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Curvilinear triangular-prism element for computation of band structure in photonic crystal slab

Koichi Hirayama[1a)], **Masatoshi Kanou**[1], **Yasuhide Tsuji**[1], **and Masanori Koshiba**[2]

[1] *Department of Electrical and Electronic Engineering, Kitami Institute of Technology*
165 Koen-cho, Kitami, Hokkaido 090–8507, Japan
[2] *Division of Media and Network Technologies, Hokkaido University*
14–8 Kita-ku, Sapporo, Hokkaido 060–0814, Japan
a) *hirakc@mail.kitami-it.ac.jp*

Abstract: The band structure of a photonic crystal slab is analyzed by the finite-element method. Here, a curvilinear mixed-interpolation-type triangular-prism element is proposed so as to express cylindrical air holes of a photonic crystal slab smoothly enough in fewer elements. The numerical results are presented to show the usefulness of the analysis method.

Keywords: Photonic crystal slab, band structure, curvilinear triangular-prism element, FEM
Classification: Electromagnetic theory

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1 Introduction

Recently, photonic crystals have much interest and have been studied theoretically and experimentally [1, 2]. 3-D photonic crystals [3, 4] have been investigated, while photonic crystal slabs have also been done because of easy fabrication and the application to optical integrated circuits.

To analyze the band structure of photonic crystals, the plane-wave expansion method [5] and the finite-difference time-difference (FDTD) method [6] are widely used. A photonic crystal slab has a two-dimensional periodic structure in plane and optical wave is confined in the slab due to the refractive index contrast. In most of photonic crystal slabs, cylindrical air holes constitute periodic structures. When analyzing such structures by the FDTD method with rectangular meshes, a fine mesh is required to express the shape of a cylinder smoothly enough and a less time step is also requested, then it causes more time-consuming.

In this letter, we analyze the band structure of a photonic crystal slab by using the finite-element method (FEM). Here we propose a curvilinear mixed-interpolation-type triangular-prism element to be able to express the cylindrical shape in fewer elements, compared with the corresponding rectilinear element. To show the usefulness of this approach, numerical results are compared with those of a rectilinear element. Also, we demonstrate that the numerical results are improved by loading an anisotropic perfectly matched layer (PML).

2 Basic Equations

We consider a photonic crystal slab, as shown in Fig. 1, where an anisotropic PML is involved at the top and bottom of the computational domain Ω for the FEM. We notice that the computational domain for the conventional FDTD [6] is twice larger than that of the FEM. In our analysis method, the regions above and below the dielectric slab with the air holes may consist of several homogeneous layers in the z direction. Domain Ω is enclosed by the top and bottom boundaries Γ_c and the side boundary Γ_p, and Γ_c is assumed to be a perfect electric conductor, while the Bloch boundary condition is applied to Γ_p.

Applying the Galerkin's method to the vector wave equation, we obtain

$$\iiint_{\Omega}\left\{(\nabla\times\tilde{\boldsymbol{E}})\cdot\left([s]^{-1}\nabla\times\boldsymbol{E}\right)-k_0^2\varepsilon_r\tilde{\boldsymbol{E}}\cdot([s]\,\boldsymbol{E})\right\}dxdydz$$
$$=\oiint_{\Gamma_c+\Gamma_p}\left\{\tilde{\boldsymbol{E}}\times\left([s]^{-1}\nabla\times\boldsymbol{E}\right)\right\}\cdot\hat{n}d\Gamma \quad (1)$$

where $\tilde{\boldsymbol{E}}$ is the test function, $\boldsymbol{E}$ is the electric field, k_0 is the wavenumber in vacuum, ε_r is the relative permittivity of material, $\hat{n}$ denotes a unit vector

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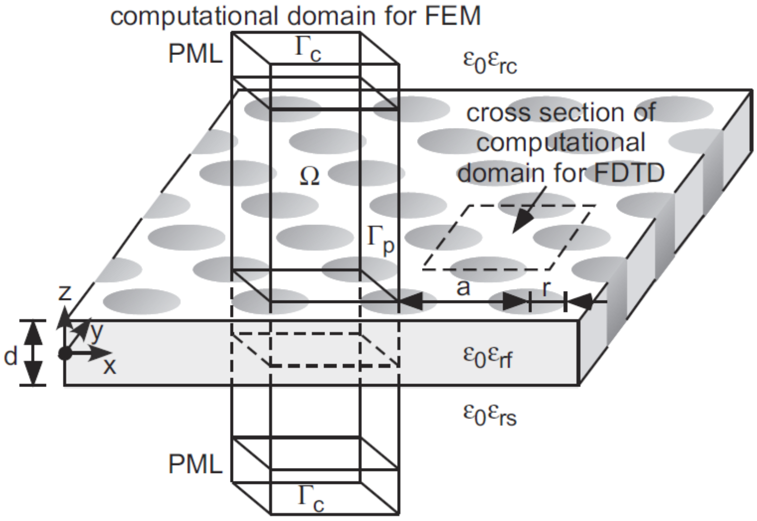

Fig. 1. Geometry of photonic crystal slab.

normal to Γ_c or Γ_p, and matrix $[s]$ represents a unit matrix outside PML and is given inside PML as follows:

$$[s] = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1/s \end{bmatrix}, \quad s = 1 - jA_z(z - z_0)^2, \quad A_z = \frac{-3\ln|R|}{2k_0\sqrt{\varepsilon_r}d_z^3} \tag{2}$$

Here d_z is the PML thickness in the z direction, z_0 denotes the z coordinate at the beginning of the PML, and $|R| = 10^{-8}$.

3 Mixed-Interpolation-Type Triangular-Prism Elements

Figure 2 shows a rectilinear and curvilinear mixed-interpolation-type triangular-prism element, whose top and bottom faces coincide with the rectilinear and curvilinear mixed-interpolation-type triangular element, respectively. The x, y, and z components of the electric field are discretized in the elements as follows:

$$E_x = \{\hat{U}\}^T\{E_t\}_e, \quad E_y = \{\hat{V}\}^T\{E_t\}_e, \quad E_z = \{\hat{N}\}^T\{E_z\}_e \tag{3}$$

with

$$\begin{aligned} \{\hat{U}\} &= \begin{bmatrix} \zeta(\zeta-1)/2 \cdot \{U\} \\ \zeta(\zeta+1)/2 \cdot \{U\} \\ (1-\zeta^2) \cdot \{U\} \end{bmatrix} \\ \{\hat{V}\} &= \begin{bmatrix} \zeta(\zeta-1)/2 \cdot \{V\} \\ \zeta(\zeta+1)/2 \cdot \{V\} \\ (1-\zeta^2) \cdot \{V\} \end{bmatrix} \\ \{\hat{N}\} &= \begin{bmatrix} (1-\zeta)/2 \cdot \{N\} \\ (1+\zeta)/2 \cdot \{N\} \end{bmatrix} \end{aligned} \tag{4}$$

where the vector $\{E_t\}_e$ has 24 components, which consist of 18 electric field components tangential to and 6 ones normal to the sides of the triangles

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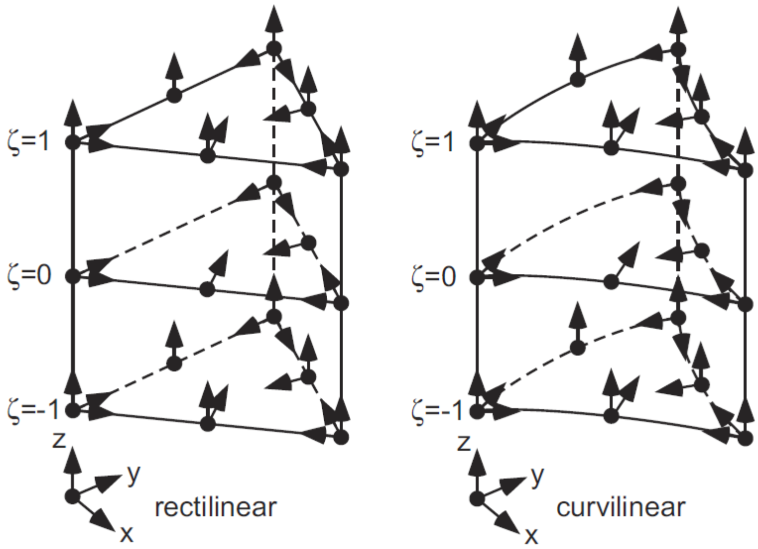

Fig. 2. Rectilinear and curvilinear mixed-interpolation-type triangular-prism elements.

of the top face ($\zeta = 1$), the bottom one ($\zeta = -1$), and the cross section ($\zeta = 0$). The vector $\{E_z\}_e$ has 12 components, which are the values of E_z at the vertices and the middle points of the sides of the triangles of the top and bottom faces. The vectors $\{U\}$, $\{V\}$, and $\{N\}$ are the shape functions of the rectilinear and curvilinear mixed-interpolation-type triangular element [7], and the superscript T denotes a transpose.

Dividing domain Ω into a number of triangular-prism elements and substituting Eq. (3) into Eq. (1), we may obtain a matrix equation. Then, applying the Bloch condition on boundary Γ_p, we can finally deduce a generalized eigenvalue problem to be solved.

4 Numerical Results

We consider a structure that there is a slab of Si in vacuum, where the relative permittivities are $\varepsilon_{rf} = 3.4^2$ and $\varepsilon_{rs} = \varepsilon_{rc} = 1$ in Fig. 1, the thickness of the slab is $d = 0.6a$ (a being the lattice constant), and the triangular lattice with the air holes of radius $r = 0.29a$ is assumed [6]. Because of the symmetry of the geometry, we assume the plane on $z = 0$ to be a magnetic wall and analyze the region only for $z \geq 0$. The PML is loaded at $3.3a \leq z \leq 4.8a$.

Figure 3 (a) shows the convergence of the eigenvalue solution against the number of the unknown electric field components for the rectilinear and curvilinear elements at the corners M and K of the irreducible Brillouin zone. We find that the computed results for the curvilinear element converge faster than those for the rectilinear element, especially in the second band. Also, if we employ the curvilinear element, we can expect that the element division with about 15000 unknowns produces the results of three significant figures.

Figure 3 (b) shows the band diagram for the photonic crystal slab of the triangular lattice. The solid lines represent the computed results by using the FEM of the curvilinear element with the PML over the slab, and the

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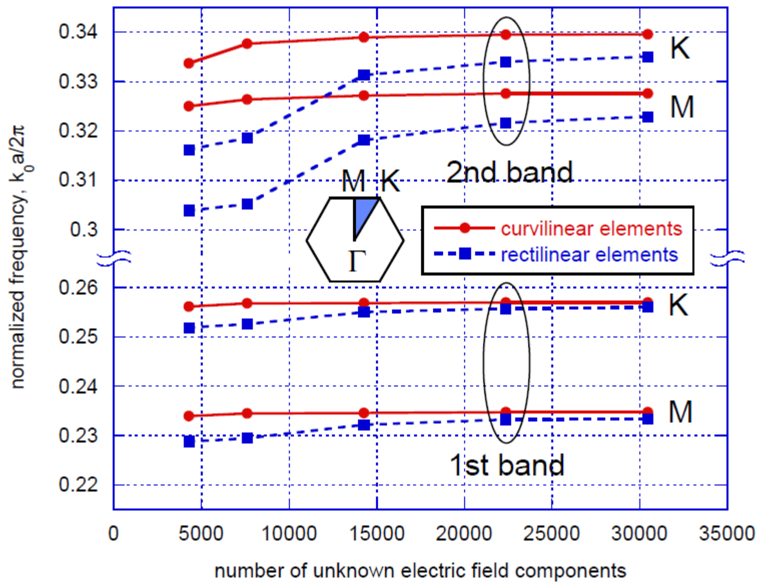
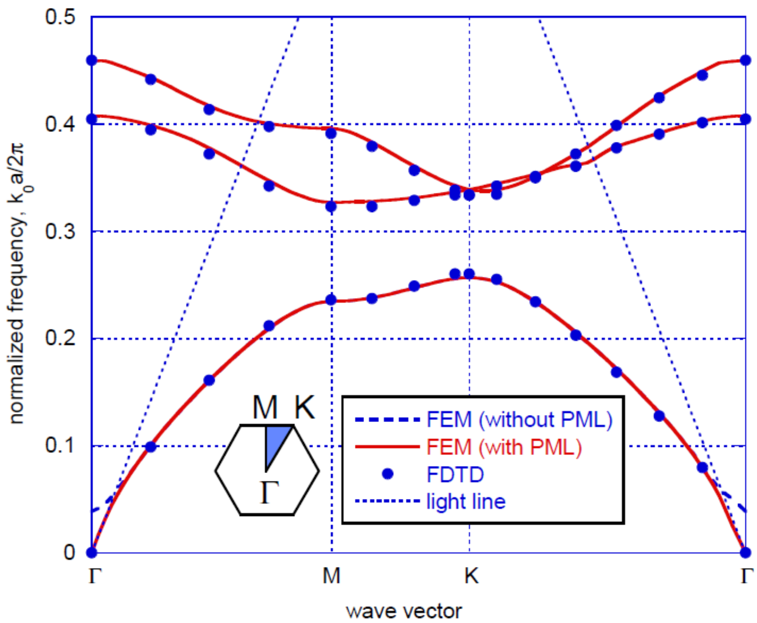


(a) Convergence of the solution against the number of the unknown electric field components.



(b) Band diagram.

Fig. 3. Convergence of computed results and band diagram for the photonic crystal slab of the triangular lattice with the air holes of radius $r = 0.29a$.

dark circles stand for the results of the FDTD [6]. Also, the broken line represents the results for the first band by using the FEM without the PML. The results of the FEM with and without the PML are almost the same over the wave vectors around the irreducible Brillouin zone, but the FEM without the PML gives wrong values near the corner Γ of the Brillouin zone. This is because the cutoff frequency for the first band exists when the PML is not loaded. We can obtain the following characteristic equation of the guided modes for a symmetric dielectric slab waveguide without air holes and closed

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by perfect electric conductors:

$$\tan\left(\frac{\kappa d}{2}\right) = \frac{\alpha}{\kappa \tanh(\alpha D)}, \quad \kappa = \sqrt{k_0^2 \varepsilon_{rf} - \beta^2}, \quad \alpha = \sqrt{\beta^2 - k_0^2} \qquad (5)$$

where β represents the phase constant of a guided mode, and D denotes the distance between the top of the slab and the perfect electric conductor. In this case $D = 4.5a$, so the normalized cutoff frequency ($\beta = k_0$) is $k_0 a/2\pi \simeq 0.042$. The value is close to and a little smaller than the normalized frequency at the intersection of the broken line with the light line in Fig. 3 (b), because the permittivity of a slab without air holes is greater than the effective permittivity of the photonic crystal slab and a cutoff frequency decreases with the permittivity of a slab.

5 Conclusion

We proposed a curvilinear mixed-interpolation-type triangular-prism element in the FEM to analyze the band structure of a photonic crystal slab. From the numerical results, we showed that the solutions with good accuracy are obtained in fewer elements, because the proposed element can express cylindrical holes of a photonic crystal slab more smoothly than the corresponding rectilinear element. Also, we demonstrated that, if one computes the band structure of a photonic crystal slab by using the FEM without the PML, the cutoff frequency for the first band appears due to the existence of the perfect electric conductor. In the future, we intend to analyze a transmission problem of a photonic crystal slab by using the FEM with a curvilinear triangular-prism element.

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