北見工業大学研究報告　Memoirs of the Kitami Institute of Technology
第20巻　第2号（昭和63年）　Vol. 20 No. 2 (1988)

Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 15)*

—Consideration of Minimum Shut-down Time Constraints—

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(Received October 13, 1988)

Abstract

An outline of already reported scheduling methods which can satisfy environmental constraints and give maximum reliability is briefly described. This scheduling method is expanded to consider the minimum shut-down time of the thermal power units in this paper. To satisfy a constraint of the minimum shut-down time, start-stop schedules are modified to improve themselves so that the thermal units at least shut-down during the minimum shut-down time, or not shut-down at all.

Using simulation results of a model power system, an usefulness of the proposed method is shown. Variations of the power supply probability and the computing time are investigated.

1. Introduction

We have continually reported the optimum scheduling method of the thermal units1). The scheduling method was considered the optimum start-stop operation for the thermal units satisfying the environmental constraints.

But the thermal units can not always re-start easily, because they require enough time to prepare for start-up. An example of the preparations to re-start is the warming up of the boiler and steam turbine, the length of which is usually about 2～6 hours depending on the types of thermal unit. This required preparation period is known as a minimum shut-down time, and the thermal units must be scheduled to satisfy this. We have already reported a scheduling method to satisfy the minimum shut-down time.

In this report, a proposed control method of line power flows is described in the first place to improve reliability, which has previously been developed2). Output power of two thermal units is controlled to satisfy the line capacities in this method. An estimation method is introduced to calculate the system state and decide the optimum start-stop schedule of the thermal units3). An already reported method is applied to this proposed method to satisfy the minimum shut-down time. When the start-stop schedules are modified to maximize reliability, they are all obtained by the least modification to the original schedule and can satisfy the minimum shut-down time.

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* Presented at I. E. E. J. Power Engineering Symposium (July 1988).

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As a typical example of environmental pollution, the nitrogen oxide (NO_x) emission from each thermal unit is considered. The usefulness of the proposed method is confirmed through various simulations in a model power system. Influences of the minimum shut-down time are shown concretely.

2. Economic Power Flows

The power flow $[i]$ of each line becomes eq. (1)[4] by the DC method.

$$[i]=[e]\,[Pb] \tag{1}$$

Where, $[i]$ is the column vector whose element i_l is the power flow of the l-th transmission line. $[e]$ is a sensitivity matrix[5], and $[Pb]$ is the column vector whose element $[Pb_n]$ represents the power of the n-th bus.

The economic power flows can be obtained by eq. (1) when the thermal output power is decided by the law of equal incremental fuel cost.

3. A Satisfaction of Line Capacities

If some flows among the economic power flows exceed the line capacities, then the outputs of two thermal units are adjusted to satisfy the line capacities in order to achieve speedy control rather than strict economy.

The increase of the power flow in the l-th line Δi_l becomes eq. (2) because of eq. (1) when the m'-th and m''-th units change their output power by $\pm\Delta g$.

$$\Delta i_l=(e_{lm'}-e_{lm''})\cdot\Delta g \tag{2}$$

That is, when the l-th power flow must be increased by Δi_l to satisfy the capacity, then the proper pair of thermal units can modify their output by Δg. The unit pair is chosen so that the absolute value of Δg becomes minimum. This indicates that the unit pair can be chosen independently of the value and the sign of Δi_l.

In the proposed method, the unit pair is chosen by weighted elements of the sensitivity matrix when overflow occurs on many lines, and Δi_l is used as the weight by which the power flow of each overflow line should be increased to satisfy the capacity. That is, the unit pair is composed of the greatest unit and the least unit as to E of eq. (3).

$$E=\sum_l \Delta i_l\cdot e_{lm} \tag{3}$$

Where, $\sum$ on the right hand of eq. (3) means the summation of the overflow lines.

The adjusted value by eq. (2) may introduce thermal output outside the lower or upper limit of power. Furthermore, even if the unit pair changes its output by estimated value, the power flow of a few lines may exceed the line capacities more than before the output change. In that case, the power adjustment may have to be repeated.

4. Satisfaction of Both the Environmental Constraints and the Line Capacities

In the proposed method, in order to achieve rapid estimation and because the probability of a system state whose elements are all available is greatest, the NO_x emission is controlled only by this all-up state to satisfy the expected final value of the NO_x emission constraint. The other system states are operated considering only line capacity without regard to the emission.

NO_x emission of the m-th thermal unit is denoted by y_m and μ is the LaGrange's multiplier concerned with the emission constraint. The emission characteristics of a thermal unit can be evaluated by dy_m/dg_m and a stricter emission constraint introduces a negative lower value of μ because of the Kuhn-Tucker conditions, eq. (3) is modified to eq. (3)′ using the weight of $-\mu$.

$$E=\sum_{l} \Delta i_l \cdot e_{lm} - \mu \cdot \frac{dy_m}{dg_m} \qquad (3)'$$

When the emission constraint must be considered, eq. (3)′ is used to decide the proper unit pair whose output should be modified instead of eq. (3).

5. A Recursive Representation of the State Probability for Start-stop Thermal Units

The state probability of a thermal unit must be considered separately depending on the different kinds of scheduled states. They are the operating states before scheduled shutdown, the scheduled shutdown state and the operating state after re-startup.

The up-state probability becomes eq. (4) when the thermal unit has not yet shut down.

$$s(s_0, \text{on}, t_1) = s_0 \cdot e^{-(p_{\text{on}}+q_{\text{on}})t_1} + \frac{q_{\text{on}}}{p_{\text{on}}+q_{\text{on}}} \cdot \left(1-e^{-(p_{\text{on}}+q_{\text{on}})t_1}\right) \qquad (4)$$

Where, s_0 is the initial condition of the up state at $t=0$. p_{on} and q_{on} are the failure rate and the repair rate during scheduled operations respectively, t_1 is the period of continuous operation.

When the thermal unit is shutting down during t_2 after continuous operation during t_1, its up state probability becomes $s(s(s_0, \text{on}, t_1), \text{off}, t_2)$ recursively. In the above, p_{off} and q_{off} are the failure rate and the repair rate during scheduled shutdown respectively.

p_{up} denotes the frequency of an outage in the period of transition leading to re-start up. When the thermal unit is re-started up and operated during t_3 after shutdown during t_2, the up-state probability is also introduced by $s((1-p_{\text{up}}) \cdot s(s(s_0, \text{on}, t_1), \text{off}, t_2), \text{on}, t_3)$ recursively.

Since the state probability after re-start depends on the period of the shut-down t_2, the Markov-like characteristic is not maintained in a start-stop schedule of the thermal units. But when the following assumptions can be made, the influence of a shutdown period can be ignored and the Markov-like characteristic can be recognized to be approximately maintained.

1. The start up operation is usually successful, that is $0 < p_{up} \ll 1$.
2. The mean time of the up state is usually sufficiently longer than the mean time of the outage state during shutdown, that is $0 < p_{off} \ll q_{off}$.
3. $s(s_0, \text{on}, t_1)$ is usually near to 1 because it is the up probability at t_1.
4. q_{off} is small because sufficient time is needed to repair the thermal unit.

6. A Consideration for a Practical Optimal DSS Operation

The usual meaning of the 'economic operation' is the cheapest operation which can satisfy each constraint[6,3]. And the operating cost is estimated by the fuel cost of the thermal units.

But when various system states are probabilistically considered by estimating each apparatus outage in the power system, and the minimum expected value of the operating cost is required, then the power supply probability tends to become low. Thus the minimum cost is usually accompanied by the lowest reliability. It is doubtful that this scheduled operation is practical, because it may cut the power supply to reduce the fuel cost.

The operating fund is mainly supplied by the income from charges for the supply of electricity by the power system. The fuel cost of the thermal units forms only a part of total necessary costs. There are many kinds of other costs, the cost of equipment, the cost of its maintenance, staff salaries, etc., which are necessary irrespectively of whether electric power is supplied or not. That is, practical economics should consider the income from electricity charges, which is only obtained when electric power is supplied.

When practical economics is strictly mathematically considered, the individual charge paid by each consumer must be estimated in the scheduling step, because such economics is the balance of income and costs. This requires the anticipation of the individual load curve because there are usually many different unit costs for making up an electric supply charge. However the anticipation of the individual load curve is neither easy nor practical. Therefore the proposed method requires an operation whose probability of power supply is greatest for the practical optimum operation.

7. An Improvement of the DSS Schedule

Using a given proper DSS scheduled pattern, output power of each thermal unit is decided by using eqs. (1)~(3) so that the power supply probability becomes maximum at each time interval. In estimating the expected value of NO_x emission, if it exceeds the constraint value of the emission, it is satisfied by controlling the emission in the fully-up system state, which is done by using eq. (3)′.

If the controlled emission can satisfy the constraint value, the DSS schedule is modified so as to increase the power supply probability. That is, the increasing value of the power supply probability is denoted by $\varDelta H_{\mathrm{mn}}$ when the operating schedule of the m-th thermal unit is changed by shutting-down or starting-up at the n-th time interval. Estimating $\varDelta H_{\mathrm{mn}}$ for each thermal unit and each time interval, let $\varDelta H_{\max}$ be maximum value of these $(M \times N)$ values of $\varDelta H_{\mathrm{mn}}$. Since $\varDelta H_{\mathrm{mn}}$ is calculated by recognizing the Markov-like characteristic to be approximately maintained, inequality (5) is considered.

$$\varDelta H_{\mathrm{mn}} \geq \begin{cases} \varDelta H_{\max} - \varepsilon_{\mathrm{pat}} & (\varDelta H_{\max} \geq 0) \\ -\varepsilon_{\mathrm{pat}} & (\varDelta H_{\max} < 0) \end{cases} \tag{5}$$

Where, $\varepsilon_{\mathrm{pat}}\,(\geq 0)$ is an approperiate margin.

The m-th thermal unit is shut-down or started-up at the n-th time interval when eq. (5) is satisfied. If there exist several $\varDelta H_{\mathrm{mn}}$'s which satisfy eq. (5), then several new DSS schedules are obtained. This means that even if $\varDelta H_{\max}$ missed the strict optimum modification because it is approximately estimated, it is possible to determine the strict optimal DSS schedule as long as calculation error of $\varDelta H_{\mathrm{mn}}$ does not exceed $\varepsilon_{\mathrm{pat}}$. New DSS schedules are searched even if $\varDelta H_{\max} < 0$ in eq. (5), because the power supply probability is not always decreased by starting-up or shutting-down the m-th thermal unit at the n-th time interval, since $\varDelta H_{\mathrm{mn}}$ is estimated by approximately maintaining the Markov-like character as described above. The concrete value of margin $\varepsilon_{\mathrm{pat}}$ has to be determined taking into account the computing speed and memory requirement because the number of DSS schedules to be considered increaseds with the value of $\varepsilon_{\mathrm{pat}}$.

When a DSS schedule can not satisfy the emission constraint, then $\varDelta H_{\mathrm{mn}}$ is given by a decreasing value of emission. That is, the denotation of $\varDelta H_{\mathrm{mn}}$ is modified to the emission decrease in which the DSS schedule is modified by the m-th thermal unit at the n-th time interval.

8. Determination of DSS Schedules Satisfying the Minimum Shut-down Time

Let us that minimum shut-down time is equal to T time intervals. In the subsequent discussion, T is assumed to be 3 as the concrete value. If a thermal unit is out of service at the time interval marked with $\overset{\downarrow}{\mathrm{x}}$ of schedule A in which the thermal unit is continually opperated in Table 1, then new DSS schedules are patterns ①, ② and ③ which are stopped at $\overset{\downarrow}{\mathrm{x}}$ and can satisfy the minimum shut-down time constraint. For each new DSS schedule ①~③ shown in Table 1, each totaled value of $\varDelta H_{\mathrm{mn}}$ of modified time interval compared with original schedule A is recognized as $\varDelta H_{\mathrm{mn}}$ of eq. (5). That is, the optimal schedule is chosen from among such schedules using totaled $\varDelta H_{\mathrm{mn}}$ and eq. (5).

A similar statement holds for a thermal unit when it is required to start-up at the time interval marked with $\overset{\downarrow}{\mathrm{o}}$ in schedule B of Table 1. Dividing schedule B into former and latter part, schedule ④, ⑤ and ⑥, ⑦ can be created in which the thermal unit starts-up at $\overset{\downarrow}{\mathrm{o}}$, and can satisfy the minimum shut-down time.

Table 1. DSS schedule change for $T=3$

Change of mode	Schedule	Time interval →								
Shutting-down case	A		○	○	○	○ ↓ ×	○	○	○	
	①		○	×	×	×	○	○	○	
	②		○	○	×	×	×	○	○	
	③		○	○	○	×	×	×	○	
Starting-up case	B	○	○	×	×	× ↓ ○	×	×	○	○
	④	○	×	×	×	○	—	—	—	—
	⑤	○	○	○	○	○	—	—	—	—
	⑥	…	…	…	…	○	×	×	×	○
	⑦	…	…	…	…	○	○	○	○	○

○ : in operation, × : out of operation, — : latter part of either ⑥ or ⑦, … : former part of either ④ or ⑤.

In short, T DSS schedules can be obtained as the maximum number in stopping a thermal unit. Four DSS schedules are obtained as the maximum number in starting a thermal unit by combining the former part and the latter part. Incidentally, to determine the stopping time definitely, we choose the first time interval at the peak load hour and assume that all the thermal units are put into service at the first time interval.

9. Simulations by a Model Power System

The proposed method was simulated variously by using the model power system[7]. A schedule in which each thermal unit is always in service at each time interval was used as the initial DSS schedule which is a starting schedule in the proposed method. The minimum shut-down time T was assumed to be three for every thermal unit. Table 2 and 3 show the thermal and probabilistic characteristic constants respectively of the model system. Table 4 is the line

Table 2. Characteristic constants of thermal stations

No.	Bus	$f_m=a_m+b_m\cdot g_m+c_m\cdot g_m^2$ [\$]			d_m $\left[\frac{\text{kg}}{\$}\right]$	$\underline{g_m}$ [MW]	$\overline{g_m}$ [MW]
		a_m	b_m	$c_m\times 1000$			
1	1	40	3.6	5.0	—	30	120
2	2	60	3.4	4.0	0.258	30	120
3	3	60	3.4	4.0	0.266	30	120
4	4	50	3.5	4.5	0.241	30	120
5	5	40	3.5	4.5	0.250	30	120

Note: No. 1 unit is not constrained by emission controls because it is constructed in a remote area.

Table 3. Failure rate and repair rate of thermal stations

Thermal unit	Scheduled operation		Scheduled shutdown		Failure frequency for start up
	failure	repair	failure	repair	
1	0.0006	0.02	0.0005	0.02	0.01
2	0.0005	0.02	0.0004	0.02	0.01
3	0.0004	0.02	0.0003	0.02	0.01
4	0.0004	0.02	0.0003	0.02	0.01
5	0.0003	0.02	0.0002	0.02	0.01

Table 4. Line data

Line	Node	X	Error rate	Line	Node	X	Error rate
1	1– 9	0.50	0.030	10	5– 6	0.36	0.024
2	1–11	0.16	0.010	11	5– 9	0.16	0.010
3	2– 3	0.50	0.030	12	7– 8	0.16	0.010
4	2– 7	0.28	0.020	13	7–10	0.24	0.016
5	2–10	0.16	0.010	14	8– 9	0.36	0.024
6	3– 4	0.24	0.016	15	8–10	0.24	0.016
7	4– 6	0.28	0.020	16	8–11	0.28	0.020
8	4– 8	0.28	0.020	17	10–11	0.36	0.024
9	4– 9	0.50	0.030				

Base : 100 MVA

Table 5. Load data

Bus	Load [MW]	Bus	Load [MW]
1	0	7	40
2	0	8	90
3	0	9	70
4	0	10	25
5	0	11	25
6	10		

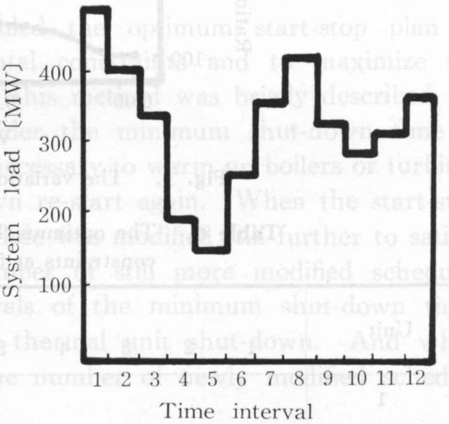

Fig. 1. Load curve.

data. The load of each bus was regarded as varying in proportion to Table 5 and Fig. 1.

The economic operation was firstly estimated whose probability of power supply is maximized by the proposed method. Setting $\varepsilon_{\text{pat}}=0$, all thermal units were in service at all time intervals in this result, and this was the same for the

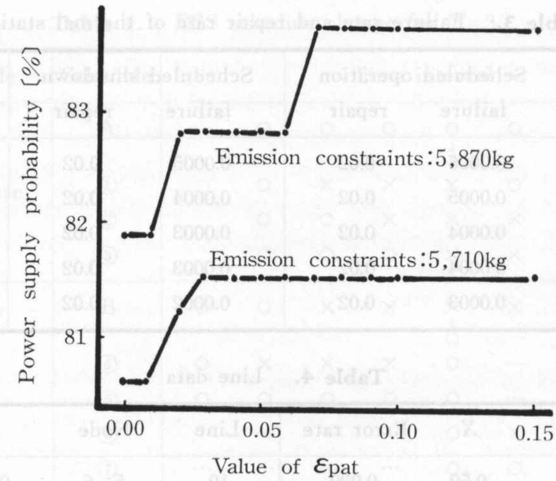

Fig. 2. The variations of the reliability.

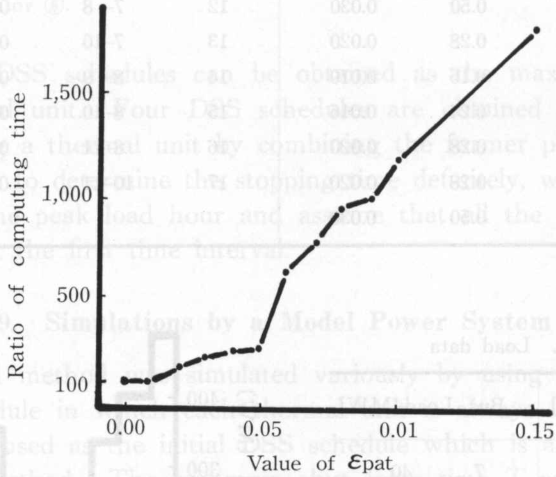

Fig. 3. The variation of the computing time.

Table 6. The optimum DSS schedule for the emission constraints at 5,870 kg ($\varepsilon_{pat}=0.08$)

Unit	Time interval											
	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3				×	×	×						
4												
5												

Space: in operation, ×: out of operation, the probability of power supply: 83.75%, the expected value of the operating cost: $28,918, the expected value of the emission is 6,306 kg.

initial schedule. The power supply probability of it was 83.83%, the expected value of NO_x emission was 6,331.1 kg and the expected value of the fuel cost was $ 29,185.

For constraining NO_x emission at 5,890 kg and 5,710 kg, each optimum schedule was estimated by various values of ε_{pat}. The power supply probability of the results is shown in Fig. 2. Other optimum schedules were obtained by many kinds of values of ε_{pat} and NO_x emission constraint. The ratio of their computing time is shown in Fig. 3.

It may be verified by Fig. 2 that the power supply probability is improved when the value of ε_{pat} increases. A reason for this is the approximation of the Markov-like characteristic of re-started up thermal units. The usefulness of proposed margin ε_{pat} is shown because it introduces the strict optimum schedule using computer ability.

An increased computing time was shown by Fig. 3 by increasing the value of ε_{pat}. It was also found that the strict optimum schedule could be estimated by using several times the computing time as in the case of $\varepsilon_{pat}=0$.

An example of an optimum DSS schedule is shown in Table 6, which can satisfy the NO_x emission constraint. This table shows the appropriateness of the proposed method for the constraint of the minimum shut-down time. And it was shown too, by comparing it with the economic schedule, that the optimum schedule can satisfy the NO_x emission constraint while not reducing the reliability of the power supply.

10. Conclusion

An already-proposed method scheduled the optimum start-stop plan of thermal units to satisfy the environmental constraints and to maximize the power supply probability. The outline of this method was briefly described.

The method was expanded to consider the minimum shut-down time in this paper, because preparing periods are necessary to warm up boilers or turbines when thermal units which have shut down re-start again. When the start-stop schedule was improved, the modified schedule was modified still further to satisfy the minimum shut-down time. The number of still more modified schedules was at most the number of time intervals of the minimum shut-down time, when the schedule was modified by the thermal unit shut-down. And when the schedule was modified by re-start, the number of newly modified schedule was four at most.

Many kinds of schedule were simulated by using the model power system. Varying the value of ε_{pat} which is the margin to search for the strict optimum schedule, the variation of the power supply probability and the computing time were concretely shown. It can be recognized from the results that the strict optimum schedule was obtained by using about ten times the computing time as in the case of the simple approximate method. As can be seen in the example of the optimum start-stop schedule which can satisfy the minimum shut-down time, the appropriateness of the proposed method for the minimum shut-down time is clear.

We would like to acknowledge various suggestions made by Dr. Toichiro Koike, the former President of Kitami Institute of Technology and the current President of Doto College. We also wish to express our sincere thanks to him.

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