

# Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 14)\* —System Reliability and Economy—

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## Abstract

When economical operation is considered as the operating cost or the fuel cost of each thermal station, then the economic operation tends to have a lower probability of power supply because reduction of the power supply decreases the fuel cost. The practical economy of power system operation is considered in this paper by using not only the operating cost but also the income received for the supply of electricity. The optimum operation is scheduled so that it can satisfy the emission constraint and the probability of power supply becomes maximum.

Using a model power system, each optimum operation is estimated considering the emission constraint. Results of simulation are reported and investigated.

## 1. Introduction

We reported previously the economical operating method when both power supply probability and environmental pollution are constrained<sup>1)</sup>. The economical operation realized the minimum expected fuel cost. But a lower expected fuel cost tended to introduce a lower probability of electric power supply.

When practical true economy is considered, not only the operating cost but also the income from each consumer should be considered. In this paper, the relation between the system reliability and economy is discussed, and the optimum operation scheduling method is reported whose probability of power supply is biggest at each emission constraint.

Because a power flow control method and a state probability estimation method were previously reported<sup>2)</sup>, this paper describes only a summary of them. The nitrogen oxide ( $\text{NO}_x$ ) emission from each thermal station is considered as the typical example of environmental pollution in this paper.

A proposed method is applied to a model system, and the results of the simulation are reported.

## 2. A Satisfying Method the Capacity of Transmission Apparatuses

### 2.1 Economical Power Flow

The initial load dispatch for each thermal station is economically decided by the law of equal incremental fuel cost. The output of each thermal station

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is introduced by eq. (1)<sup>2)</sup>.

$$\frac{df_m}{dg_m} = \lambda \quad (1)$$

Where,  $f_m$  and  $g_m$  are the fuel cost and the output power of the  $m$ -th station, and  $\lambda$  is the LaGrange's multiplier which is concerned with the power supply and demand balance.

Each power flow becomes eq. (2) when the transmission loss can be neglected and the DC method is used<sup>3)</sup>.

$$[i] = [e] [Ps] \quad (2)$$

Where, an element of  $[i]$  is  $i_l$  and it denotes the power flow of the  $l$ -th line.  $[e]$  is the sensitivity matrix<sup>4)</sup>. An element  $Ps_n$  of  $[Ps]$  is the electric power on the  $n$ -th bus.

When  $[Ps]$  is economically decided by eq. (1), then the power flow  $[i]$  can be recognized as an economical flow. If this economical power flow can satisfy every power line's capacity, then power supply is possible to the whole electrical power system by the economical output power. If some power flows do not satisfy the line's capacities, then the power dispatch of the thermal stations must be modified so as to satisfy every capacity.

## 2.2 Power Flow Control

When the output of the  $m'$ -th thermal station is increased by  $\Delta g$  and the  $m''$ -th station decreases its output by the same value to balance the power, then the increase of power flow  $\Delta i_l$  in the  $l$ -th line becomes eq. (3) because of eq. (2).

$$\Delta i_l = (e_{lm'} - e_{lm''}) \cdot \Delta g \quad (3)$$

Where,  $e_{lm'}$  and  $e_{lm''}$  are the elements of  $[e]$  respectively. If the  $l$ -th power flow is required to increase its value by  $\Delta i_l$  to satisfy the capacity, the  $m'$ -th and  $m''$ -th stations can change their output power by  $\Delta g$  from eq. (3). Our proposed method selects the combination of the  $m'$ -th and  $m''$ -th stations where the absolute value of  $\Delta g$  becomes as small as possible.

When many line's capacities are simultaneously considered, then the combination of power changing stations is decided by using a weight  $\Delta i_l$  to  $e_{lm}$  as in eq. (4).

$$E = \sum_l \Delta i_l \cdot e_{lm} \quad (4)$$

The station whose  $E$  is maximum increases output power, and the station of minimum  $E$  decreases output power. The modification of output power might be repeated<sup>5)</sup> when the capacities are many and the upper and lower limits of output power are considered.

To estimate rapidly, the proposed method controls emission of  $\text{NO}_x$  from each thermal station only when all elements are up in a power system. When both the line capacities and the emission constraint are considered, the combination of stations adjusting power is selected by eq. (5) which is modified eq. (4)<sup>6)</sup>.

$$E = \sum_i \Delta i_i \cdot e_{im} - \mu \cdot \frac{dy_m}{dg_m} \quad (5)$$

Where,  $y_m$  is the  $\text{NO}_x$  emission from the  $m$ -th thermal station. And  $\mu$  is the LaGrange's multiplier which is concerned with the  $\text{NO}_x$  emission constraint. Eq. (5) adds a term of weight  $\mu$  to eq. (4) by reason of a Kuhn-Tucker condition by which less  $\mu$  means a stricter emission constraint.

### 3. State Probability of Thermal Stations

#### 3.1 A recursive expression method

When the thermal station is operating during  $t_1$ , its up state probability becomes eq. (6)<sup>7</sup>.

$$s(s_0, \text{on}, t_1) = s_0 \cdot e^{-(p_{\text{on}} + q_{\text{on}})t_1} + \frac{q_{\text{on}}}{p_{\text{on}} + q_{\text{on}}} \cdot (1 - e^{-(p_{\text{on}} + q_{\text{on}})t_1}) \quad (6)$$

Where,  $s_0$  is the initial condition of the up state at time 0.  $p_{\text{on}}$  and  $q_{\text{on}}$  are the failure rate and the repair rate during scheduled operation respectively.

When the thermal station is stopped from  $t_1$  during  $t_2$ , the up state probability becomes  $s(s(s_0, \text{on}, t_1), \text{off}, t_2)$  recursively. off means that the failure rate  $p_{\text{off}}$  and the repair rate  $q_{\text{off}}$  should be used, which are the data of the scheduled shutdown period instead of  $p_{\text{on}}$  and  $q_{\text{on}}$  of eq. (6) respectively. The proposed method neglects the frequency of outage in the period of transition leading to shutdown.

#### 3.2 Re-startup of thermal station

The up state probability of a re-started thermal station at  $t_1 + t_2$  can also be estimated recursively. It becomes  $s((1 - p_{\text{up}}) \cdot s(s(s_0, \text{on}, t_1), \text{off}, t_2), \text{on}, t_3)$  when it is operated during  $t_3$  after re-startup.  $p_{\text{up}}$  means the frequency of outage in the period of transition leading to re-startup.

Since the up state probability of a re-started thermal station depends on the length of the shutdown period too, the Markov-like character is not kept strictly. However, It is thought to be approximately kept because  $p_{\text{up}} \ll 1$  and  $p_{\text{off}} \ll q_{\text{off}}$  are usually recognized<sup>7</sup>.

### 4. Optimum DSS Pattern

#### 4.1 An optimal operation

In the past, economical operation was considered to be the lowest fuel cost of thermal stations. But, when the probabilistic outage of each apparatus is considered in the power system, then lower fuel cost means lower power supply probability. In the case of strictly, mathematically, and economically satisfying the reliability constraint, it is even possible to stop the power supply despite the power can be supplied if the supplier wants. This can not be thought practical even if mathematical.

The operating fund is mainly supplied by the income from charges for the supply of electricity by the power system. That is, practical economy should consider the income from electric charges, which is only obtained when the electric power is supplied. When this practical economy is strictly mathematically considered, the individual charge paid by each consumer must be estimated in the scheduling step, because the practical economy is the balance of the income and the charge. This requires the anticipation of the individual load curve because there are usually many different unit costs for the electric supply charge. However the anticipation of the individual load curve is neither easy nor practical. Therefore the proposed method requires the operation whose probability of power supply is greatest for the practical optimum operation.

4.2 An estimation of the optimum operation

A proper initial DSS pattern is set and it is repeatedly improved as follows.

The output power of each thermal station is decided so as to satisfy each line capacity and the NO<sub>x</sub> emission constraint by using eqs. (3)~(5) for up state and each down state of the set DSS pattern. When the set DSS pattern can probabilistically satisfy the emission constraint, then the purpose of the pattern modification is the improvement of the system's reliability, and when the set pattern can not satisfy the emission constraint, then its purpose is an emission decrease.

Depending on these purposes, an improvement quantity is estimated when the set DSS pattern is modified by the startup or shutdown of each thermal station at each time interval. At this time, the Markov-like character is regarded to be kept<sup>7)</sup> as described before.

5. Simulations by a Model Power System

Tables 1 and 2 are the thermal and probabilistic characteristic constants respectively of the model system<sup>8)</sup>. Table 3 is the line data. The load of each bus was regarded as varying in proportion to Table 4 and Fig. 1.

Each line's capacity was fixed at 150% of the economic power flow at the

Table 1. Characteristic constants of thermal stations

No.	Bus	$f_m = a_m + b_m \cdot g_m + c_m \cdot g_m^2$ [\$]			$d_m$ [ $\frac{\text{kg}}{\$}$ ]	$\overline{g_m}$ [MW]	$\overline{g_m}$ [MW]
		$a_m$	$b_m$	$c_m \times 1000$			
1	1	40	3.6	5.0	—	30	120
2	2	60	3.4	4.0	0.258	30	120
3	3	60	3.4	4.0	0.266	30	120
4	4	50	3.5	4.5	0.241	30	120
5	5	40	3.5	4.5	0.250	30	120

Note: No. 1 unit is not constrained for emission because it is constructed in a remote area.

Table 2. Failure rate and repair rate of thermal stations

Thermal unit	Scheduled operation		Scheduled shutdown		Failure frequency for start up
	failure	repair	failure	repair	
1	0.0006	0.02	0.0005	0.02	0.01
2	0.0005	0.02	0.0004	0.02	0.01
3	0.0004	0.02	0.0003	0.02	0.01
4	0.0004	0.02	0.0003	0.02	0.01
5	0.0003	0.02	0.0002	0.02	0.01

Table 3. Line data

Line	Node	X	Error rate	Line	Node	X	Error rate
1	1- 9	0.50	0.030	10	5- 6	0.36	0.024
2	1-11	0.16	0.010	11	5- 9	0.16	0.010
3	2- 3	0.50	0.030	12	7- 8	0.16	0.010
4	2- 7	0.28	0.020	13	7-10	0.24	0.016
5	2-10	0.16	0.010	14	8- 9	0.36	0.024
6	3- 4	0.24	0.016	15	8-10	0.24	0.016
7	4- 6	0.28	0.020	16	8-11	0.28	0.020
8	4- 8	0.28	0.020	17	10-11	0.36	0.024
9	4- 9	0.50	0.030				

Base : 100 MVA

Table 4. Load data

Bus Load [MW]		Bus Load [MW]	
1	0	7	40
2	0	8	90
3	0	9	70
4	0	10	25
5	0	11	25
6	10		

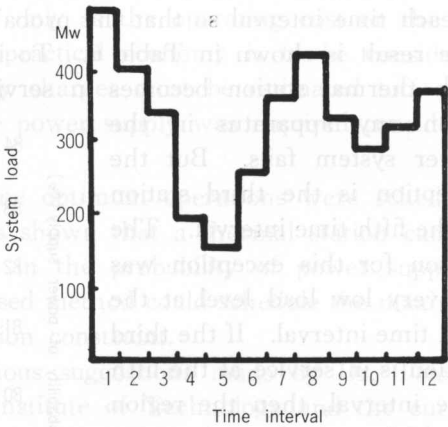


Fig. 1. Load curve.

first time interval in every station service. In the initial DSS pattern, every thermal station was assumed to be always in service.

The economic pattern was deterministically estimated by using the ordinal economic load dispatch. The result is shown in Table 5. In the table, × means out of service and space means in service respectively for each thermal station.

Table 5. Economic operation

Unit	Time interval											
	1	2	3	4	5	6	7	8	9	10	11	12
1				×	×	×				×	×	
2				×	×	×				×	×	
3							×					
4			×		×				×			
5												

The probability of power supply is 77.47%, the expected value of the operating cost is 26,431 \$, the expected value of the emission is 5,926 kg.

Table 6. The highest reliability operation

Unit	Time interval											
	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3					×							
4												
5												

The probability of power supply is 83.98%, the expected value of the operating cost is 29,113 \$, the expected value of the emission is 6,306 kg.

The highest reliability operation was estimated by selecting the unit commitment at each time interval so that the probability of power supply becomes maximum. The result is shown in Table 6. To increases the probability of power supply, each thermal station becomes in service because power can be supplied even when any apparatus in the power system fails. But the exception is the third station at the fifth time interval. The reason for this exception was the very low load level at the fifth time interval. If the third station is in service at the fifth time interval, then the region in which each thermal station can modify its output power becomes small and the probability of power supply becomes less. This shows that each thermal station can not simply be in service to increase the

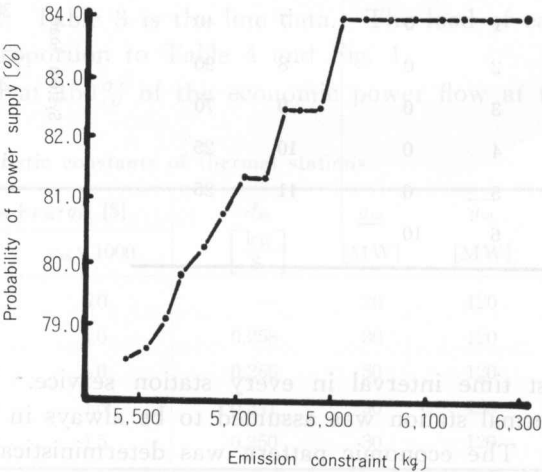


Fig. 2. The probability of power supply by the optimum operations.



probability of power supply.

From the highest reliability operation to the least emission operation, the optimum operation was estimated so as to become the biggest probability of power supply satisfying each emission constraint. The result is shown in Fig. 2. From a 6,306 kg emission constraint to a 5,910 kg, the highest reliability start stop pattern of Table 6 could satisfy each emission constraint, and an emission constraint of less than 5,910 kg was satisfied by a modified pattern of the highest reliability pattern. For example, the optimum operation is shown in Table 7 which can satisfy a 5,500 kg emission constraint.

**Table 7.** Optimum operation for the emission constraint of 5,500 kg

Unit	Time interval											
	1	2	3	4	5	6	7	8	9	10	11	12
1												
2				×		×	×					
3									×	×	×	×
4			×									
5												

The probability of power supply is 78.62%, the expected value of the operating cost is 27,073 \$.

## 6. Conclusion

It was described that the probability of operation of power supply tends to be lowered when economy is considered only as the operating cost or the fuel cost, and it was also explained that if practical economy is to be considered, then the income from the electric supply charges must be estimated too. The maximum probability of operation of the power supply was required considering each emission constraint in this paper.

Using the model power system, some optimum operations were scheduled. By investigations using simulation, it was shown that a thermal station can not simply be inservice, even if an increase in the probability of power supply is wanted, and it was shown that the proposed method could schedule the maximum reliability operation satisfying each emission constraint.

We would like to acknowledge various suggestions made by Dr. Toichiro Koike, the former President of Kitami Institute of Technology and the current President of Doto College. We wish to express our sincere thanks to him.

## References

- 1) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 19, No. 2, p. 197-207 (March 1988).
- 2) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 18, No. 1, p. 1-10 (November 1986).

- 3) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 16, No. 2, p. 89-97 (March 1985).
- 4) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 16, No. 1, p. 15-24 (November 1984).
- 5) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 18, No. 2, p. 173-180 (March 1987).
- 6) Nakamura, Y. and Yamashiro, S.: I. E. E. J. Power Engineering Symposium, PE-86-93, p. 21-30 (August 1986).
- 7) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 18, No. 2, p. 181-186 (March 1987).
- 8) T. Ueda, et al.: I. E. E. J. Power Engineering Symposium, PE-81-23, p. 51-60 (July 1981).

Table 1. Optimum operation for the emission constraint of 5,500 kg											
Time interval											
Emission constraint											
Unit											
1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1

The probability of power supply is 78.82%, the expected value of the operating cost is 37,074.8.

Conclusion

It was described that the probability of operation of power supply tends to be lowered when economy is considered as the operating cost. On the other hand, it was also explained that the probability of operation is to be considered. Then, the optimum operation for the emission constraint was estimated. The maximum probability of operation of the power supply was estimated considering each emission constraint in this paper.

Using the model power system, some optimum operations were estimated. By investigations and simulation, it was shown that a thermal station can not simply be inserted, even if an increase in the probability of power supply is wanted, and it was shown that the proposed method could estimate the maximum reliability operation satisfying each emission constraint. We would like to acknowledge various suggestions made by Dr. Toshiro Kake, the former President of Kitami Institute of Technology, and the current President of Daito College. We wish to express our sincere thanks to them.

References

- 1) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 16, No. 2, p. 89-97 (March 1985).
- 2) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 16, No. 1, p. 15-24 (November 1984).
- 3) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 18, No. 2, p. 173-180 (March 1987).