Analysis of surface deterioration of Concrete in Cold Sea Environment by Reliability Analysis*1

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Abstract

Predictions of the service life of concrete structures are an important part of maintenance, management, and durability design. One essential aspect of this is deterioration prediction. The purpose of this study is to theoretically examine the question of deterioration prediction using reliability theory. As an example, reliability theory is applied to the problem of scaling using survey data taken from exposure tests in a cold marine environment.

The exposed specimens have a concrete cover of 80 mm, and the scaling deterioration limit is assumed to be 20 mm. Hazard levels are set up at 1/10 and 1/20 of this limit, corresponding to 2 mm and 1 mm, respectively.

From this examination, we conclude that the Weibull distribution gives the best fit to the frequency distribution of scaling that exceeds the hazard level in depth. Covariate variables of the reliability function were extracted, and water-cement ratio and Fe_2O_3 were selected for study since their significance level was within 1%. The coefficient for water-cement ratio was found to be negative, while that for Fe_2O_3 was positive. The time taken for scaling to reach the hazard level was found to be approximately the same as required to reach the 50 % reliability level, thus confirming the validity of this method of reliability analysis.

1 Introduction

The prediction and evaluation of a concrete structure's serviceable life is essential to the maintenance and design of durable concrete structures. One aspect of this is the ability to predict the deterioration of concrete structures. The objective of this study is to predict the deterioration of concrete theoretically through reliability analysis, based on data regarding scaled concrete depth as measured during exposure tests in a cold marine environment.

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2. Details of study

2.1 Study method

2.2 Procedure

Figure 1 shows the procedure used in this study. First, we define deterioration and fix deterioration limits and "hazard" levels representing states prior to the limit state. Theoretical equations are also derived. Then measurements collected during the experiment described later (in Section 2.2) are input, and the main factors influencing deterioration are determined using multiple regression analysis. These deterioration factors are then treated as covariant variables, and those that reach a set "hazard" level are assumed to represent faults. Probability density functions are obtained using a reliability analysis (the analytical program SAS. LIFEREG); reliability functions are generated by integrating these probability density functions.

An appropriate probability density function and reliability function is selected. Finally, the shape of the reliability function is examined by varying the covariant variables of the reliability function and the change in reliability over time is discussed.

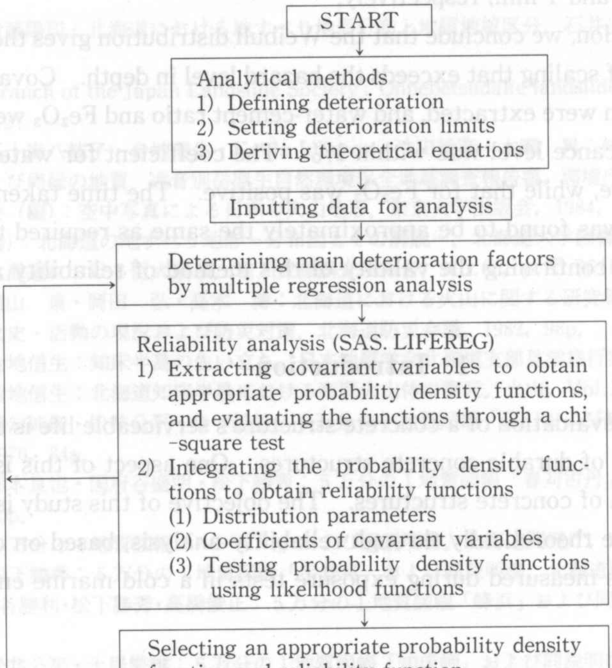

Figure 1 Study procedure

2.1.2 Setting deterioration limits

The concrete cover on specimens used in the exposure test and the allowable deterioration limit (the maximum permissible depth of scaled concrete) are assumed to be 80 mm and 20 mm, respectively. Table 1 lists the absolute depth of scaling corresponding to hazard levels of 1/1, 1/10, and 1/20 of the limit value. Hazard levels of 1/10 and 1/20 (2 mm and 1 mm) are adopted for analysis in this study.

Table 1 Set values (set up ratios) of deterioration limit

Hazard level	1/1	1/10	1/20
Scaled concrete depth (mm)	20	2	1

2.1.3 Theoretical equations for reliability analysis

There are three main probability density functions with the following probability distributions, and they were fitted as shown in Fig. 6.

(1) Exponential distribution

If events can be assumed to occur randomly in a poisson process, the time, t, (a random variable) at which an event first occurs obeys an exponential distribution given by the following equation:

$$\text{Probability density function: } f(t)=\alpha\exp(-\alpha t) \quad \cdots\cdots \text{Eq. (1)}$$

where $\alpha=\exp(-\mu)$, and μ is the product of the covariant vector and the unknown parameter vector described later in Eq. (9).

(2) Logarithmic-normal distribution

If there is a random variable, t, and log(t) obeys a normal distribution, the probability distribution of t is called a logarithmic-normal distribution. It is given by the following equation:

$$\text{Probability density function: } f(t)=\frac{1}{\sqrt{2\pi}\sigma t}\exp\left(\frac{(\log t-\mu)^2}{2\sigma^2}\right) \quad \cdots\cdots \text{Eq. (2)}$$

where σ is a scaling population parameter as described later.

(3) Weibull distribution [1]

If the variate representing the time of fault occurrence (the time when a hazard arises) is t, then assuming that the probability density function has a Weibull distribution (which is capable of representing frequently used engineering probability distributions) and taking into account the covariant variables, the reliability function is as given below.

These equations represent the relationships between the reliability function, population parameters of the probability density function, and the covariant variables.

$$\text{Reliability function: } R(t)=\exp\left[-\left(\frac{t}{\beta}\right)^{\alpha}\right] \quad \cdots\cdots \text{Eq. (3)}$$

where $\alpha=1/\sigma$, σ: scaling population parameter, and the unreliability function, F(t), is given by R(t) = 1−F(t).

$$\text{Probability density function: } f(t) = [F(t)]' = \frac{\alpha t^{\alpha-1}}{\beta^{\alpha}} \exp\left(-\left(\frac{t}{\beta}\right)^{\alpha}\right) \quad \cdots\cdots\text{Eq. (4)}$$

$$\beta = \exp(\mu) \quad \cdots\cdots\text{Eq. (5)}$$

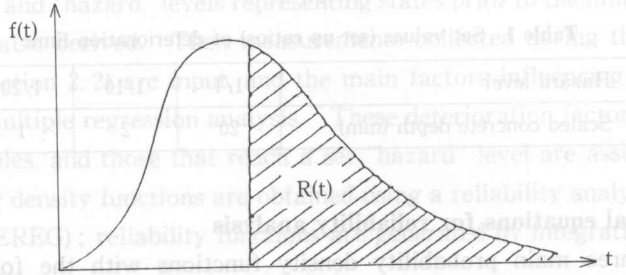

Figure 2 Relationship between probability density function f(t) and reliability function R(t)

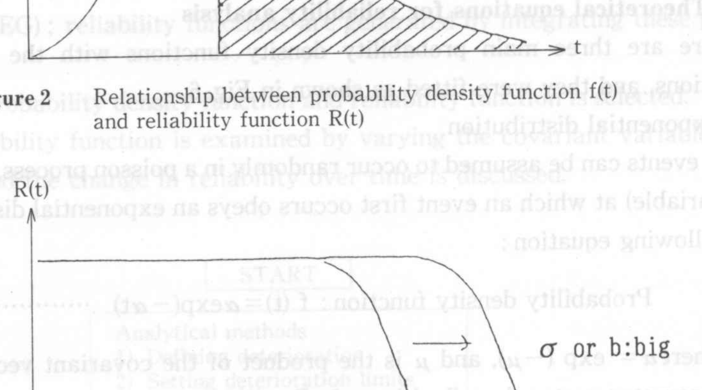

Figure 3 Relationship between shape of the reliability function R(t) and parameter σ

Figure 2 shows the relationship between the probability density function and the reliability function. The relationship between the shape of the reliability function and the parameters, σ and b, as shown Eq. (9), is such that R(t) increases as σ or b increases if $t/\beta \geqq 1$ and σ or $b > 0$, as shown in Figure 3.

To predict the probability that the scaled depth of concrete on a deteriorated concrete structure exceeds a particular value as time passes, a reliability analysis is conducted using LIFEREG. Let us regard the time of failure in LIFEREG as the number of years that pass before the scaled depth of concrete exceeds the hazard value, To. Then let the deterioration factors influenced by materials, composition, environment, etc. be X and the associated coefficient be b ; the time of failure associated with a specimen if these deterioration factors are zero (the number of years until the scaled depth of a specimen reaches the hazard value) be to : associated coefficients be a scaling population parameter, σ ;

Assume that the random variable, t, can be expressed as the logarithm of to, as given in Eq. (8). Also assume that the logarithm of To, or the number of years before scaled depth of concrete exceeds the set value, be T as given in Eq. (9). Then the this relationship is modeled by the following equation :

$$T=\mu+\sigma \quad t \qquad \text{Eq. (6)}$$

The variables can be expressed by the following equations in vector form :

$$T=\ln T_0 \qquad \text{Eq. (7)}$$

$$t=\ln t_0 \qquad \text{Eq. (8)}$$

where log denotes the natural logarithm.

A covariant vector, x, and an unknown paramete vector, b, are defined as follows :

$$\mu=b_0+\Sigma xb \qquad \text{Eq. (9)}$$

Selecting k factors of the covariant vector x, which may have high correlation and become main factors, through a multiple regression analysis, etc., letting associated unknown parameters be $b_1, \ldots, b_k$, and assuming that the covariant vector has n combinations, derive the following equation from Eq. (9)

$$\mu_i=b_0+\sum_{j=1}^{k} x_{ij}b_{ij} \qquad \text{Eq. (10)}$$

From Eq. (6), the t_i corresponding to the i-th data point in the n sets of data can be written as below.

$$t_i=(T_i-\mu_i)/\sigma \qquad \text{Eq. (11)}$$

The probability of the simultaneous occurrence of $t_1, \ldots, t_i, \ldots, t_n$ is generally given by the following equation.

$$f(t_1) \times \cdots \times f(t_i) \times \cdots \times f(t_n) \qquad \text{Eq. (12)}$$

The logarithmic likelihood function is defined using the unknown parameter, s, and $f(t_i)$ as a function of b to directly derive the optimum parameter vector.

$$L=\sum_{i=1}^{n} \log\{f(t_i)/\sigma\} \qquad \text{Eq. (13)}$$

To make use of data for scaled depths not exceeding the hazard value, a survival function, S(t), is used. The relationship between S(t) and f(t) is represented by the following equation :

$$f(t)=-\frac{\partial S(t)}{\partial t} \qquad \text{Eq. (14)}$$

The logarithmic likelihood function taking into account this survival function is given by the following equation :

$$L=\sum_{i=1}^{n}[\delta_i 1_n\{f(t_i)/\sigma\}+(1-\delta_i)1_n\{S(t_i)\}] \qquad \text{Eq. (15)}$$

where $d_i = 1$ for data where the scaled depth of concrete exceeds the hazard value, and zero for data which does not exceed the hazard value. The number of unknowns is (k+2) factors of $b_0, b_1, \ldots, b_k$.

The maximum likelihood estimate is given by the following equations:

$$\frac{\partial L}{\partial b_0}=\sum_{i=1}^{n}\left[\delta_i\frac{f'(t)}{f(t_i)}-(1-\delta_i)\frac{f(t_i)}{S(t_i)}\right]\frac{1}{\sigma}=0 \quad \text{Eq. (16)}$$

$$\frac{\partial L}{\partial b_j}=\sum_{i=1}^{n}\left[\delta_i\frac{f'(t)}{f(t_i)}-(1-\delta_i)\frac{f(t_i)}{S(t_i)}\right]\frac{X_{ij}}{\sigma}=0 \quad \text{Eq. (17)}$$

$$\frac{\partial L}{\partial \sigma}=\sum_{i=1}^{n}\left[\delta_i\frac{f'(t)}{f(t_i)}-(1-\delta_i)\frac{f(t_i)}{S(t_i)}\right]\frac{t_i}{\sigma}-\frac{m}{\sigma}=0 \quad \text{Eq. (18)}$$

where m is the number of not censored data for scaled depths exceeding the set value, and $b_0, b_1, \ldots, b_k$, and σ can be obtained using the Newton-Raphson method or similar.

2.1.4 Method of testing covariant variables

The significance of estimated coefficients, b, of the covariant variables is tested using the chi-square distribution. In this test, judgments are made on the basis of significance levels of 1% and 5%.[2]

2.2 Experimental method

2.2.1 Type of specimen

Tables 2 lists the types of specimens. Table 3 gives the specified mix proportions, properties (when fresh), and compressed strength of the various cements. Normal Portland cement (N), Fly-ash cement Types A, B, and C (FA, FB, and FC), and Portland blast furnace slag cement Class B (BB) were used in the experiments. Table 4 lists the properties and chemical compositions of the cements by cement class. In this table, C denotes cement type, W/C is the water-cement ratio, CC is the curing conditions, and CD is the curing period. In addition, F0 denotes that no curing took place, F5 is curing with fresh water sprays for 5 days, F14 is curing with fresh water sprays for 14 days, and S5 is curing with sea water sprays for 5 days after placement.[3]

Table. 2 Kind of specimens

No.	C•W/C•CC•CD	No.	C•W/C•CC•CD	No.	C•W/C•CC•CD	No.	C•W/C•CC•CD
1	N 55 F* 0	6	N 45 F 14	11	FB 55 S 5	16	BB 55 F 5
2	N 55 F 5**	7	FA 55 F 5	12	FB 45 F 5	17	BB 55 F 14
3	N 55 F 14	8	FB 55 F 0	13	FB 45 F 14	18	BB 55 S 5
4	N 55 S* 5	9	FB 55 F 5	14	FC 55 F 5	19	BB 45 F 5
5	N 45 F 5	10	FB 55 F 14	15	BB 55 F 0	20	BB 45 F 14

Note;

* ; F: Fresh water curing, S: Sea water curing

** : Specimen measuring temperature is the same mix proportion as N55F5 Specimen

C: cement, W/C: water cement ratio, CC: curing condition, CD: number of curing days,

N: Normal portrand cement, FB: Fly-ash cement type B, BB: Blast furnace cement type B

Table. 3 Type, property and chemical compound of cement

Case		Specific gravity	admixture (%)	Strength of cement (kgf/cm²)	Chemical compoun's cement (%)							
					ig.loss	insol	SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	SO_3
N	Normal portland cement	3.17	0	433	0.6	0.0	22.1	5.4	3.1	64.8	1.5	1.9
FA	Fly-ash cement Type A	3.09	8	—	0.3	4.6	21.8	5.2	2.9	61.5	1.4	1.4
FB	Fly-ash cement Type B	2.91	15	344	0.6	12.0	20.0	5.2	2.9	55.5	1.3	1.7
FC	Fly-ash cement Type C	2.49	22	—	0.6	14.0	16.6	5.7	2.4	47.4	1.3	1.8
BB	Blast-furnance slag cement	3.05	40*₁	368	0.6	0.1	25.0	9.1	1.8	55.8	3.4	2.6

Note ; chemical compound to report in CAJ 1974. *1 ; approximately

Table. 4 Mix proportion of concrete

Case	Type of cement	Specified mix					Property of fresh concrete			Compressive Strength				
		W/C	W	C	S	G	Slump	Air content	Temperature	Standard curing	In site curing (kgf/cm²)			
		(%)	(kg)	(kg)	(kg)	(kg)	(cm)	(%)	(℃)	(kgf/cm²)	F5	F0	S5	F14
N55	N	55	136	248	770	1166	6.6	5.6	27	280	198	238	240	270
N45	N	45	135	300	718	1177	3.9	4.5	28	372	—	340	—	316
FA55	FA	55	123	224	790	1194	4.3	6.5	22	298	—	259	—	—
FB55	FB	55	122	222	790	1194	8.5	4.3	22	254	207	250	247	268
FB45	FB	45	123	274	790	1205	8.0	5.0	24	325	—	263	—	288
FC55	FC	55	121	220	790	1194	6.5	4.3	21	231	—	222	—	—
BB55	BB	55	128	233	780	1182	2.9	4.0	25	258	192	225	196	211
BB45	BB	45	130	289	722	1186	6.0	3.0	23	338	—	285	—	311

2.2.2 Exposure conditions

The specimens were exposed at a test site 40 m from the shoreline. The average number of freeze-and-thaw cycles at the test site is about 57 annually.

2.2.3 Method of measurement

Figure 4 gives the dimensions of a large specimen at Monbetsu. Measurements were made on the top, and at upper, middle, and lower points on the north, south, east, and west faces for each specimen. Scaled depth was measured after the type of scaling was classified as either scaling or pop-up. Up to 35 measuring points were selected where scaling type occurred and 12 for pop-up. To accurately trace the changes in scaled depth with time, measurements were taken at exactly the same measuring point annually.[3]

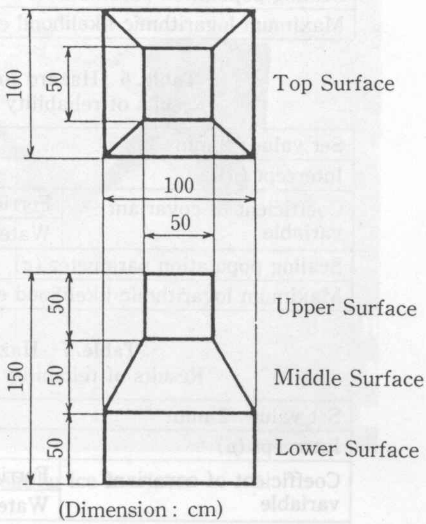

Fig. 4 Form of large specimens of Monbetsu

3. Results of analysis and discussion

3.1 Results of analysis

3.1.1 Deterioration factors regarded as covariant variables

A multiple regression analysis was carried out to extract the deterioration factors which had relatively high correlation and which could be regarded as covariant variables in the subsequent reliability analysis. The factors extracted included water-cement ratio, compressive strength of concrete cured in the field for 28 days, chemical constituents of the cement, such as CaO, SiO_2, Fe_2O_3, Al_2O_3, and MgO, and the height of measuring points above the ground. Tables 5, 6, and 7 list the results of reliability analysis carried out using the exponential distribution, logarithmic-normal distribution, and Weibull distribution, respectively, for a set value of 2 mm.

On the basis of these tables, water-cement ratio and ferric oxide content were extracted as covariant variables with coefficients within a significance level of 5%. The coefficient of the covariant water-cement ratio was negative, and that of ferric oxide positive. Both coefficients were within a significant level of 1%. The result agrees with the observed result that the durability of concrete falls as the water-cement ratio increases. It also agrees with the fact that cement containing more ferric oxide, i.e. cement with a lower iron modulus (Al_2O_3/Fe_2O_3), produces more C_4AF and exhibits better resistance to corrosion in sea water[4].

Table. 5 Hazard for exponential distribution:
Results of reliability analysis at a scaled depth of 2 mm

Set value: 2 mm		Estimated values	PR>CHI
Intercept (μ)		3.8238	0.0001
Coefficient of covariant variable	Ferric oxide	0.3438	0.0030
	Water-cement ratio	−0.4811	0.0014
Scaling population parameter (σ)		1.0000	—
Maximum logarithmic likelihood estimation		−332.3259	

Table. 6 Hazard for logarithmic normal distribution:
Results of reliability analysis at a scaled depth of 2 mm

Set value: 2 mm		Estimated values	PR>CHI
Intercept (μ)		3.4600	0.0001
Coefficient of covariant variable	Ferric oxide	0.3348	0.0001
	Water-cement ratio	−0.0456	0.0001
Scaling population parameter (σ)		0.6990	—
Maximum logarithmic likelihood estimate		−287.0354	

Table. 7 Hazard for Weibull distribution:
Results of reliability analysis at a scaled depth of 2 mm

Set value: 2 mm		Estimated values	PR>CHI
Intercept (μ)		3.3448	0.0001
Coefficient of covariant variable	Ferric oxide	0.2665	0.0001
	Water-cement ratio	−0.3444	0.0001
Scaling population parameter (σ)		0.5545	—
Maximum logarithmic likelihood estimation		−284.0681	—

Photo 1. Exposed specimens in summer

Photo 2. Exposed specimens and floating ice in winter

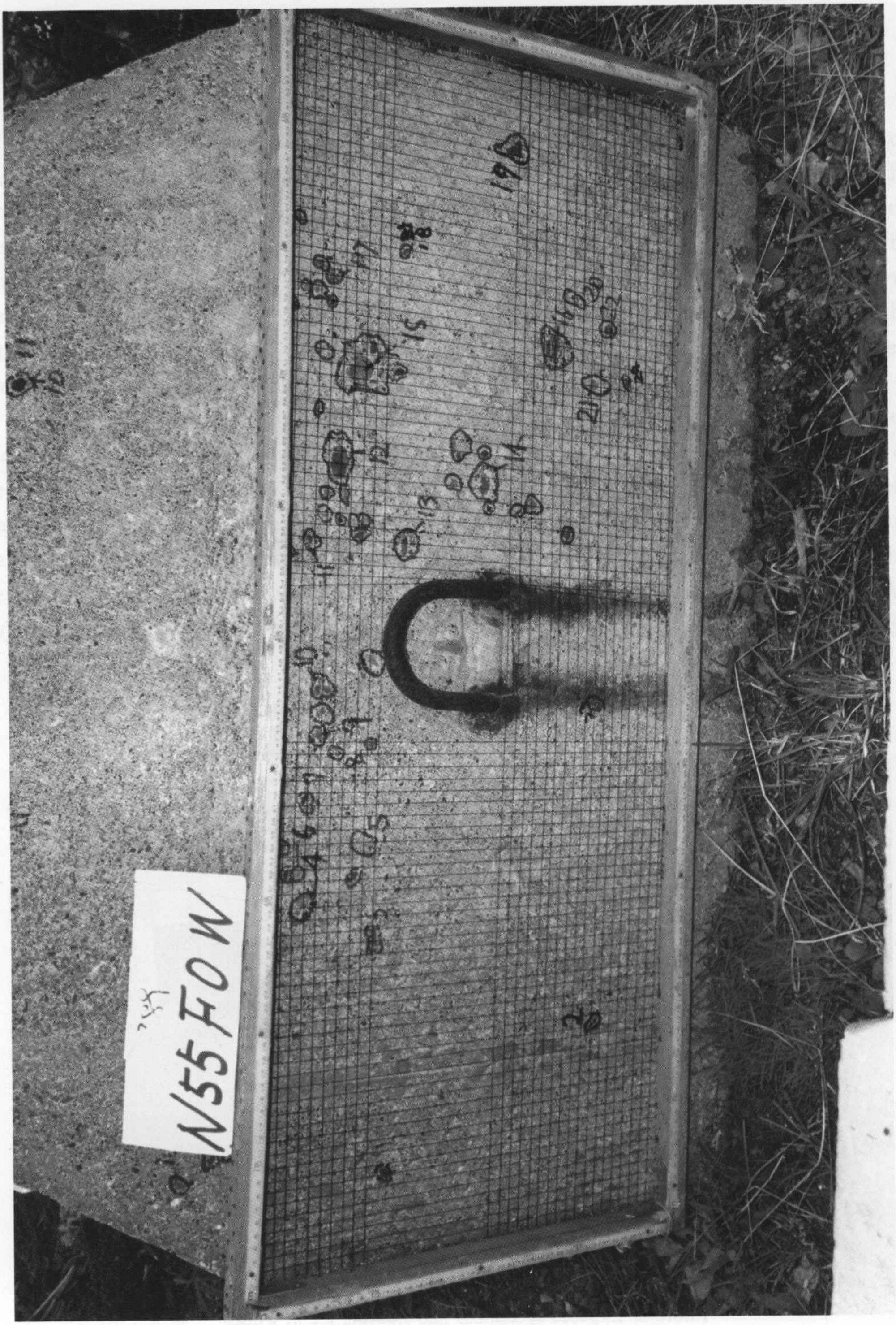

Photo 3. Surface scalling on specimens at Lower surface

3. 1. 2 Selection of suitable probability density function

Figure 5 shows the frequency distribution for scaled depth exceeding the hazard value of 2 mm. Figure 6 shows the probability density functions based on the exponential distribution, the logarithmic-normal distribution, and the Weibull distribution. Figure 5 shows a high frequency at 10 years (the exponential distribution, logarithmic-normal distribution) and the Weibull distribution in Figure 6 also has a peak at 10 years; that is, the Weibull distribution fits best.

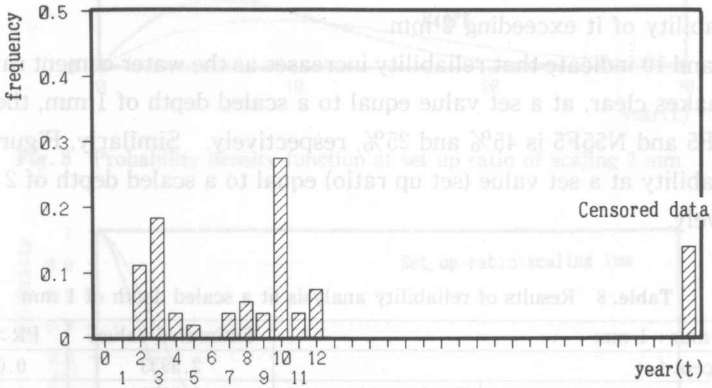

Fig. 5 The frequency distribution of scaling exceeded set up ratio of scaling 2mm

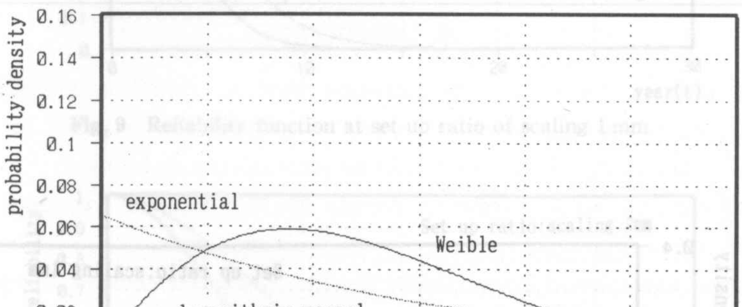

Fig. 6 Probalility density function of exponential distribution, logarithmic normal distribution and Weible distribution

3. 2 Discussion

3. 2. 1 Prediction of deterioration based on results of reliability analysis

Table 8 shows the results of a reliability analysis carried out using a hazard value equivalent to a scaled depth of 1 mm. The significance levels of the coefficients for ferric oxide and water-cement ratio were within 1% at the set value. As is clear from 3.1.1, water-cement ratio is one of the deterioration factors influencing reliability. Therefore, we decided to predict deterioration by focusing on one particular covariate: water-cement ratio.

Figures 7 and 8 show the probability density functions of test specimens N45F5 and N55F5 for set values equal to scaled depths of 1 and 2 mm (as listed in Table 5) respectively. Figures 9 and 10 show the reliability functions of the same specimens at the same set values.

As Figure 7 makes clear, the probability density functions for N45F5 and N55F5 at a set value equal to a scaled depth of 1 mm peak at 3.5 and 2.5 years, respectively. And Figure 8 shows that the probability density functions for N45F5 and N55F5 for a set value equal to a scaled depth of 2 mm reach their peaks at 10 and 6 years, respectively. Accordingly, it can be concluded that the probability of scaling exceeding the set value of 1 mm peaks earlier than the probability of it exceeding 2 mm.

Figures 9 and 10 indicate that reliability increases as the water-cement ratio is decreased. As Figure 9 makes clear, at a set value equal to a scaled depth of 1 mm, the reliability at 5 years for N45F5 and N55F5 is 45% and 25%, respectively. Similarly, Figure 10 shows that the 5-year reliability at a set value (set up ratio) equal to a scaled depth of 2 mm is 85% and 35%, respectively.

Table. 8 Results of reliability analysis at a scaled depth of 1 mm

Set value : 1 mm		Estimated values	PR>CHI
Intercept (μ)		2.5933	0.0001
Coefficient of covariate	Ferric oxide	0.2276	0.0004
	Water-cement ratio	−0.0351	0.0001
Scaling population parameter (σ)		0.5944	—
Maximum logarithmic likelihood estimate		−261.5004	—

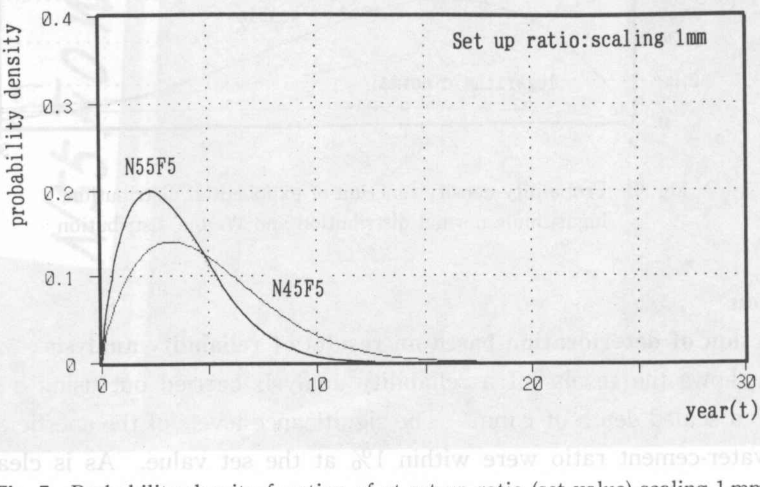

Fig. 7 Probability density function of at set up ratio (set value) scaling 1 mm

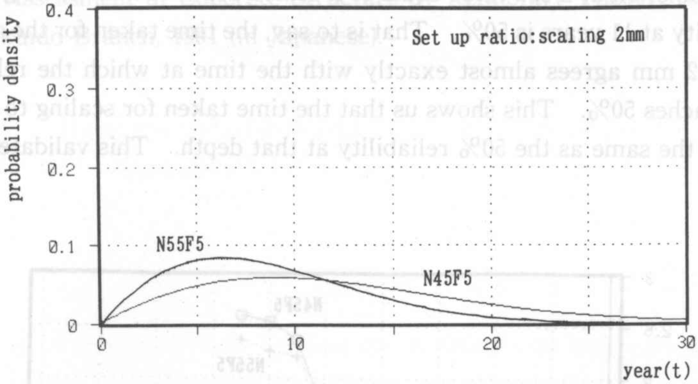

Fig. 8 Probability density function at set up ratio of scaling 2 mm

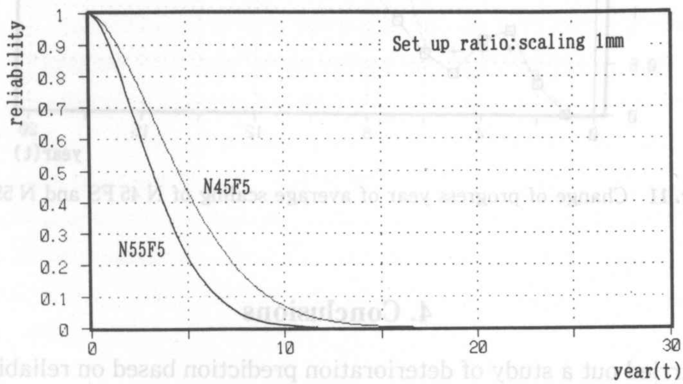

Fig. 9 Reliability function at set up ratio of scaling 1 mm

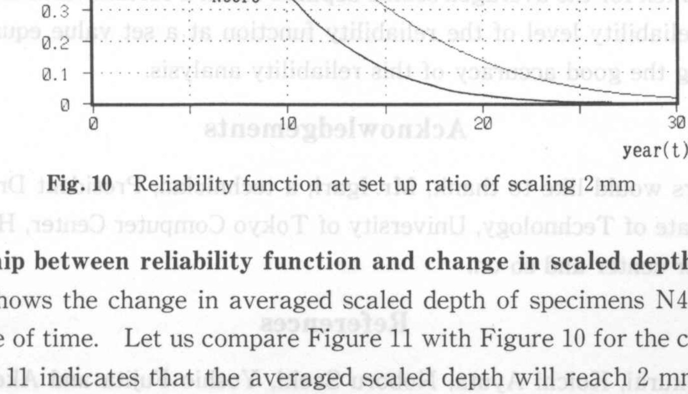

Fig. 10 Reliability function at set up ratio of scaling 2 mm

3.2.2 Relationship between reliability function and change in scaled depth with time

Figure 11 shows the change in averaged scaled depth of specimens N45F5 and N55F5 with the passage of time. Let us compare Figure 11 with Figure 10 for the case of specimen N45F5. Figure 11 indicates that the averaged scaled depth will reach 2 mm in 10.5 years.

The reliability function at a set value equal to a scaled depth of 2 mm in Figure 10 indicates that the reliability at 11 years is 50%. That is to say, the time taken for the averaged scaled depth to reach 2 mm agrees almost exactly with the time at which the reliability at that hazard level reaches 50%. This shows us that the time taken for scaling to reach a certain depth is almost the same as the 50% reliability at that depth. This validates the reliability analysis.

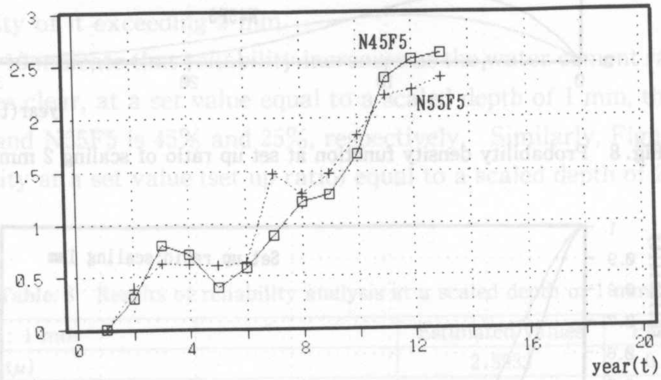

Fig.11 Change of progress year of average scaling of N 45 FS and N 55 F5

4. Conclusions

We have carried out a study of deterioration prediction based on reliability theory using experimental data on scaled depth of concrete for specimens exposed to a cold marine environment. The results of this study can be summarized as follows:

(1) A Weibull distribution of probability density fits well with the frequency distribution of scaled depth exceeding a set, or hazard, value (set up ratio).

(2) As covariant variables of the reliability function, the water-cement ratio and ferric oxide content were extracted at significance levels of less than 1%. Regarding the coefficients of these covariate, that for water-cement ratio is negative and that for ferric oxide positive.

(3) The time taken for the averaged scaled depth to reach a certain value is almost the same as the 50% reliability level of the reliability function at a set value equal to that depth, demonstrating the good accuracy of this reliability analysis.

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