Concrete Durability Assessment for Scaling by Reliability Theory*

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Abstract

The proper prediction of concrete deterioration is important for estimating and evaluating the service life of concrete structures and for maintaining and managing them. It is necessary to establish the limit of deterioration in management (the deterioration limit) and understand the probability of exceeding that limit.

In this study, the introduction of reliability theory to predict and evaluate concrete deterioration was tried in order to determine if it could be used for these purposes, predicting surface scaling, which is a typical form of surface deterioration of concrete structures located in cold regions and an important factor in the maintenance of the beauty of value-added concrete structures, and the deterioration of cover concrete of steel.

1. INTRODUCTION

The proper prediction of concrete deterioration is important to estimate and evaluate the service life of concrete structures and to maintain and manage concrete structures. The establishment of limits of deterioration in management (the deterioration limit) and an understanding of the probability that these limits will be exceeded are needed.

In this study, reliability theory to predict and evaluate concrete deterioration was introduced to determine if it could be used for these purposes.

2. METHOD

2. 1 Process of Study

The Process of Study is shown in Fig. 1.

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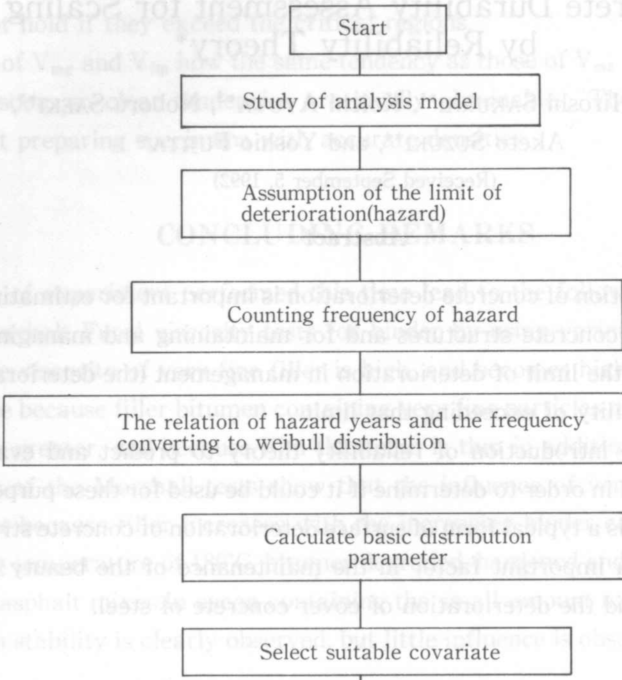

Fig. 1 The method of study

2. 2 Analysis Method

2. 2. 1 Analysis theory

(1) Reliability function

The probability density function (PDF) of lifetime distribution, f(x), and the failure (hazard) rate function is assumed as follows:

The average hazard rate:

λ_i = (The number of hazards in the interval: i)/(The number of survivors when i begins) ......Eq. (1)

The hazard rate over a very short period, dt, is expressed as a function of t: f(t). The relationship between λ(t) and the PDF of the lifetime distribution, f(t), is expressed as follows:

$$\lambda(t) = \frac{f(t)}{\int_t^{\infty} f(x)\,dx} \quad \text{......Eq. (2)}$$

As in the above equation, if the PDF of the lifetime distribution, f(t), is obtained, the instant failure rate function (hazard rate function), λ(t), can be induced arbitrarily.

If the PDF of the lifetime distribution is expressed as f(t), the probability of realization of more than duty time (up time), t_0, is called reliability, $R(t_0)$. The $R(t_0)$ is expressed as follows:

$$R(t_0) = \int_{t0}^{\infty} f(x)\,dx \quad \text{Eq. (3)}$$

Reliability can be expressed as a function of time, t and R(t). It is called the reliability function. On the other hand, the unreliability function, F(t), which cannot survive in the required time is expressed as follows:

$$F(t) = 1 - R(t) = \int_0^t f(x)\,dx \quad \text{Eq. (4)}$$

In the equation (2), the variable, t, of both sides of this is replaced with x. Further, the equation (3) is put into this. And the equation is integrated 0 to t as follows:

$$\int_o^t \lambda(x)\,dx = \int_0^t \frac{f(x)\,dx}{R(x)} \quad \text{Eq. (5)}$$

The expression of R(t) for equation (5) is obtained as follows:

$$R(t) = \int_t^{\infty} f(x)\,dx = exp\,(-\int_0^t \lambda(x)\,dx) \quad \text{Eq. (6)}$$

Both sides of this equation are differentiated by x as follows:

$$f(t) = -\lambda(t) \cdot exp\ (-\int_0^t \lambda(x)\,dx) \quad \text{Eq. (7)}$$

As in the above expression, the PDF of the lifetime distribution can be expressed by the hazard rate function, $\lambda(t)$.

The cumulative hazard function, H(t), is expressed with the hazard rate function, $\lambda(t)$, as follows:

$$H(t) = \int_0^t \lambda(t)\,dt \quad \text{Eq. (8)}$$

The relationship between the reliability function, R(t), and the cumulative hazard function H(t), can be expressed as follows:

$$R(t) = exp\,(-\int_0^t \lambda(x)\,dx) = exp\,(-H(t)) \quad \text{Eq. (9)}$$

(2) Accelerated hazard model

The accelerated hazard model shows that the factor X affects the time to failure, making it longer or shorter than the standard time to failure, t_0.

In this case, the logarithmic linear model can be applied. The accelerated hazard model, in which the factors affecting the hazard rate, $\lambda(t)$, are the covariate, X, and the vector, X, this constitutes a multiple covariate, and is expressed as follows:

$$\lambda(X, t) = a \cdot \lambda_0(t) \cdot \psi(X) \quad \text{Eq. (10)}$$

where $\lambda_0(t)$ is the standard hazard rate function and $\psi(x)$ is the function of vector X. For the purpose of simplicity, a = 1.

In this case, hazard rate function is assumed to be the Weible distribution for two population parameters. This is because the function is commonly used as a hazard rate function and it closely matches the distribution of variables with no minus values and asymmetric distribution. It is expressed as follows:

$$F(t) = exp\,(-(\frac{t}{\beta})^{\alpha}) \quad \text{Eq. (11)}$$

where, α is the shape population parameter and β is the scale population parameter. The PDF of Equation (11) is expressed as follows:

$$f(t) = \frac{\alpha}{\beta} \cdot \left(\frac{t}{\beta}\right)^{\alpha-1} exp\left(-\left(\frac{t}{\beta}\right)^{\alpha}\right) \quad \text{Eq. (12)}$$

Accordingly, the standard hazard function, $\lambda_0(t)$, is expressed as follows:

$$\lambda_0(t) = \frac{\alpha}{\beta} \cdot \left(\frac{t}{\beta}\right)^{\alpha-1} exp\left(-\left(\frac{t}{\beta}\right)^{\alpha}\right) \quad \text{Eq. (13)}$$

where, α is the shape population parameter and β is the scale population parameter. As well, $\psi(X)$ is the exponential distribution and the regression coefficient, b, is the vector b.

$$\psi(X) = exp(X'b) = exp(b_1 \cdot X_1 + b_2 \cdot X_2 + \cdots\cdots + b_n \cdot X_n) \quad \text{Eq. (14)}$$

This model is called the accelerated hazard model because the covariate affects the change of the scale of a standard hazard function. In this model, for each covariate, it is assumed that the effect is multiplication of time to failure. If the random variable: t_0 is the time to failure for a sample in which the covariate: $X = 0$, with a standard hazard distribution, the time to failure, t, with a covariate vector, X, is assumed to be as follows:

$$t = exp(X'b) \cdot t_0 \quad \text{Eq. (15)}$$

where, $Y = \ln(t)$ and $Y_0 = \ln(t_0)$, then the expression is as follows:

$$Y = Xb + Y_0 \quad \text{Eq. (16)}$$

The equation (16) is a logarithmic linear model with an error term, Y_0. The above model includes the constant (intercept) term and the coefficient indicating scale. In the time to failure before converting the logarithmic function, the constant term affects the change of scale and the coefficient indicating the scale multiplies the time to failure. Therefore, the equation is assumed with the coefficients, μ, and σ as follows:

$$Y = \mu + \sigma Y_0 \quad \text{Eq. (17)}$$

The relation t and t_0 is expressed as follows:

$$t = exp(\mu) \cdot t_0^{\sigma} \quad \text{Eq. (18)}$$

The accelerated hazard model is expressed with the probability of survival (the probability of no hazard) as follows:

$$Prob(t > t|X) = Prob(t_0 > (exp(-x'b))\,t) \quad \text{Eq. (19)}$$

The left side of equation (19) is the rate which is calculated in the assumption at the covariate X. The right side of the equation is the estimate which is calculated as the hazard distribution after multiplying time, t, by a constant with the value of the covariate.

Thus, because of the relationship between equation (13) and equation (14) the relation between the coefficients can be expressed as follows:

$$\sigma = 1/\alpha \quad \mu = l\eta\beta \quad \text{Eq. (20)}$$

(3) Estimation of parameters by degree of most likelihood

The time to failure (lifetime), including data with no hazards in the observed time (censored on right) is assumed. When estimating the parameters, a method is used which sets the likelihood function with censored data. This function is maximized. The censoring is done given that the hazard data follows the PDF and the censored data follows the reliability function. If R(t) and f(t), in which i is assumed to be an indicator of hazard ($\delta i = 1$) and censoring ($\delta i = 0$), the extreme value of the distribution is converted, and the likelihood function is expressed as follows:

$$L(b, \sigma) = \Pi_{i=1,n} f(t_i)^{\delta_i} R(t_i)^{1-\delta_i} \quad \cdots\cdots (21)$$

where, equation (21) is converted the logarithm likelihood function.

Then the parameters: b and σ, which are calculated to the extreme value of equation (19), is estimated with the first partial differential by b, σ: $\partial \ln L(b, \sigma)/\partial b$, $\partial \ln L(b, \sigma) / \partial \sigma$ and the second partial differential: $\partial^2 \ln L(b, \sigma) / \partial b_1 \partial b_h$, $\partial^2 \ln L(b, \sigma) / \partial \sigma^2$ and $\partial_2 \ln L(b, \sigma) / \partial bj \partial \sigma$ and by the Newton-Lapson method.

In addition, examination of the null hypothesis of b is done by examination of chai squarem, χ_2, distribution.

2. 3 Introducing a Method for Assessing the Durability of Concrete

2. 3. 1 Object

In this study, the introduction of reliability theory for assessing concrete durability was attempted using an example for predicting surface scaling. Surface scale is a typical surface deterioration of concrete structures located in cold regions and an important factor in the aesthetics of concrete studies which are nowadays often demanded as a value-added feature.

2. 3. 2 Assumption of the set rate

The event exceeding a level of the scaling area ratio was assumed to be the hazard. Then, the deterioration limit was set at 25 % of the scaling area ratio. It is assumed that the occurrence of the hazard is 2.5 %, which is one-tenth of the deterioration limit, and a repair is required. This is assumed to be the setting up rate. Accordingly, the event exceeding the setting up rate: 2.5 % is analyzed with reliability theory.

2. 3. 3 Data studied and the method of calculation

The data studied is the test data of exposed specimens which were tested in a cold marine environment (Monbetsu city in North Japan) from 2 to 11 years, as shown in Table 1. This analyzed data was calculated to estimate σ, μ and b of the mathematical parameter as shown in 2. 2. 1. The examination of the analyzed results is that of the degree of logarithmic likelihood which is effective for small sample populations.

Therefore, calculations were done using the LIFEREG procedure of the SAS (Statistical Analysis System).

Table. 1 The external and internal factor of specimens

External factor		Internal factor					
Cycles of freeze-thaw per year (cycles)	Distance from sea (sea side) (m)	No.	C•W/C•CC•CD	No.	C•W/C•CC•CD	No.	C•W/C•CC•CD
59.4	From 30 to 50	1	N 55 F* 0	7	FB 65 F 0	13	BB 55 F 0
(8 years average)	(Seasonal change)	2	N 55 F 5**	8	FB 55 F 5	14	BB 55 F 5
		3	N 55 F 14	9	FB 55 F 14	15	BB 55 F 14
		4	N 45 S* 5	10	FB 45 F 0	16	BB 45 F 0
		5	N 45 F 5	11	FB 45 F 5	17	BB 45 F 5
		6	N 45 F 14	12	FB 45 F 14	18	BB 45 F 14

*; F : Fresh water curing, S: Sea water curing

** ; Specimen measuring temperature in the same mix proportion as N55F5 Specimen.

Note; C: Cement, W/C: Water cement ratio, CC: Curing condition, CD: Number of curing days
N : Normal portland cement, FB: Fly ash cement type B, BB: Blast furnace cement type B

3. RESULTS

A histogram of the frequency exceeding the set-up rate in each lapse year of all specimens is shown in Fig. 2. And the data, which did not exceed the set-up rate until 11 years (censored data) is shown to the right in Fig. 2. The data which did not exceed the set-up rate was included and analyzed as the censored data. The data exceeding the set-up rate were numerous in the early period and tended to decrease in later years.

The results of the analysis are shown in Table 2. The parameters and the examination rate of chai square to the Weible distribution without covariates and those with covariates are shown in Table 2. In the cases of some covariate combinations, the case, which the level of significance of σ and coefficient b was within 1 % of the null hypothesis and the degree of likelihood was high was selected as Case-4. The relation of the estimate quantile value year of the reliability (the hazard rate) 50 %, which is estimated with the parameter and the coefficient selected by Equation (10) and Equation (9), and the measured year is shown in Fig. 3.

The lapse year and the shapes of the hazard PDF, f(t), and the reliability function, in which the set-up rate (hazard) analyzed were 1 mm and 2 mm at the scaling depth, are shown in Fig. 4. When the set-up rate was at 1 mm, the peak of the PDF was at 8 years. The reliability decreases from about 5 years, radically in 8 years and to nearly zero in 10 years. In the set-up rate at 2 mm, the peak of the PDF was 17 years, which is apparently quite different from the case of 1 mm.

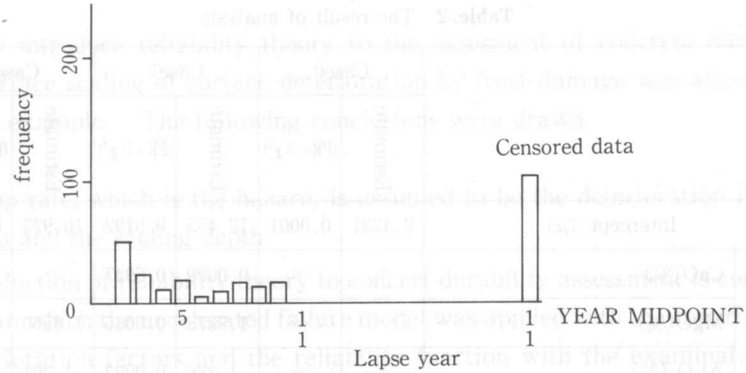

Fig. 2 Histgram of lapse year and the frequency

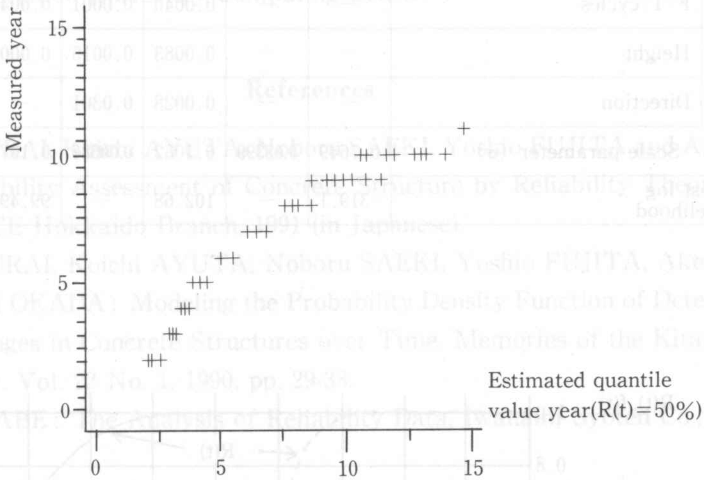

Fig. 3 The relationship between of estimated quantile value year (R(t)=50 %) and measured year

Table. 2 The result of analysis

		Case0		Case1		Case4	
		Estimate	(PR : $>\chi^2$)	Estimate	(PR : $>\chi^2$)	Estimate	(PR : $>\chi^2$)
Intercept (μ)		2.4231	0.0001	12.455	0.0192	10.923	0.0014
Covariate parameter (b)	CaO(%)	—	—	0.0099	0.6947	—	—
	MgO(%)	—	—	1.9373	0.0056	1.8560	0.0040
	Al_2O_3(%)	—	—	-1.386	0.0093	-1.291	0.0030
	Fe_2O_3(%)	—	—	-1.320	0.0492	-1.311	0.0015
	w/c(%)	—	—	-0.046	0.1443	-0.033	0.0040
	Curing water	—	—	-0.011	0.1629	—	—
	Curing day	—	—	-0.065	0.0001	-0.065	0.0001
	28 days Strength	—	—	-0.019	0.0014	-0.017	0.0004
	Surface Strength	—	—	0.0813	0.0001	0.0801	0.0001
	F-T/cycles	—	—	0.0040	0.0001	0.0040	0.0001
	Height	—	—	0.0083	0.0015	0.0007	0.0042
	Direction	—	—	0.0028	0.0301	—	—
Scale parameter (σ)		0.7649	0.05399	0.1052	0.00586	0.1074	0.0060
Most log likelihood		-319.1	—	102.68	—	99.493	—

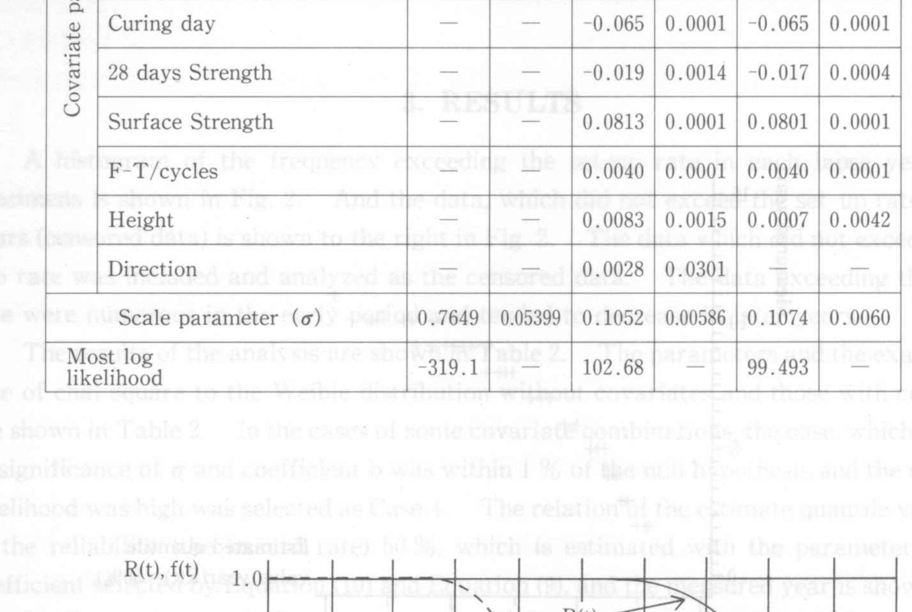

Fig. 4 Between lapse year and reliability function (R(t)) and probability density function (f(t)) to hazard of the average scalling depth (1 mm and 2 mm)

4. CONCLUSION

In order to introduce reliability theory to the assesment of concrete durability the prediction of surface scaling of surface deterioration by frost damage was attempted with reliability as an example. The following conclusions were drawn.

(1) The set-up rate, which is the hazard, is assumed to be the deterioration limit of the scaling ratio and the scaling depth.

(2) The introduction of reliability theory to concert durability assessment is confirmed by the method in which the accelerated failure model was applied with the covariate of each of the deterioration factors and the reliability function with the examination of chai squares and that of the degree of likelihood.

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