Simplified method to include the tensor contribution in α-cluster model

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We propose a simplified model to directly take into account the contribution of the tensor interaction (SMT) for light nuclei by extending the α-cluster model. In ^{8}Be, the energy curve with respect to the relative distance between the two ^{4}He clusters suggests that the cluster structure persists even though the tensor interaction contributes strongly. In addition to SMT, a simplified method to take into account the strong spin-orbit contribution is introduced and the coupling effects of these two models is shown to be important in ^{12}C, in contrast to ^{8}Be.

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I. INTRODUCTION

It has been shown for more than four decades that cluster models succeed in predicting and reproducing various, even exotic properties of light nuclei [1–3]. In these cluster models, the nucleon-nucleon interactions adopted have been effective interactions instead of bare ones, constructed by renormalizing the short-range core and tensor parts in the central part so as to be used in model spaces of the cluster models, where each nucleon is usually described by Gaussian-type wave functions. However, recently, theoretical investigations for light nuclei based on realistic nucleon-nucleon interactions became feasible [4–6]. It is significant that quantum Monte Carlo calculations have microscopically shown the appearance of α-α cluster structure in ^{8}Be based on a realistic nucleon-nucleon interaction. Such systematic calculations based on the Green's function Monte Carlo method have been performed up to ^{12}C.

Despite this situation, it should be mentioned that cluster models are still useful in describing very exotic cluster structures [7] or intruder states of excess neutrons [8]. Therefore, now is the time to further refine the effective interaction used in conventional cluster models to include the nature of the original nucleon-nucleon interaction, extending the model space to describe both exotic cluster and shell-like structures. One of the key issues is the direct treatment of the tensor terms. Recently, several attempts of directly taking into account the tensor part of the interaction in microscopic cluster models have begun [9–13], and its strong contribution has been discussed, for example in the ^{4}He nucleus.

The purpose of the present study is to establish a simplified method to take into account the tensor contribution and show its applicability to ^{4}He, ^{8}Be, and ^{12}C. In the previous work, we have developed a simplified method to include the spin-orbit contribution (SMSO) in the α-cluster model [14], and now we try to extend this idea for the inclusion of the second-rank tensor interaction. When the tensor interaction is directly treated, the central part of the effective interaction has to be completely modified to remove the renormalized part of the interaction. Establishing a simple method to take the tensor interaction into account is also important for making new interactions, which is the next step of the study.

In this article, we study a fundamental aspect of the mechanism for the appearance of cluster structure. In the 1960s, a microscopic α-α potential was derived based on meson theory [15,16], and the appearance of the cluster structure was explained by saying that the contribution of the one-pion exchange potential (OPEP) vanishes from the direct terms when each α cluster is described as a $(0s)^4$ configuration. This idea has been generalized as a threshold rule and proposed as a mechanism for explaining the appearance of various cluster structures [17]. If the α cluster is described as a simple $(0s)^4$ configuration, neither the spin-orbit nor the tensor interactions contribute. However, it is important to show that the picture of "weak interactions" between the two ^{4}He is still valid, although the model space is extended. The tensor interaction may not act strongly enough to change the relative motion between the two ^{4}He nuclei, and cluster structure may survive.

Furthermore, we focus on the competition between tensor and spin-orbit contributions. We have previously shown the breaking up of α cluster(s) resulting from spin-orbit interaction in ^{12}C, although the α-cluster structure essentially remains in ^{8}Be [18]. In the present analysis, we switch on the tensor interaction and discuss the difference between the contributions of these two noncentral interactions in ^{8}Be and ^{12}C.

This paper is organized as follows. In Sec. II, the framework is shown, and in Sec. III, numerical results for light nuclei are presented. The conclusion is given in Sec. IV.

II. FRAMEWORK

A. Hamiltonian

The Hamiltonian operator $\hat{H}$ has the following form:

$$\hat{H} = \sum_{i=1}^{A} \hat{t}_i - \hat{T}_{\text{c.m.}} + \sum_{i>j}^{A} \hat{v}_{ij}, \tag{1}$$

TABLE I. The parameters of the present interactions. Units of V_1^c, V_2^c, V_0^{ls}, V_1^t, V_2^t, and V_3^t are MeV, those of c_1 and c_2 are fm, and those of d_1, d_2, μ_1, μ_2, and μ_3 are fm^{-2}.

V_1^c	V_2^c	c_1	c_2	M				
−60.65	61.14	1.8	1.01	0.6				
V_0^{ls}	d_1	d_2						
2000	5.0	2.778						
V_1^t	V_2^t	V_3^t	μ_1	μ_2	μ_3	H_1	H_2	H_3
−16.96	−369.5	1688	0.53	1.92	8.95	0.6723	0.5898	0.5

where a two-body interaction ($\hat{v}_{ij}$) includes the central, spin-orbit, tensor, and Coulomb parts. We adopt the Volkov (No. 2) interaction [19] for the central part,

$$V_c = (W - M P^\sigma P^\tau) \sum_{i=1}^{2} V_i^c \exp\left(-r^2/c_i^2\right), \qquad (2)$$

where $W = 1 - M$. For the spin-orbit part, we adopt the G3RS potential [20]

$$V_{ls} = V_0^{ls}\left(e^{-d_1 r^2} - e^{-d_2 r^2}\right)\vec{L}\cdot\vec{S}P(^3O), \qquad (3)$$

where $P(^3O)$ is a projection operator onto a triplet-odd channel, the operator $\vec{L}$ stands for the relative angular momentum, and $\vec{S}$ is the spin ($\vec{S}_1 + \vec{S}_2$). In Ref. [21], it is shown that the $\alpha + n$ and $\alpha + \alpha$ scattering phase shifts are well reproduced by using the parameters listed in Table I, when each α cluster is described as a $(0s)^4$ configuration.

In conventional cluster models, the tensor terms are mocked up by renormalized central terms of the potential. However, here, we directly include the tensor terms:

$$V_t = \sum_{i=1}^{3}(W_i - H_i P^\tau) S_{12} V_i^t r^2 \exp(-\mu_i r^2), \qquad (4)$$

where $W_i = 1 - H_i$ and S_{12} is a standard tensor operator given as

$$S_{12} = 3\frac{(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^2} - \vec{\sigma}_1\cdot\vec{\sigma}_2. \qquad (5)$$

We adopt the Furutani potential [22] parametrized in Table I. This potential reproduces the tensor part of the OPEP around the interaction range of $1 \sim 3$ fm, as shown in Fig. 1.

B. Wave function

The total wave function is fully antisymmetrized and is given by a superposition of the basis states (Slater determinants $\{\Psi_k\}$) with coefficients $\{c_k\}$:

$$\Phi = \sum_k c_k P^\pi P^J_{\mathrm{MK}} \Psi_k, \qquad (6)$$

$$\Psi_k = \mathcal{A}[(\psi_1\chi_1)(\psi_2\chi_2)\cdots]_k. \qquad (7)$$

Projection onto good parity (P^π) and angular momentum (P^J_{MK}) are performed numerically (with $16^3 = 4096$ mesh points for the Euler angle integral) and the coefficients $\{c_k\}$ are determined by diagonalizing the Hamiltonian matrix after this projection. Each Slater determinant (Ψ_k) consists of A nucleons and each nucleon function ($\psi_i\chi_i$ $i = 1$–A) has a Gaussian form,

$$\psi_i = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} \exp\left[-\nu(\vec{r} - \vec{z}_i/\sqrt{\nu})^2 + \vec{z}_i^{\,2}/2\right], \qquad (8)$$

where $\{\vec{z}_i\}$ are complex parameters and $\{\chi_i\}$ represent the spin-isospin eigenfunctions. The oscillator parameter is set equal to $\nu = 1/2b^2$, with $b = 1.46$ fm, which is common for all nucleons to exactly remove the c.m. kinetic energy.

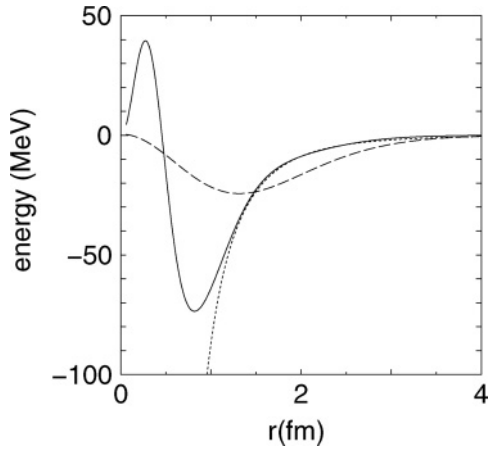

FIG. 1. The strength of the tensor interaction for the triplet-even channel without the S_{12} factor. The solid line and the dotted line correspond to the Furutani potential and the one-pion exchange potential, respectively. The dashed line shows the strength of the Volkov interaction for the even-parity channel.

III. RESULTS

A. Stochastic variational method calculation for ^{4}He

Before introducing our simplified model, we first perform a calculation for ^{4}He based on the idea of the stochastic variational method (SVM) [23] to estimate the tensor contribution. Here, 1000 Slater determinants with a total spin of both $S_z = 0$ and $S_z = 1$ are introduced, and in each Slater determinant, the Gaussian-center parameters for the four nucleons are randomly generated. The ground 0^+ energy of ^{4}He is shown in Fig. 2 as a function of the number of the Slater determinants introduced. To reduce the number of basis states, the basis state is adopted only when the inclusion of this Slater determinant decreases the sum of the energies of the ground and second 0^+ states by more than 0.02 MeV. The energy almost converges at −37.3 MeV, which is 9.8 MeV lower than that for the $(0s)^4$ configuration. Here, the expectation value of the tensor interaction is −16.2 MeV.

However, the tensor contribution here (−16.2 MeV) is much smaller than in the calculations based on the realistic nucleon-nucleon interactions. For example, in Refs. [24,25], the contribution is calculated to be around −68 MeV. There are mainly two reasons for this difference. One is the tensor interaction adopted here itself. The tensor term of the Furutani interaction is determined from an analysis of ^{3}He + p

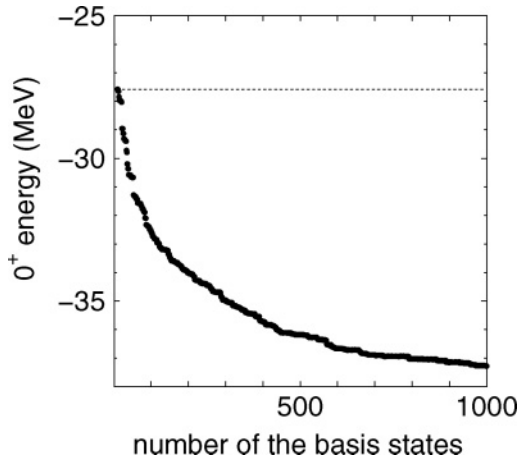

FIG. 2. The ground 0^+ energy of ^{4}He as a function of the number of the Slater determinants introduced. In each Slater determinant, the positions of the Gaussian-center parameters are randomly generated. The dotted line shows the energy for the $(0s)^4$ model space.

$(T = 1)$ scattering assuming a $3N[(0s)^3] + N$ configuration. Therefore, there is no guarantee that the interaction works properly for the $T = 0$ state of ^{4}He when four nucleons are treated independently. The other reason is the range of the central interaction. In the present calculation, the central interaction used (Volkov) is an effective one, which widely spreads in space, as shown in Fig. 1, and the interaction range is much larger than that of the tensor terms. When we multiply the interaction by r^2, the central and tensor parts have the lowest values at $r = 2.0$ and 0.8 fm, respectively. Because of this difference, the relative wave function between nucleons does not have a large amplitude, where the contribution of the tensor interaction becomes maximum. However, the effective interaction derived from the realistic one by solving the G matrix [26,27] shows the optimal distance between nucleons at around 1 fm, when multiplied by r^2. Therefore, to compare the tensor contribution with the calculations using the realistic nucleon-nucleon interactions, it is also necessary to modify the central part of the interaction completely.

It should be mentioned that when the tensor interaction is switched on, obtaining a converged solution requires a large number of basis states. In the SVM calculation here for ^{4}He, 1000 Slater determinants are prepared, and in total 236 are adopted. Therefore, a simplified model is needed for the analysis of heavier systems.

B. Introduction of a simplified cluster model to take into account the tensor contribution

Next, we define our simplified cluster model to take into account the tensor contribution in ^{4}He. When the α cluster is described as the simplest $(0s)^4$ configuration, it is a spin-zero system, and noncentral interactions of either spin-orbit or tensor type do not contribute. Here, four nucleons (spin-up proton, and spin-down proton, spin-up neutron, and spin-down neutron) are described by Gaussian wave functions centered at the same position. Furthermore, the tensor interaction does not act attractively even if we break the α cluster with the $(0s)^4$ configuration to two cluster systems of $p + t$ or $n + {}^3$He. Suppose that the position of the spin-up proton is shifted from the other three nucleons, which form a triton (t) cluster. The tensor interaction acts between this spin-up proton and spin-up neutron in the t cluster, since they have a deuteron-like $S = 1, T = 0$ component. However, in the triton cluster, a spin-down neutron occupies the same spatial orbit as the spin-up neutron. Thus, the tensor contribution between the spin-up proton and spin-up neutron is canceled by the presence of this spin-down neutron, which has the same spatial wave function as the spin-up neutron but with the opposite spin direction.

This discussion suggests that we must break the α cluster not into two clusters as $p + t$ or $n + {}^3$He but at least into three clusters such as $p + n + {}^2$H. As for the coordinate system, the energy surface of the deuteron system [10] suggests that the tensor interaction acts attractively when a spin-up proton and spin-up neutron stay along the spin direction. In our framework, the spin direction is defined parallel to the z axis. Therefore, one of the promising ways to break the α cluster centered at the origin is to shift the central position of the spin-up proton ($\vec{z}_{p\uparrow}$) in the z direction and that of the spin-up neutron, ($\vec{z}_{n\uparrow}$) in the $-z$-direction:

$$\vec{z}_{p\uparrow}/\sqrt{\nu} = d\vec{e}_z, \tag{9}$$

$$\vec{z}_{p\downarrow}/\sqrt{\nu} = 0, \tag{10}$$

$$\vec{z}_{n\uparrow}/\sqrt{\nu} = -d\vec{e}_z, \tag{11}$$

$$\vec{z}_{n\downarrow}/\sqrt{\nu} = 0. \tag{12}$$

Here, $p\uparrow$, $p\downarrow$, $n\uparrow$, and $n\downarrow$ express the spin-up proton, spin-down proton, spin-up neutron, and spin-down neutron, respectively, and $\vec{e}_z$ is a unit vector along the z axis. This wave function corresponds to a deuteron-like two-particle two-hole (2p2h) excitation of the spin-up proton and neutron from the $(0s)^4$ core, which is known to be a dominant source of the tensor contribution in ^{4}He. The central positions of the spin-up proton and that of the spin-up neutron are shifted by introducing a parameter d, which, for simplicity, is common for these nucleons. This d value is changed from 0 to 7 fm in steps of 0.7 fm, and in total 11 Slater determinants are generated. Since the tensor interaction contributes mostly at short relative distances between the proton and neutron [11,12], we need to superimpose a rather large number of basis states to improve the description of this part. This model gives a 0^+ energy of -31.03 MeV compared to -27.57 MeV for the $(0s)^4$ model space. However, it is found that some part of the tensor correlation between the spin-up proton and neutron is compensated by the spin-down neutron, which is located between them with the opposite spin direction.

Therefore, we introduce a more effective model. We remove the spin-down neutron from the spin-up proton-neutron pair. The Gaussian-center parameters of the four nucleons ($\vec{z}_i$) are defined as

$$\vec{z}_{p\uparrow}/\sqrt{\nu} = d\vec{e}_z, \tag{13}$$

$$\vec{z}_{p\downarrow}/\sqrt{\nu} = 0, \tag{14}$$

TABLE II. The calculated total (E), kinetic (T), central potential (V^c), Coulomb ($V^{\rm clm}$), spin-orbit (V^{ls}), and tensor (V^t) energies (in units of MeV) and r.m.s. radius (in fm) of ^{4}He. The SMT results and those of the $(0s)^4$ configuration are compared with the experimental values (Exp.).

	E	T	V^c	$V^{\rm clm}$	V^{ls}	V^t	r.m.s. radius
SMT	−33.83	53.23	−78.10	0.83	0.14	−9.92	1.66
$(0s)^4$	−27.57	43.77	−72.14	0.79	0	0	1.73
Exp.	−28.297						1.63 ± 0.03[a]
							1.40 ± 0.05[b]

[a]Reference [28].
[b]Reference [29].

$$\vec{z}_{n\uparrow}/\sqrt{\nu} = 0, \tag{15}$$

$$\vec{z}_{n\downarrow}/\sqrt{\nu} = -d\vec{e}_z. \tag{16}$$

The spin-up proton-neutron pair have the component of $S = 1, T = 0$; similarly, the spin-down proton and neutron have an $S = 1, T = 0$ component. Here, the spin-down neutron (spin-up proton) does not stay inside the spin-up proton-neutron pair (spin-down proton-neutron pair), so the tensor correlation is not disturbed. Hereafter, the transformations in Eqs. (13)–(16) are called the simplified method to include the tensor contribution (SMT). Here, ^{4}He is described as the linear combination of 11 Slater determinants ($d = 0$–7 fm), and the calculated 0^+ energy E is −33.83 MeV. The 0^+ energy is lower by 6.3 MeV following the inclusion of the tensor terms, which is a little smaller than the results of Sec. III A and other more sophisticated calculations [11,12]. However the method turns out to be a reasonable approximation. In Table II, the kinetic (T), central potential (V^c), Coulomb ($V^{\rm clm}$), spin-orbit (V^{ls}), and tensor (V^t) energies and r.m.s. radius are presented, together with the results of the $(0s)^4$ configuration and the experimental values. The tensor interaction contributes within the present model space by −9.92 MeV, and some part is compensated for by the increase of the kinetic energy from the $(0s)^4$ model space. It should be noted that although the d values employed in our model are rather large (up to 7 fm), the calculated r.m.s. radius of ^{4}He is smaller than that of the $(0s)^4$ model. Because of this shrinkage effect, the central part of the potential energy is stronger than that of the $(0s)^4$ model space by about 6.3 MeV.

The effect of charge projection is discussed in Ref. [11] and we also estimate it in a simple way. In our model, charge projection for a two-nucleon subsystem can be performed by incorporating basis states, in which positions of the proton and neutron are interchanged. By adding these basis states, these two nucleons can have the optimal isospin. We prepare the basis states in which the positions of the spin-up (down) proton and spin-up (down) neutron are interchanged. Basis states with the positions of the spin-up proton and neutron and spin-down proton and neutron simultaneously interchanged are also generated. By incorporating these basis states, the energy of ^{4}He becomes −34.12 MeV. This means that the effect of charge projection contributes by decreasing the energy by about 300 keV and our original model is a good approximation. Thus, this projection is ignored in the following calculations.

C. Application to ^{8}Be and ^{12}C

Similarly, for ^{8}Be, our simplified method to include the tensor interaction (SMT) is adopted. Each ^{4}He nucleus is described as in Eqs. (13)–(16), and 2p2h excitations (in the z direction) are taken into account. If we put the centers of mass of the two ^{4}He on the z axis, the 2p2h excitation in each ^{4}He nucleus is strongly suppressed because of the presence of another ^{4}He nucleus, and the tensor interaction does not work strongly. To avoid this problem, the c.m. of each ^{4}He is placed on the x axis [$(R/2)\vec{e}_x$ or $(-R/2)\vec{e}_x$]. The Gaussian-center parameters of the α cluster centered at $(R/2)\vec{e}_x$ are taken as

$$\vec{z}_{p\uparrow}/\sqrt{\nu} = (R/2)\vec{e}_x + d_1\vec{e}_z, \tag{17}$$

$$\vec{z}_{p\downarrow}/\sqrt{\nu} = (R/2)\vec{e}_x, \tag{18}$$

$$\vec{z}_{n\uparrow}/\sqrt{\nu} = (R/2)\vec{e}_x, \tag{19}$$

$$\vec{z}_{n\downarrow}/\sqrt{\nu} = (R/2)\vec{e}_x - d_1\vec{e}_z, \tag{20}$$

and those of the α cluster centered at $(-R/2)\vec{e}_x$ are taken as

$$\vec{z}_{p\uparrow}/\sqrt{\nu} = (-R/2)\vec{e}_x + d_2\vec{e}_z, \tag{21}$$

$$\vec{z}_{p\downarrow}/\sqrt{\nu} = (-R/2)\vec{e}_x, \tag{22}$$

$$\vec{z}_{n\uparrow}/\sqrt{\nu} = (-R/2)\vec{e}_x, \tag{23}$$

$$\vec{z}_{n\downarrow}/\sqrt{\nu} = (-R/2)\vec{e}_x - d_2\vec{e}_z. \tag{24}$$

Here, R is the relative distance between the two clusters, and the d_1 and d_2 values employed are the same as the d values of ^{4}He. Since each ^{4}He nucleus is described as a linear combination of 11 Slater determinants, there exist $11 \times 11 = 121$ Slater determinants for each ^{4}He-^{4}He distance, and 66 of these are independent after performing the angular momentum projection. To reduce the number of the basis states, we only retain the important ones in the same way as for antisymmetrized molecular dynamics—superposition of selected snapshots (AMD triple-S) [30]: When the inclusion of one Slater determinant decreases the sum of the energies of the ground and second 0^+ states by more than 0.05 MeV, this Slater determinant is adopted.

In Fig. 3, the 0^+ energy curves of ^{8}Be are shown as a function of the distance between the two ^{4}He nuclei. Both

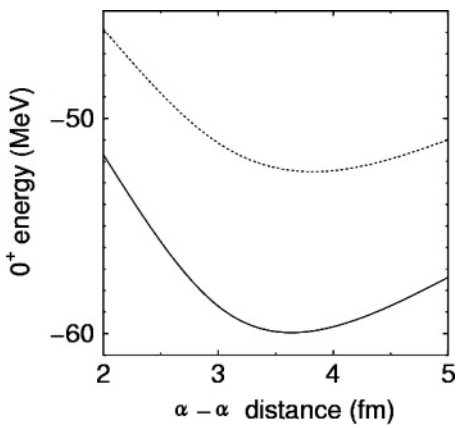

FIG. 3. The 0^+ energy curves of ^{8}Be as a function of the distance between the centers of the two ^{4}He. Both the results of SMT (solid line) and of the $(0s)^4$ configuration for each α cluster (dotted line) are presented.

TABLE III. The total energy, tensor, and spin-orbit expectation values of the 0^+ state of ^{8}Be (in units of MeV). These values for the $(0s)^4$ model space, SMT, and SMSO are presented, together with the combined model space (SMSO+SMT).

	Total	Tensor	Spin-orbit
$(0s)^4$	−54.19	0.00	0.00
SMT	−62.28	−11.66	0.14
SMSO	−54.54	0.08	−0.71
SMSO+SMT	−62.66	−11.64	−0.59

the results of SMT (solid line) and of the $(0s)^4$ configuration for each α cluster (dotted line) are presented, and they show that the energy difference is large (∼8 MeV) owing to the tensor contribution. The result also shows that, even though the tensor interaction contributes strongly, the contribution is restricted to the inside of each ^{4}He cluster, and the relative distance between two ^{4}He nuclei is still large enough and the cluster structure persists. The expectation value of the tensor interaction is −11.52 MeV at the ^{4}He-^{4}He distance of 4 fm; however, it decreases to −8.31 MeV at 2 fm. This is explained by recognizing that the 2p2h excitation of one ^{4}He (in the z direction) is Pauli-blocked owing to the 2p2h excitation of the ^{4}He at short relative distances, even if the centers of mass of the two ^{4}He are placed on the x axis, perpendicular to the direction of the 2p2h excitation.

When we superimpose states with respect to the relative distance between the two ^{4}He (from 2 to 5 fm in steps of 1 fm), the calculated 0^+ energy becomes −62.28 MeV, as shown in Table III. Since ^{4}He has been calculated as −33.83 MeV by using SMT, the energy of ^{8}Be is a little higher than twice the ^{4}He energy, in contrast to the experimental result. This fact may suggest the necessity of double projection. In this calculation of ^{8}Be, only the wave function of the total system is projected to 0^+; however, the wave function of each subsystem (^{4}He) is not projected to the eigenstate of angular momentum around its center.

SMT is further applied to ^{12}C. Three α clusters have an equilateral-triangular configuration on the xy-plane, and here, for simplicity, only two of them are transformed to take into account the tensor contribution as in Eqs. (13)–(16). (Transforming three α clusters is possible but the result does not change drastically.) In Fig. 4, the 0^+ energy curves of ^{12}C as a function of the distance between each ^{4}He-^{4}He pair are presented. The difference between SMT (solid line) and the $(0s)^4$ configuration for each α cluster (dotted line) is large (∼6 MeV) owing to the tensor contribution. The result also shows that the relative distance between two ^{4}He is still large enough to prevent the cluster structure from being washed out by the tensor interaction. As shown in Table IV, the energy of the 0^+ ground state of ^{12}C is calculated to be −93.21 MeV by using SMT when we superimpose the states with the ^{4}He-^{4}He distances of 2, 2.5, and 3 fm. This is lower than that of the $(0s)^4$ model space by 6.5 MeV, and here the expectation value of the tensor interaction is −7.18 MeV. However, the spin-orbit interaction, which plays an important role in ^{12}C [18], gives a small contribution for this model space (0.07 MeV).

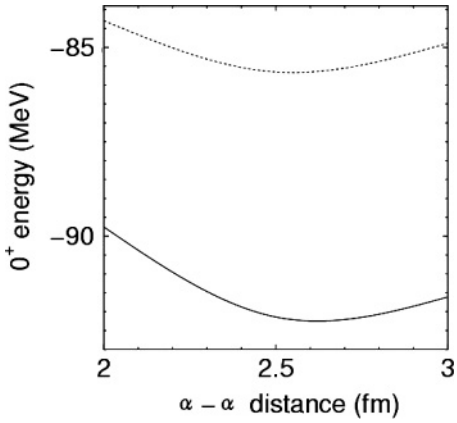

FIG. 4. The 0^+ energy curves of ^{12}C with the equilateral triangular configuration of 3α as a function of the distance between the centers of two ^{4}He. Both the results of SMT (solid line) and of the $(0s)^4$ configuration for each α cluster (dotted line) are shown.

We have previously shown that the breaking up of the α cluster(s) to take into account the spin-orbit interaction is important in ^{12}C [18]; however, this is not described within SMT. Therefore, here, in addition to SMT, SMSO [14] is introduced, and the interference between these two model spaces is examined. As shown in Ref. [14], using SMSO, we can prepare a basis state, in which one of the α clusters is broken up into four independent nucleons owing to the spin-orbit interaction. In SMSO, two α clusters with the $(0s)^4$ configuration are placed at $R/2$ and $-R/2$ on the z axis, and the last one is placed at $\sqrt{3}R/2$ on the x-axis so that they form an equilateral-triangular configuration. Next, the Gaussian centers of nucleons in the last α cluster on the x axis are changed from $\sqrt{3}R/2\ \vec{e}_x$ to $\sqrt{3}R/2\ (\vec{e}_x + i\Lambda\vec{e}_y)$ for the spin-up nucleons as

$$\vec{z}/\sqrt{\nu} = (\sqrt{3}R/2)(\vec{e}_x + i\Lambda\vec{e}_y) \tag{25}$$

and to $\sqrt{3}R/2\ (\vec{e}_x - i\Lambda\vec{e}_y)$ for the spin-down nucleons as

$$\vec{z}/\sqrt{\nu} = (\sqrt{3}R/2)(\vec{e}_x - i\Lambda\vec{e}_y), \tag{26}$$

where, $\vec{e}_x$ and $\vec{e}_y$ are unit vectors for the x and y directions and Λ is an "order parameter" of cluster dissolution. By introducing these imaginary parts to the Gaussian centers, we can mimic the spherical harmonics. Namely, when Λ is equal to 1, the wave functions for the spin-up nucleons, which are excited to the p shell owing to the Pauli principle, correspond exactly to the shell-model wave function of $(x + iy)\exp[-\nu r^2] \sim rY_{11}\exp[-\nu r^2]$, and those for the spin-down nucleons correspond to $(x - iy)\exp[-\nu r^2] \sim rY_{1-1}\exp[-\nu r^2]$, at the limit of $R \to 0$. Since the directions of the spin and orbital parts of the angular momentum become parallel, the spin-orbit interaction acts attractively.

The expectation values for the 0^+ state of ^{12}C with SMSO ($R = 2$ fm) are shown in Fig. 5. The contributions of the tensor interaction (solid line), spin-orbit interaction (dotted line), and kinetic energy (dashed line) are represented as functions of the Λ value. The kinetic energy is shifted to zero at $\Lambda = 0$. The dotted line shows that the spin-orbit interaction acts attractively with increasing Λ value, and the kinetic energy with the

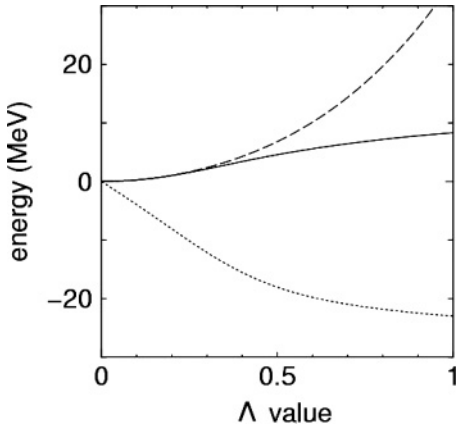

FIG. 5. The expectation values of the tensor interaction (solid line), the spin-orbit interaction (dotted line), and the kinetic energy (dashed line) for the 0^+ states of ^{12}C with SMSO. The relative distance of ^{4}He-^{4}He is 2 fm. The kinetic energy curve is shifted so as to be at zero at $\Lambda = 0$.

opposite sign also increases as shown by the dashed line. The total energy becomes lowest at around $\Lambda = 0.3$ owing to the cancellation of these two components. Here, it is important that the expectation value of the tensor interaction (solid line) is 2.4 MeV, which partially cancels the attractive spin-orbit interaction. A mechanism to explain the repulsive tensor effect between such j-upper protons and j-upper neutrons has recently been proposed by Otsuka *et al*. [31].

Mixing these two models of SMT and SMSO is important. As shown in Table IV, the energy becomes −96.70 MeV after coupling these model spaces, which is lower than the energy for the $(0s)^4$ configuration for each α cluster by about 10.0 MeV. Here, the spin-orbit and tensor interactions contribute −8.42 MeV and −5.80 MeV, respectively. The ^{4}He-^{4}He distances employed are 2, 2.5, and 3 fm for both models, and the Λ values for SMSO are from 0 to 0.4 in steps of 0.1.

This strong coupling between SMT and SMSO is a significant feature of ^{12}C. However, in ^{8}Be the spin-orbit interaction does not act strongly as we have already discussed in Ref. [18]. To quantitatively estimate this effect, we apply SMSO to ^{8}Be: One α cluster with the $(0s)^4$ configuration is centered at the origin and the other one is centered at R on the x axis. Next, the Gaussian centers of nucleons in the second α cluster are changed from $R\,\vec{e}_x$ to $R\,(\vec{e}_x + i\Lambda\vec{e}_y)$ for the spin-up nucleons as

$$\vec{z}/\sqrt{\nu} = R(\vec{e}_x + i\Lambda\vec{e}_y) \tag{27}$$

and to $R\,(\vec{e}_x - i\Lambda\vec{e}_y)$ for the spin-down nucleons as

$$\vec{z}/\sqrt{\nu} = R(\vec{e}_x - i\Lambda\vec{e}_y). \tag{28}$$

The adopted Λ values are from 0 to 0.4 in steps of 0.1, and the R values are changed from 2 to 5 fm in steps of 1 fm. As shown in Table III, the expectation value of the spin-orbit interaction of ^{8}Be is very small in both the results of SMSO (−0.71 MeV) and SMSO + SMT (−0.59 MeV). This is because the optimal distance between the two ^{4}He nuclei (∼3.5 fm) is much larger than the interaction range of the spin-orbit interaction. Thus, the contribution of the spin-orbit interaction for the relative motion between the two ^{4}He nuclei is not significant in ^{8}Be, in contrast to ^{12}C. However, owing to the large distance between two ^{4}He, the contribution of the tensor interaction (−11.64 MeV) is larger than that in ^{12}C (−5.80 MeV). This is because the 2p2h excitation of any one of the α clusters is not blocked by the other α cluster.

TABLE IV. The total energy, tensor, and spin-orbit expectation values of ^{12}C (in units of MeV). These values for the $(0s)^4$ model space, SMT, and SMSO are presented, together with the combined model space (SMSO+SMT).

	Total	Tensor	Spin-orbit
$(0s)^4$	−86.68	0.00	0.00
SMT	−93.21	−7.18	0.07
SMSO	−91.19	1.42	−11.43
SMSO+SMT	−96.70	−5.80	−8.42

IV. CONCLUSION

We have proposed a method (SMT) to directly treat the contribution of the tensor force for the α-cluster model in a simplified way. In ^{8}Be, the results show a large increase in the binding energy (∼8 MeV) from the $(0s)^4$ model space owing to the tensor contribution. However, even though the tensor interaction contributes strongly, the contribution is restricted to the inside of each ^{4}He cluster, and the relative distance between the two ^{4}He nuclei is large enough for the cluster structure to remain. Next, SMT has been applied to ^{12}C and similar factors appear to be important. In ^{12}C, not only the tensor interaction but also the spin-orbit interaction contributes strongly, and SMSO is introduced to incorporate the spin-orbit contribution. The mixing of SMT and SMSO is important: When coupling these model spaces, the energy becomes −96.70 MeV, 10.0 MeV lower than the result of the $(0s)^4$ model space.

From these analyses, two roles of tensor contributions have been clarified. One role is to bind the ^{4}He cluster. The tensor interaction acts strongly inside each ^{4}He nucleus; however, it is not strong for the relative motion between ^{4}He nuclei. The tensor interaction contributes to increase the binding energy of each ^{4}He; however, the cluster structure persists. The other role of the tensor interaction is to give a correct spin dependence for the motion of the single particles rotating around the core nucleus. This tensor contribution is relatively weak compared to the first one, but it acts repulsively between the j-upper proton and neutron (attractive between the j-upper proton and j-lower neutron). To take these two effects of the tensor interaction into account, coupling the two model spaces of SMT and SMSO is necessary.

As a next step, it is necessary to modify the central part of the effective interaction and remove the renormalized part of the tensor interaction for a quantitative understanding of these nuclei. Also, the strength of the spin-orbit interaction

should be corrected when the tensor contribution can be taken into account, as suggested in Ref. [12]. After modifying these interactions, we will study a long-standing problem of the α-cluster model. When we use the effective interaction that reproduces the binding energy of the 3α system, the 4α system becomes over bound by about 20 MeV. In contrast, if the binding energy of 4α is reproduced, the 3α system becomes underbound by about 10 MeV. We have previously discussed the lack of full reproducability of the experimental binding energy difference between ^{12}C and ^{16}O, even if we utilize a finite-range and density-dependent interaction [32], when the model space is restricted to α clusters. Now it is shown that by incorporating the α-breaking component, the binding energy of ^{12}C becomes deeper by about 10 MeV owing to the spin-orbit and tensor contributions. It is worthwhile to investigate whether these noncentral interactions are the important keys to resolve this long-standing problem.

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