Characteristics of Evanescent Waves in the Nonstandard FDTD Method

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***Abstract* — This paper investigates the characteristics of evanescent waves in the generalized nonstandard finite difference time domain (NS-FDTD) method and derives the dispersion equation and stability condition for two-dimensional space. The validity of the dispersion equation is confirmed by comparison with results of numerical simulations. The NS-FDTD method is applied to the analysis of an optical switching device using a photon tunneling effect. It is shown that the NS-FDTD method has highly accurate characteristics not only for propagating waves, but also for evanescent waves compared with the standard FDTD method.**

***Index Terms* — evanescent wave, FDTD methods, NS-FDTD methods, numerical dispersion, photon tunneling effect, stability condition.**

I. INTRODUCTION

THE nonstandard finite difference time domain (NS-FDTD) method [1]-[4] is a highly accurate method for analysis at a fixed frequency that reduces the phase error of propagating waves in the FDTD method. In applying the method to real problems, it is important to evaluate dispersion error. In addition, evanescent waves as well as propagating waves play important roles in microwave and optical elements.

In the past, the characteristics of numerical dispersion for the propagating waves in the NS-FDTD method have been investigated [3]. However, the characteristics of evanescent waves in the method have not been shown. Therefore, it is necessary to investigate the behavior of evanescent waves in the NS-FDTD analysis.

In this paper, the dispersion equation and stability condition for evanescent waves in the generalized NS-FDTD method are derived for two dimensions. Furthermore, the validity of the derived dispersion equation is confirmed by comparison with results of numerical simulations. The evanescent characteristics of the method are then compared with those of the FDTD method. The NS-FDTD method has been applied to the analysis of an optical switching device based on a resonant photon tunneling effect [5], [6]. It is shown that the NS-FDTD method has highly accurate characteristics not only for propagating waves but also for evanescent waves when compared to the standard FDTD method.

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II. DISPERSION EQUATION AND STABILITY CONDITION FOR EVANESCENT WAVES IN THE NS-FDTD METHOD

For an evanescent wave propagating in the *x* direction and decaying in the *y* direction, the analytical evanescent wave can be expressed as follows:

$$\phi(x,y,t)=\phi_0 e^{j\omega t - jk_x x - k_y y}, \tag{1}$$

$$k^2 = k_x^2 - k_y^2, \tag{2}$$

where ϕ_0 is the amplitude, ω is the angular frequency, t is the time. k_x and k_y are the wave number in the x and y directions, and $k=\omega/c$ with c the speed of light in the medium. As can be easily verified, (2) is satisfied if the wave numbers are expressed as:

$$k_x = k\cosh\chi, \tag{3}$$

$$k_y = k\sinh\chi, \tag{4}$$

where χ is a parameter associated with the propagation angle of the evanescent wave [7]. By using (3) and (4), we can express the dispersion equation and stability condition of the NS-FDTD method for evanescent waves in the following manner.

A. Numerical Dispersion Equation

The difference equations of the generalized NS-FDTD method in the two-dimensional space are expressed for the TM and the TE modes as follows [4].

For the TM mode;

$$\begin{aligned}&\tilde{\mathbf{d}}_{\mathbf{t}} E_z(x,y,t+\Delta t/2)\\&=\frac{S_{\omega_c}(\Delta t)}{\varepsilon(\mathbf{r_p})}\left[\frac{\tilde{\mathbf{d}}_x^{(1)}}{S_{k_c}(\Delta x)}H_y(x,y,t+\Delta t/2)\right.\\&\qquad\left.-\frac{\tilde{\mathbf{d}}_y^{(1)}}{S_{k_c}(\Delta y)}H_x(x,y,t+\Delta t/2)\right],\end{aligned} \tag{5}$$

$$\tilde{\mathbf{d}}_{\mathbf{t}} H_x\left(x, y-\Delta y/2, t\right) = -\frac{S_{\omega_c}(\Delta t)}{\mu(\mathbf{r_p})}\left[\frac{\tilde{\mathbf{d}}_y^{(0)}}{S_{k_c}(\Delta y)} E_z\left(x, y-\Delta y/2, t\right)\right], \quad (6)$$

$$\tilde{\mathbf{d}}_{\mathbf{t}} H_y\left(x-\Delta x/2, y, t\right) = -\frac{S_{\omega_c}(\Delta t)}{\mu(\mathbf{r_p})}\left[-\frac{\tilde{\mathbf{d}}_x^{(0)}}{S_{k_c}(\Delta x)} E_z\left(x-\Delta x/2, y, t\right)\right]. \quad (7)$$

For the TE mode;

$$\tilde{\mathbf{d}}_{\mathbf{t}} E_x\left(x-\Delta x/2, y, t+\Delta t/2\right) = \frac{S_{\omega_c}(\Delta t)}{\varepsilon(\mathbf{r_p})}\left[\frac{\tilde{\mathbf{d}}_y^{(1)}}{S_{k_c}(\Delta y)} H_z\left(x-\Delta x/2, y, t+\Delta t/2\right)\right], \quad (8)$$

$$\tilde{\mathbf{d}}_{\mathbf{t}} E_y\left(x, y-\Delta y/2, t+\Delta t/2\right) = \frac{S_{\omega_c}(\Delta t)}{\varepsilon(\mathbf{r_p})}\left[-\frac{\tilde{\mathbf{d}}_x^{(1)}}{S_{k_c}(\Delta x)} H_z\left(x, y-\Delta y/2, t+\Delta t/2\right)\right], \quad (9)$$

$$\tilde{\mathbf{d}}_{\mathbf{t}} H_z\left(x-\Delta x/2, y-\Delta y/2, t\right) = \frac{S_{\omega_c}(\Delta t)}{\mu(\mathbf{r_p})}\left[\frac{\tilde{\mathbf{d}}_x^{(0)}}{S_{k_c}(\Delta x)} E_y\left(x-\Delta x/2, y-\Delta y/2, t\right) - \frac{\tilde{\mathbf{d}}_y^{(0)}}{S_{k_c}(\Delta y)} E_x\left(x-\Delta x/2, y-\Delta y/2, t\right)\right], \quad (10)$$

where,

$$S_{\omega_c}(\Delta t) = 2\sin\left(\omega_c \Delta t/2\right)/\omega_c , \quad (11)$$

$$S_{k_c}(\Delta\xi) = 2\sin\left(k_c \Delta\xi/2\right)/k_c . \quad (12)$$

Furthermore, $\tilde{\mathbf{d}}_{\mathbf{t}}$ and $\tilde{\mathbf{d}}_{\xi}^{(\mathbf{i})}\left(i=0,1,\ \xi=x,y\right)$ are the time and spatial difference operators [4]. $\varepsilon(\mathbf{r_p})$ and $\mu(\mathbf{r_p})$ are the permittivity and permeability as functions of the position $\mathbf{r_p}$. Δt is the time increment. Δx and Δy are the spatial increments in the x and y directions, respectively. ω_c and k_c are the angular frequency and the wave number at the given frequency.

Substituting (1) into (5)-(10), the dispersion equation of the NS-FDTD method is obtained [8] as follows:

$$\frac{1}{c^2}\left(\frac{\sin\left(\omega\Delta t/2\right)}{S_{\omega_c}(\Delta t)}\right)^2 - \left(\frac{\sin\left(k_{xnum}\Delta x/2\right)}{S_{k_c}(\Delta x)}\right)^2 \rho_x + \left(\frac{\sinh\left(k_{ynum}\Delta y/2\right)}{S_{k_c}(\Delta y)}\right)^2 \rho_y = 0, \quad (13)$$

where

$$\rho_x = \alpha_{0x} + \left(1-\alpha_{0x}\right)\cosh\left(k_{ynum}\Delta y\right), \quad (14)$$

$$\rho_y = \alpha_{0y} + \left(1-\alpha_{0y}\right)\cos\left(k_{xnum}\Delta x\right), \quad (15)$$

$$k_{xnum} = k_{num}\cosh\chi , \quad (16)$$

$$k_{ynum} = k_{num}\sinh\chi . \quad (17)$$

Here, k_{num} is the numerical wave number. α_{0x} and α_{0y} are optimizing parameters of the NS-FDTD method for Δx and Δy respectively at the given frequency [1], [4].

B. Stability Condition

Substituting the following waveform into the NS-FDTD difference equations (5)-(10)

$$\phi^n(x,y) = Z_0^n e^{-jk\cosh\chi x - k\sinh\chi y} , \quad (18)$$

where, Z_0^n is the amplitude, and n is the time step, we obtain the following stability condition

$$S_{\omega_c}(\Delta t) \le \frac{1}{c}\left[\left(\frac{\sin\left(k_{xnum}\Delta x/2\right)}{S_{k_c}(\Delta x)}\right)^2 \rho_x - \left(\frac{\sinh\left(k_{ynum}\Delta y/2\right)}{S_{k_c}(\Delta y)}\right)^2 \rho_y\right]^{1/2} . \quad (19)$$

If $S_{k_c}(\Delta x) = \Delta x$, $S_{k_c}(\Delta y) = \Delta y$ and $\alpha_{0x} = \alpha_{0y} = 1$ are used in (13) and (19), then these equations yield the dispersion and stability condition of the FDTD method for evanescent waves.

III. Numerical Results

First, the numerical wave number of the NS-FDTD method was studied. Next, the validity of the derived dispersion equation was confirmed by comparison with results of numerical simulations. Finally, the NS-FDTD method was applied to the analysis of an optical switching device using a photon tunneling effect.

A. Numerical Wave Number in the Evanescent Wave

Fig. 1 shows the numerical wave number k_{num} in free space calculated using (13) as a function of χ. For comparison, the results of the FDTD method are also presented. The time increment is $\Delta t = \Delta/2c_0\ \left(\Delta = \Delta x = \Delta y\right)$. In the figure, the vertical axis represents k_{num} normalized by $k_0 = \omega/c_0$ with c_0 the speed of light in the free space. Here, $\omega_c = \omega$ and $k_c = k_0$ were used in the correction functions (11) and (12). This shows that the NS-FDTD method has highly accurate characteristics for evanescent wave when compared with the FDTD method.

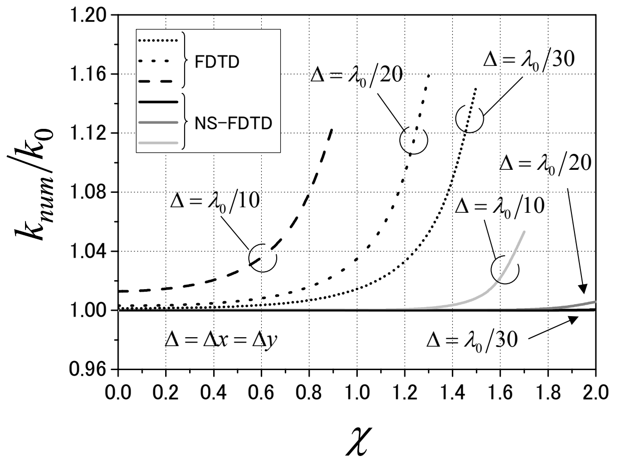

Fig. 1. Numerical wave number in free space as a function of χ obtained by NS-FDTD and FDTD methods.

B. Comparison with Simulated Results

In this part, the values of k and χ obtained from (13) are compared with those obtained from a numerical simulation to confirm the validity of (13).

Evanescent waves are generated when a plane wave is incident on the boundary between two media. As shown in Fig. 2, we assume that the boundary between the two media is parallel to x-axis and that the plane wave is incident on the boundary from the ε_{r2} area, and evanescent waves are generated in the ε_{r1} area.

In the two media problem, the numerical values χ_{num} and k_{num} using (13) can be obtained as follows;

$$\chi_{num} = \tanh^{-1}\left(k_{y\,num} / k_{x\,num}\right), \tag{20}$$

$$k_{num} = k_{x\,num} / \cosh \chi_{num}, \tag{21}$$

where

$$k_{x\,num} = k_0 \sqrt{\varepsilon_{r2}} \sin\theta_i . \tag{22}$$

(22) is obtained by the continuity of the field at the boundary. The value of $k_{y\,num}$ in (20) is obtained by substituting (22) into (13). On the other hand, the simulated values χ_{sim} and k_{sim} are obtained from the numerical simulation as follows;

$$\chi_{sim} = \tanh^{-1}\left(k_{y\,sim} / k_{x\,sim}\right), \tag{23}$$

$$k_{sim} = k_{x\,sim} / \cosh \chi_{sim}, \tag{24}$$

where $k_{x\,sim}$ and $k_{y\,sim}$ are the observed wave numbers in the x and y direction in the ε_{r1} area, respectively.

Figs. 3 and 4 show k/k_0 and χ/χ_0 characteristics for the evanescent waves. Here, $k_0 = \omega/c_0$ and χ_0 is the physical value of χ. In these figures, the angle θ_i represents incident angle of a plane wave to the boundary measured from the y-axis. For comparison, the results of the FDTD method are also presented. In order to ensure the same boundary conditions for both the NS-FDTD and the FDTD simulations, the electromagnetic fields in the ε_{r2} area are computed using the NS-FDTD method. The figure shows results for relative

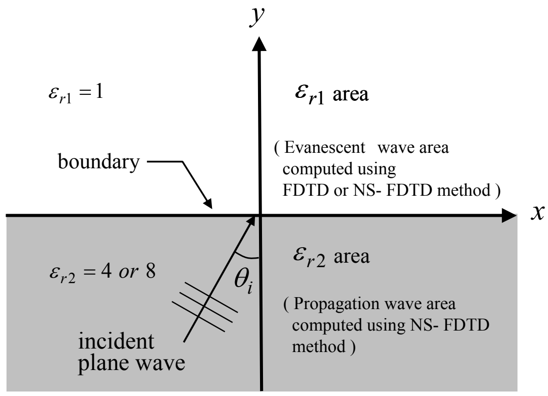

Fig. 2. Boundary between two media where generates evanescent waves.

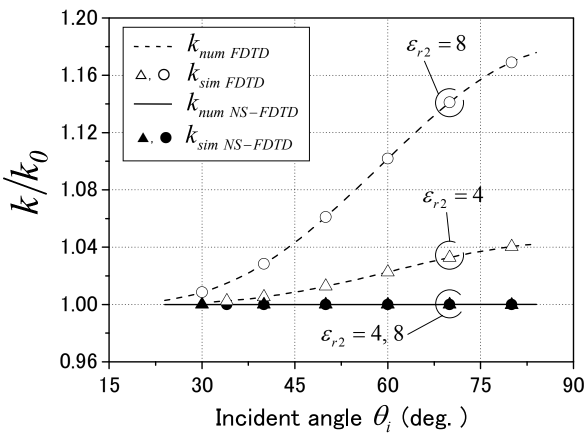

Fig. 3. k/k_0 versus incident angle θ_i obtained by various methods.

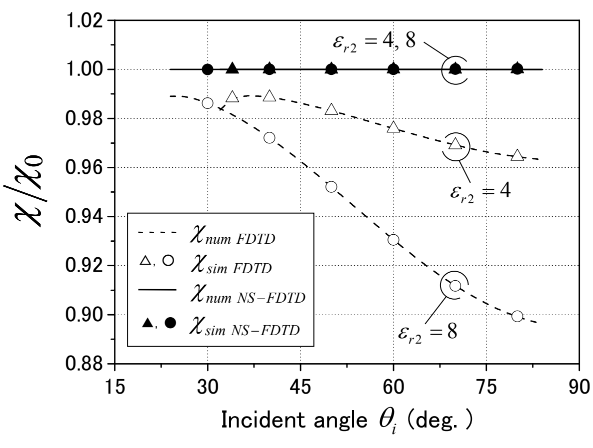

Fig. 4. χ/χ_0 versus incident angle θ_i obtained by various methods.

permittivities of $\varepsilon_{r1}=1$, and $\varepsilon_{r2}=4$ and 8. The spatial and time increments were chosen as $\Delta x=\Delta y=\lambda/30$ and $\Delta t=\Delta x/2c_0$, respectively. For these spatial and time increments, the phase error of the NS-FDTD calculation for incident plane wave is negligible in the ε_{r2} area [1]. The physical value χ_0 in Fig. 4 is given as follows [9];

$$\chi_0=\tanh^{-1}\left(k_0\sqrt{\varepsilon_{r2}\sin^2\theta_i-\varepsilon_{r1}}\Big/k_x\right), \tag{25}$$

where $k_x=k_0\sqrt{\varepsilon_{r2}}\sin\theta_i$.

As shown in the figures, k and χ obtained from (13) agree well with the simulated results using NS-FDTD and FDTD simulations, respectively. From these results, the validity of derived dispersion equation (13) is confirmed. And, it is clear that the errors of k and χ in the NS-FDTD method are smaller than those obtained by the FDTD method at all incident angles.

C. Analysis of an Optical Switching Device Using a Photon Tunneling Effect

Fig. 5 shows an optical switching device based on a photon tunneling switching effect. In this analysis, Δx and Δy are set to $\lambda_0/39$ where λ_0 is the wavelength in the free space. Fig. 6 shows the transmission rate of the switching device. In the figure, the results obtained using the FDTD method and by the theory based on the transfer matrix method [9] are also shown. The results using the NS-FDTD method agree well with the theoretical results.

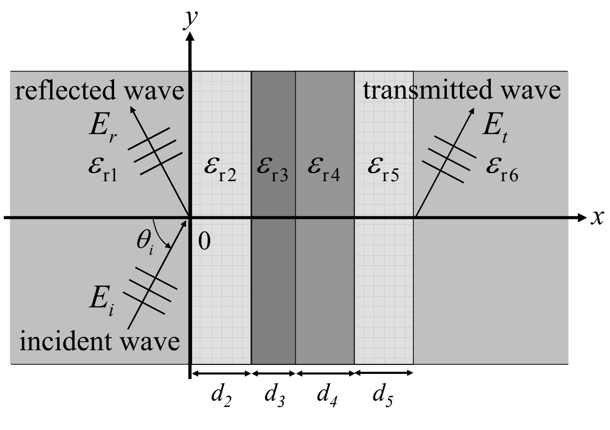

Fig. 5. Switching device based on the photon tunneling effect. ($\varepsilon_{r1}=\varepsilon_{r6}=10.89$, $\varepsilon_{r2}=\varepsilon_{r5}=9.0$, $\varepsilon_{r3}=13.69$, $\varepsilon_{r4}=11.56$, $\lambda_0=780$ nm, $d_2=d_5=500$ nm, $d_3=220$ nm, $d_4=240$ nm)

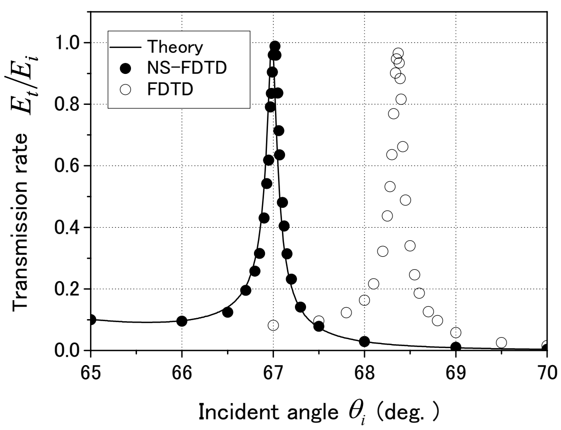

Fig. 6. Transmission rates of the switching device obtained by NS-FDTD and FDTD methods with theoretical values.

IV. Conclusion

The dispersion equation and stability condition for evanescent waves in the generalized NS-FDTD method were derived for two-dimensional space. Furthermore, the validity of the dispersion equation was confirmed by comparison with the results of numerical simulation. As a result, it was shown that the NS-FDTD method has highly accurate characteristics not only for propagating waves but also for evanescent waves when compared with the standard FDTD method. Next, the NS-FDTD method was applied to an optical device based on a resonant photon tunneling effect. Finally, the efficiency of the NS-FDTD method for evanescent wave was shown.

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