4.11$ e^2fm^4. Therefore, the neutron part of the ^{16}C is deformed, and the proton part is considered to be spherical. The experimental $B(E2)$ value can be explained by introducing effective charge ($\sim 0.4e$) for the neutron part, however the value is slightly larger compared to the recent core+n+n model calculation [21], which consistently explains the $B(E2)$ values of ^{15}C and ^{16}C. The calculated B(E2) values of ^{12}C and ^{14}C are also listed in this Table. The calculated proton and neutron $B(E2)$ values of ^{14}C are 2.99 and 0.03 (e^2fm^4), respectively, and contrary to ^{16}C, the normal $B(E2)$ value is much larger than that for the neutrons due to the shell effect. In ^{14}C, the small neutron $B(E2)$ value shows that the neutrons have the closed-shell configuration of the p-shell, and the 2^+-excitation comes from the proton part.

TABLE II: Calculated $B(E2; 2^+_1 \to 0^+_1)$ values of the carbon isotopes. The $B(E2)_n$ values for ^{14}C and ^{16}C are calculated by inverting the charge as $e_p = 0$ and $e_n = e$. Units of R_1 and $B(E2)$ are fm and e^2fm^4, respectively. Experimental data are taken from Ref. [24] (^{12}C and ^{14}C) and Ref. [23] (^{16}C).

	(R_1, Λ)	$B(E2)$	$B(E2)_n$
^{12}C	(1.5, 0.4)	3.51	3.51
	Exp.	8.2 ± 0.1	
^{14}C	(0.5, 0.8)	2.99	0.03
	Exp.	3.74 ± 0.5	
^{16}C	(0.5, 0.8)	0.01	4.11
	Exp.	0.63	

The mechanism for the small $B(E2)$ value of ^{16}C is considered in the following way: (1) Due to the small relative α-α distance corresponding to the parameter R_1, the deformation of the proton part becomes small. (2) Since Λ is large, and hence an α cluster is dissolved into four independent particles, the 2^+_1 state is no longer a collective (rotational) state. (3) A strong proton-neutron correlation makes the protons deeply-bound in neutron-rich nuclei, thus the 2^+-excitation is mostly caused by the neutrons outside the ^{12}C core. The effects of (1) and (2) can be confirmed by comparing the $B(E2)$ value of ^{12}C with that of ^{14}C. In ^{12}C and ^{14}C, the optimal R_1 and Λ values change from (1.5 fm, 0.4) to (0.5 fm, 0.8), and the $B(E2)$ value decreases from 3.51 to 2.99 (e^2fm^4). However, the value for ^{14}C is still much larger than that

of ^{16}C. Therefore, the remaining effect must come from the origin (3). The relatively large neutron $B(E2)$ value of ^{16}C supports this picture, which is also suggested in Ref. [21].

IV. SUMMARY AND DISCUSSION

We have studied carbon isotopes with the combined frameworks of AMD triple-S and SMSO. As the neutron number increases, the 3α-cluster structure of the ^{12}C core disappears as shown in terms of the decrease of R_1 and increase of Λ. We have also studied the $B(E2; 2_1^+ \to 0_1^+)$ values of the carbon isotopes and analyzed the mechanism to make the $B(E2)$ value of ^{16}C very small in terms of the cluster-shell competition. By comparing with the values of ^{12}C with various R_1 and Λ parameters, it has been shown that the shrinkage effect of the 3α-structure and dissolution of an α make the $B(E2)$ value small. The ^{12}C core of ^{16}C is more shell-model-like structure judging from the small R_1 (0.5 fm) and large Λ (0.8) values. However, in ^{16}C, another origin, the item (3) in the last paragraph of the previous section, plays an important role that the excitation of the valence neutrons is the main source of the 2^+-excitation. Since the protons are strongly bound in neutron-rich nuclei, the $1p$-$1h$ excitation of the protons needs higher energy compared with the excitation of the neutrons.

Although the shell-model-like component is dominant in the ground state of ^{16}C, a cluster state appears in the excited states. The energy of the 0_3^+ state significantly decreases if we add pure cluster states with $R_1 = 2.5$ (fm) and $\Lambda = 0$, whereas the energies of the ground and second 0^+ states do not change drastically. Therefore, it can be considered that the 3α-cluster structure with an equilateral triangle shape appears in $E_x = 10$ MeV region of ^{16}C, in analogy with the crystallization of the clusters in ^{14}C [22].

As a future work, we proceed to study heavier carbon isotopes and also the other isotopes with the present framework of SMSO plus AMD triple-S for the consistent understanding of the cluster-shell competition.

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