Overlap Algorithm for the Nonstandard FDTD Method Using Non-uniform Mesh

Tadao Ohtani, Kenji Taguchi, Tatsuya Kashiwa, *Member*, *IEEE*, and Yasushi Kanai, *Member*, *IEEE*

***Abstract*— In this paper, the overlap algorithm is applied to the nonstandard FDTD (NS-FDTD) method using a non-uniform mesh in two-dimensional space. The characteristics of the overlap algorithm are numerically examined and compared with a no overlap algorithm. The results show that the reflection rate from the interface of the two meshes using the overlap algorithm is less than –80 dB, which is smaller than that of the NS-FDTD method without the overlap. Consequently, it is shown that the overlap algorithm is more suitable for the NS-FDTD method when using a non-uniform mesh. The overlap algorithm is successfully applied to the analysis of a dielectric flat panel and a corrugated surface.**

***Index Terms*— FDTD method, Non-uniform mesh, NS-FDTD method, numerical analysis, scattering.**

I. INTRODUCTION

THE nonstandard FDTD (NS-FDTD) method is a highly accurate method for a fixed frequency analysis that reduces the phase error of propagating waves in the FDTD method [1]–[4]. The NS-FDTD method generally uses a uniform mesh. Therefore, when the structure is locally complex, smaller space increments must be used for the whole region in order to treat the localized fine structure. But, the use of a smaller mesh size causes an increase in the calculation time and memory, that is, in the cost in the NS-FDTD simulation. One solution to the problem is to use a non-uniform mesh [5], [6]. However, non-uniform mesh leads to un-symmetrical arrangement of electromagnetic nodes, and numerical reflection arises on the boundary of different cell sizes. This means that when ordinal non-uniform connection is applied the symmetry will be lost and the accuracy will be lower. It is especially serious in the NS-FDTD method because the symmetrical electromagnetic node positions are indispensable to derive high accuracy in this method. Therefore, if an overlap algorithm using symmetrical nodes is applied, a highly accurate connection of two meshes with different cell sizes will be possible, and the NS-FDTD analysis with high accuracy using non-uniform mesh may be achieved. However, the overlap algorithm for non-uniform meshes has not been studied in relation to the NS-FDTD method. Therefore, it is important to verify the accuracy of the overlap algorithm for the NS-FDTD method using non-uniform meshes.

In this paper, the overlap algorithm which is used in the subgrid technique [4], [7] is applied to the NS-FDTD method using a non-uniform mesh in two-dimensional space. The characteristics of the overlap algorithm are examined and compared with that of a version with no overlap. The reflection rate from the boundary of two meshes is also compared with the FDTD method. From these evaluations, it is found that the reflection rate of the overlap algorithm is less than -80 dB, while that of the NS-FDTD method without the overlap is less than -30 dB. For FDTD, the reflection rate of the overlap algorithm is less than -40 dB, while that without the overlap is less than -30 dB. Consequently, it is shown that the overlap algorithm is a more accurate calculation technique for non-uniform meshes than the no overlap one. Furthermore, it is shown that NS-FDTD method using the overlap algorithm has better reflection characteristics than the FDTD method. The algorithm is applied to the analysis of a dielectric flat panel and a corrugated surface to confirm the suitability for practical problems.

II. CALCULATION ALGORITHM OF NON-UNIFORM MESH

In this section, two calculation algorithms for the NS-FDTD method using a non-uniform mesh are described in the two-dimensional space.

A. No overlap Algorithm for Non-uniform Mesh

Fig. 1 shows the positions of the field components for the

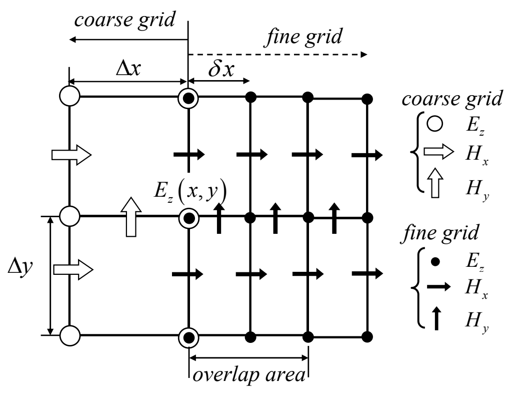

Fig. 1. Positions of the field components for no overlap algorithm.

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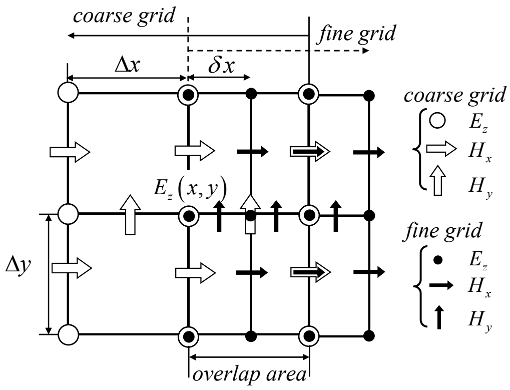


Fig. 2. Positions of the field components for the overlap algorithm.

no overlap algorithm with a grid size ratio $\Delta x/\delta x = 2$. In the no overlap algorithm, the E_z field components at the mesh interface are calculated using the following difference equations;

$$
\begin{aligned}
&E_z^{n+1}(x,y) = E_z^n(x,y) \\
&+\frac{S_\omega(\Delta t)}{\varepsilon}\Big[\big\{H_y^{n+1/2}(x+\delta x/2,y) - H_y^{n+1/2}(x-\Delta x/2,y)\big\}\big/S_k(\Delta x') \\
&\qquad -\big\{H_x^{n+1/2}(x,y+\Delta y/2) - H_x^{n+1/2}(x,y-\Delta y/2)\big\}\big/S_k(\Delta y)\Big],
\end{aligned} \tag{1}
$$

where,

$$S_{\omega_c}(\Delta t) = 2\sin(\omega_c \Delta t/2)/\omega_c, \tag{2}$$

$$S_{k_c}(\Delta\xi) = 2\sin(k_c \Delta\xi/2)/k_c \quad (\Delta\xi = x, y). \tag{3}$$

Here, ε is the permittivity, n is the time step and Δt the time increment, Δx and Δy the spatial increments in the x and y directions, respectively, δx is the spatial increment in the fine grid region in the x direction. ω_c and k_c are the angular frequency and the wave number at the given frequency. α_{0x} and α_{0y} are the optimizing parameters of the NS-FDTD method for Δx and Δy at the given frequency, respectively [1]. In (1), $\Delta x' = (\Delta x + \delta x)/2$ is used. In this manner, the E_z component can be calculated using the magnetic field values that are adjacent to the mesh interface. The H_x component on the interface can be also calculated in a similar way. Thus, in the no overlap algorithm the central finite difference is not maintained at the mesh interface because of the difference in the two spatial increments Δx and δx .

B. Overlap Algorithm for Non-uniform Mesh

The overlap algorithm is used in the subgrid technique for a non-uniform mesh [4], [7]. Fig. 2 shows the positions of the field components for the overlap algorithm with a grid size ratio $\Delta x/\delta x = 2$. In the overlap algorithm, the different grid size meshes are overlapped at the interface. Consequently, the E_z field components on the mesh interface are calculated using the following difference equations;

$$
\begin{aligned}
&E_z^{n+1}(x,y) = E_z^n(x,y) \\
&+\frac{S_\omega(\Delta t)}{\varepsilon}\Big[\big\{H_y^{n+1/2}(x+\Delta x/2,y) - H_y^{n+1/2}(x-\Delta x/2,y)\big\}\big/S_k(\Delta x) \\
&\qquad -\big\{H_x^{n+1/2}(x,y+\Delta y/2) - H_x^{n+1/2}(x,y-\Delta y/2)\big\}\big/S_k(\Delta y)\Big],
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
&E_z^{n+1}(x+\Delta x,y) = E_z^n(x+\Delta x,y) \\
&+\frac{S_\omega(\Delta t)}{\varepsilon}\Big[\big\{H_y^{n+1/2}(x+\Delta x+\delta x/2,y) \\
&\qquad -H_y^{n+1/2}(x+\Delta x-\delta x/2,y)\big\}\big/S_k(\delta x) \\
&\qquad -\big\{H_x^{n+1/2}(x+\Delta x,y+\Delta y/2) \\
&\qquad -H_x^{n+1/2}(x+\Delta x,y-\Delta y/2)\big\}\big/S_k(\Delta y)\Big],
\end{aligned} \tag{5}
$$

Thus, in the overlap algorithm, the central finite difference is maintained at all nodes on the mesh interface.

III. Numerical Evaluation

In this section, the reflection characteristics of both the overlap and the no overlap algorithms described in Section II are evaluated numerically.

A. Reflection Characteristics of Non-uniform Mesh

Figs. 3 and 4 show the reflection rate from the interface between the two meshes for the FDTD and NS-FDTD methods, respectively. Grid size ratios in the range from 2 to 5 were evaluated. Here, the Sine-Cosine method [5] is used for evaluation to realize periodic conditions for angles less than 60 degrees, while the reflected wave from the mesh interface was directly observed by using a point source for angles greater than 60 degrees. The values of the optimization parameters α_{0x} and α_{0y} are set to be 5/6 [1]. For this example, the spatial increments were set to $\Delta x = \Delta y = \lambda/10$, where λ is the wave length, Δt is set to be $\delta x/2c_0$ with c_0 the speed of light. In Fig. 3, the reflection rate of the FDTD method using the overlap algorithm is less than -40 dB, while that of the FDTD method using the no overlap algorithm is less than -30 dB. On the other hand, as shown in Fig. 4, the reflection rate

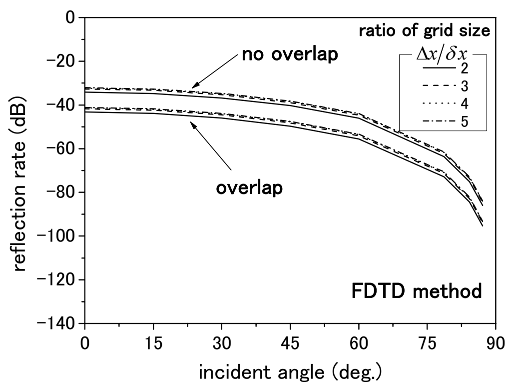


Fig. 3. Reflection rates from the mesh interface in the FDTD methods with overlap and no overlap.

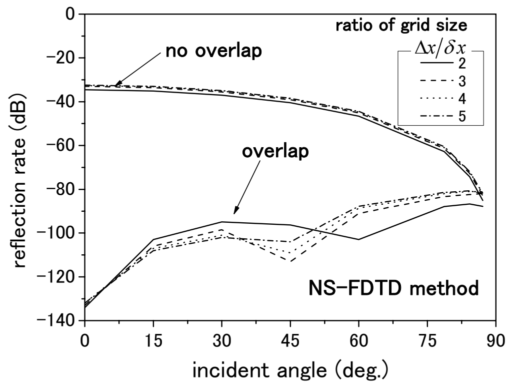


Fig. 4. Reflection rates from the mesh interface in the NS-FDTD method with overlap and no overlap.

of the NS-FDTD method using the overlap algorithm is less than -80 dB and that of the no overlap version is less than -30 dB. From these results, it is seen that the overlap algorithm increases calculation accuracy for a non-uniform mesh in the NS-FDTD method compared to using the no overlap algorithm.

The reflection at the mesh interface is caused by both the difference in numerical phase velocity in two meshes, and the loss of the central finite difference. The overlap algorithm maintains the central finite difference, unlike the no overlap algorithm. The numerical phase velocity is nearly equal to the physical one irrespective of grid size in the NS-FDTD method. Conversely, the numerical phase velocity varies with grid size in the FDTD method. Consequently, the reflection from the interface of the two meshes is very small in the NS-FDTD method using the overlap algorithm.

B. Application to the Analysis of Dielectric Flat Panel

The dielectric flat panel is used as a radome. The NS-FDTD method using both the overlap algorithm and the no overlap algorithm was applied to the analysis of the reflection rate from the dielectric flat panel. Fig. 5 shows the two dimensional model of the dielectric flat panel which is composed of three dielectric layers. The relative dielectric constants are $\varepsilon_s = 2$ for the surface layers and $\varepsilon_c = 1.2$ for the

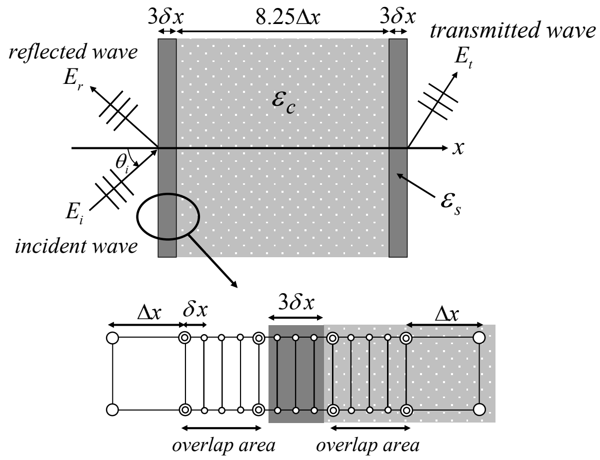


Fig. 5. Two dimensional model of the dielectric flat panel.

core. Here, the grid size ratio is $\Delta x/\delta x = 4$. The spatial and time increments are $\Delta x = \Delta y = \lambda/11.5$ and $\Delta t = \delta x/2c_0$, respectively. Fig. 6 shows the reflection rate from the dielectric flat panel. For comparison, the results of the FDTD

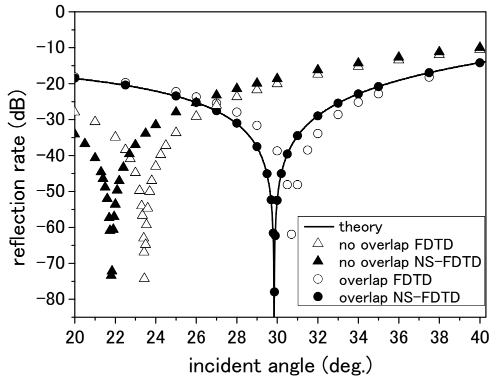
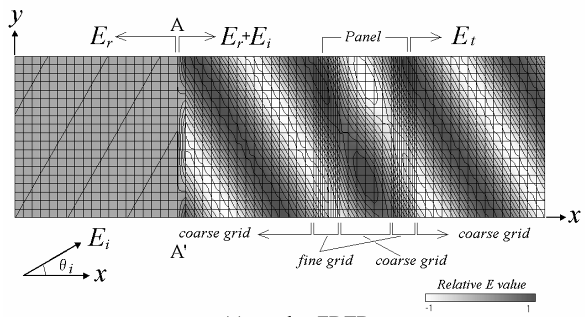
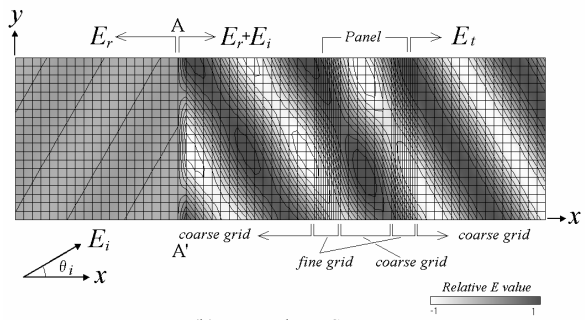


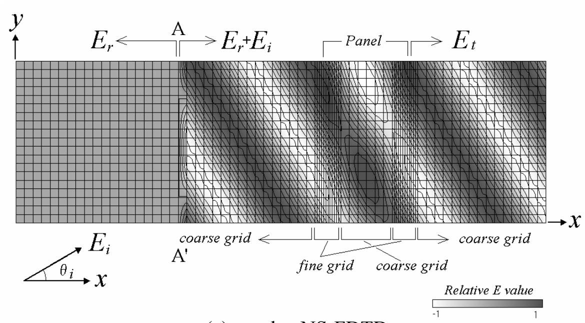
Fig. 6. Comparison of reflection rates from the dielectric flat panel obtained by various numerical methods. Theoretical values are also shown.



(a) overlap FDTD



(b) no overlap NS-FDTD



(c) overlap NS-FDTD

Fig. 7. The snapshots of the contour plot of E fields.. $\theta_i = 29.86$ degrees.

method are also presented. For the evaluation of the reflection rate at oblique incidence, the Sine-Cosine method [5] is used. Theoretical values were obtained using the transfer matrix method [8]. The result of the NS-FDTD method using the overlap algorithm agrees well with the theoretical value. Fig. 7 shows the snapshots of the contour plot of normalized E fields in the plane shown in Fig. 5. In the figure, E_i, E_r, and E_t denote the incident, reflected, and transmitted waves, respectively. The gray scale shows the normalized E value and the contour shows the equi-electrical field line. Note that incident angle θ_i is 29.86 degrees. On the boundary A-A', total field/scattered field technique [5] is applied, therefore, theoretically only reflected wave exists on left hand side of A-A', and incident and scattered waves exist on right hand side of A-A'. Reflected wave is observed in overlap FDTD and no overlap NS-FDTD solutions on left hand side of A-A', which is against the theory. While in overlap NS-FDTD solution, reflected wave can be negligible.

Consequently, it is shown that the NS-FDTD method using the overlap algorithm is a more accurate and effective analysis technique than the FDTD method in practical use.

C. Reflection Analysis of Corrugated Surface

Corrugated surfaces are used for the suppression of scattered waves. The effectiveness of the overlap algorithm for the NS-FDTD method is confirmed by using a reflection analysis of a corrugated surface. Fig. 8 shows the cross-sectional view of the corrugated surface. The corrugated surface is made of a perfect electric conductor. Periodic length Lp to y axis is λ. Here, the grid size ratio is $\Delta x/\delta x = 20$. The spatial and time increments are $\Delta x = \Delta y = \lambda/10$ and $\Delta t = \delta x/2c_0$, respectively. The fin width of the corrugated surface is $3\Delta y$. The incident angle θ_i of the incident plane wave is 30 degrees. The Sine-Cosine method [5] is used for the oblique incidence analysis.

Fig. 9 shows the simulation results of the reflection rate from the corrugated surface presented in Fig. 8. The reference value is the result of the NS-FDTD method using a uniform mesh of δx and Δy in all space. As shown in Fig. 9, the results of the NS-FDTD method using the overlap algorithm agree with the reference value well, even with a large grid size ratio of 20. Consequently, it is also confirmed that the overlap algorithm is a more effective technique than the no overlap algorithm for the NS-FDTD method with large grid size ratios.

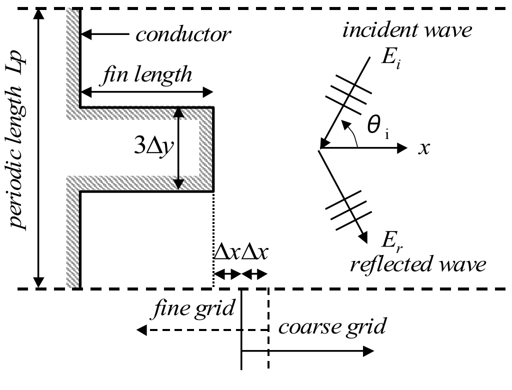


Fig. 8. Cross-sectional view of the corrugated surface.

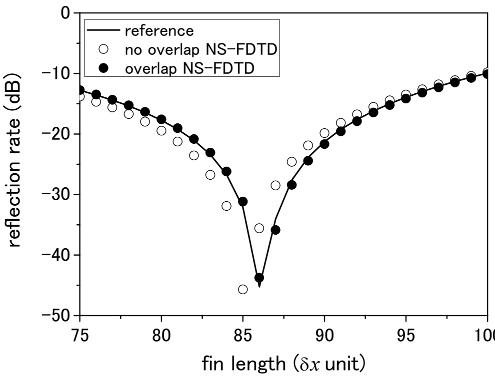


Fig. 9. Comparison of reflection rates from the corrugated surface obtained by NS-FDTD methods with overlap and no overlap. Reference values are also shown.

IV. Conclusion

The overlap algorithm was applied to the non-uniform NS-FDTD method in two-dimensional space. The characteristics of the overlap algorithm were numerically examined and compared with the no overlap algorithm. The results showed that the reflection rate of the overlap algorithm in the NS-FDTD method was less than -80 dB. The reflection rate was smaller than that of the FDTD method and the NS-FDTD method using the no overlap algorithm. Furthermore, the algorithm was applied to the analyses of a dielectric flat panel and the corrugated surface. As a result, it was shown that the NS-FDTD method is a highly accurate and effective analysis tool in not only with a uniform mesh but also with a non-uniform mesh.

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