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Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 13)*

—A Practical Satisfaction of the Constraint—

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Abstract

The optimum start stop scheduling method is reported considering the constraints of both environmental pollution and the probability of power supply, because big and inflexible power stations are being built in spite of load increase and bad load factor of the power system. The proposed method controls the output power of two thermal stations so as to satisfy the line capacities and to make a schedule rapidly although the estimated schedule may not be strictly economical. The emission from each thermal station is controlled only when the whole system state is completely up in order to avoid huge estimations. If the power supply must be stopped to reduce the emission, then it is practically done only when the system includes some down elements. It is shown that the influence of the state probability is sufficiently little by variation of the length of the off service period of the thermal station and that the multistage choice process can be used.

The usefulness of this method is concretely shown by estimating the optimum schedule of a model system. The estimating error is investigated by comparing results.

1. Introduction

Electric power demand is increasing and the load factor worsening, while big and inflexible power stations are being built, a typical example being a nuclear power station. This means that the thermal stations are required to provide not only wide control of the output power but also proper daily start stop (DSS) operation considering the whole system. Optimum scheduling becomes important for the electric power system.

The schedule must be practical. This paper reports a speedy scheduling method for optimum operation considering constraints of both power supply probability and environmental pollution by the thermal stations. The proposed method can make a start stop schedule for each thermal station and its output at any time so that both constraints are satisfied most economically.

From a scheduling position power supply is considered possible only when all capacities in the system are satisfied. The output power of every thermal station must be controlled to satisfy the capacities strictly economically[1], and this introduces huge calculations[2]. The proposed method approximately controls the output power of two thermal stations so as to make an operation schedule rapidly.

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The failure rate of each element is assumed to have been previously provided for the whole system. The environmental constraint is satisfied by the expected value because the system state is varied depending on which element is down. When we must try to satisfy the environmental constraint strictly economically, then each system state must always control the emission and this introduces huge calculations. Because the probability of a completely up state is usually greater than other states, the proposed method controls the emission approximately only when every element in the system is up. Although it is possible to cut the power supply to decrease the emission, a practical scheduling method is considered. The proposed method considers the nitrogen oxide (NO_x) emission from each thermal station as the typical example of environmental pollution.

The state probability of a thermal station which was re-started up depends on the length of the off service period. Because this means a loss of the Markov-like character, the optimum start stop pattern can not be exactly decided by the multistage choice process. But it is shown that the influence of this period is sufficiently small, and the proposed method uses the multistage choice process approximately.

Finally, a model system is used to simulate the method, and the usefulness of the proposed method is investigated.

2. A Control Method for Each Power Flow

Firstly, the output of each thermal power station is simply decided by the law of equal incremental fuel cost so that the total fuel cost of the thermal stations is at its most economical. The power flow of each line is estimated by using this economical power dispatch. If every powerline's capacity is satisfied by this economical power flow, then power supply is possible in the whole electric system.

If some power flows do not satisfy the line's capacities, then the power dispatch of the thermal stations must be modified so as to satisfy every capacity.

2.1 A Power Flow Estimation

Each power flow is estimated by the DC method, because this can calculate power flow conveniently and very rapidly[3] The power flow of each line becomes eq. (1) when the transmission loss can be neglected[4].

$$[i]=[e]\,[Ps] \tag{1}$$

Where, $[i]$ is a row vector whose element i_l is the power flow of the l-th line. $[e]$ is the sensitivity matrix[5], and its element becomes eq. (2) which is described later. $[Ps]$ is a column vector whose element Ps_n is the electric power on the n-th bus.

$$e_{ln}=\frac{(b'_{jn}-b'_{kn})}{x_l} \tag{2}$$

Where, b'_{jn} and b'_{kn} are the elements of the inverse matrix[6] of the susceptance matrix, and the direction of the positive power flow in the l-th line is assumed from the j-th bus to the k-th bus. x_l means the reactance of the l-th power line.

2.2 Satisfaction of only one line's capacity

When the output of the m'-th thermal station is increased by Δg, then the output of the m''-th station must be decreased by the same value to balance the power. At this time, the increase of power flow in the l-th line becomes eq. (3) because of eq. (1).

$$\Delta i_l = (e_{lm'} - e_{lm''}) \cdot \Delta g \tag{3}$$

Eq. (3) means that the m'-th and m''-th stations can change their output power by Δg when the l-th power flow must be increased by Δi_l to satisfy the capacity. A combination of the m'-th and m''-th stations is selected so that the absolute value of Δg becomes as small as possible in this proposed method. When the sign of Δg is set equally to the sign of Δi_l, then the combination of stations is decided by selecting the maximum $e_{lm'}$ and the minimum $e_{lm''}$ because of eq. (3).

2.3 Satisfaction of every line's capacity

When many line's capacities are simultaneously considered, then the output power of a certain station might be increased to satisfy one line's capacity, but it might be decreased to satisfy another line's capacity. The combination of power-changing stations is decided by using a weight Δi_l to e_{lm} as in eq. (4) in the proposed method.

$$E = \sum_l \Delta i_l \cdot e_{lm} \tag{4}$$

The combination consists of two stations which have the maximum E and minimum E.

2.4 Repetition of the modification to satisfy every line's capacity

Every line's capacity is not always satisfied by the modification of one pair of stations as described previously. Besides, the upper and lower limits of thermal output must also be considered, and it is necessary to repeatedly modify the output power of the thermal stations. But the direction of the output modification is fixed when each station is selected by eq. (4) because oscillatory or divergent repetition must be avoided. This means that the station selected to increase power by eq. (4) is excluded when selecting which station should decrease power.

2.5 The consideration of the environmental constraint

The emission is controlled only when all elements are up in a power system under the proposed method because then a schedule can be estimated rapidly.

The objective function for power dispatch becomes eq. (5) when the NO_x emission is also considered.

$$\phi = \sum_{m=1}^{M} f_m + \lambda \cdot \left(Ps - \sum_{m=1}^{M} g_m\right) + \mu \cdot \left(Y - \sum_{m=1}^{M} y_m\right) \tag{5}$$

Where, M is the number of thermal stations in the power system, and f_m and y_m are the fuel cost and the NO_x emission of the m-th thermal station. These are estimated by quadratic functions[7] of the output power g_m. λ and μ are the LaGrange's multipliers which are concerned with the power supply and demand balance and with the NO_x emission constraint respectively. Ps and Y are the system load and the constraint value of NO_x respectively.

When the line capacities are not satisfied by the power flow which is estimated by eqs. (5) and (1), then the combination of stations adjusting power is estimated by eq. (4)′ which is modified eq. (4)[8].

$$E=\sum_{l}\Delta i_l \cdot e_{lm}-\mu\cdot\frac{dy_m}{dg_m} \tag{4$'$}$$

3. State Probability of Start Stop Thermal Stations

3.1 A recursive expression of the state probability

The up state probability becomes eq. (6) at time t_1 when the thermal unit has not yet shut down[9].

$$s(s_0,\mathrm{on},t_1)=s_0\cdot e^{-(p_{\mathrm{on}}+q_{\mathrm{on}})\cdot t_1}+\frac{q_{\mathrm{on}}}{p_{\mathrm{on}}+q_{\mathrm{on}}}\cdot(1-e^{-(p_{\mathrm{on}}+q_{\mathrm{on}})\cdot t_1}) \tag{6}$$

Where, s_0 is the initial condition of the up state at time 0. p_{on} and q_{on} are the failure rare and the repair rate during scheduled operation respectively.

The up state probability becomes $s(s(s_0, \mathrm{on}, t_1), \mathrm{off}, t_2)$ recursively when the thermal station is stopping from t_1 during t_2. At this time, the frequency of outage in the period of transition leading to shutdown is neglected. Off means that the failure rate p_{off} and the repair rate q_{off} should be used, which are data of the scheduled shutdown period in stead of p_{on} and q_{on} respectively.

When the thermal station is operated during t_3 after re-startup at t_1+t_2, then the up state probability can be also estimated recursively and it is denoted S_3 as eq. (7).

$$\begin{aligned}S_3&=s\big((1-p_{\mathrm{up}})\cdot s\big(s(s_0,\mathrm{on},t_1),\mathrm{off},t_2\big),\mathrm{on},t_3\big)\\&=(1-p_{\mathrm{up}})\cdot\Big[\Big\{s_0\cdot e^{-(p_{\mathrm{on}}+q_{\mathrm{on}})t_1}\\&\quad+\frac{q_{\mathrm{on}}}{p_{\mathrm{on}}+q_{\mathrm{on}}}\cdot(1-e^{-(p_{\mathrm{on}}+q_{\mathrm{on}})\cdot t_1})\Big\}\cdot e^{-(p_{\mathrm{off}}+q_{\mathrm{off}})\cdot t_2}\\&\quad+\frac{q_{\mathrm{off}}}{p_{\mathrm{off}}+q_{\mathrm{off}}}\cdot(1-e^{-(p_{\mathrm{off}}+q_{\mathrm{off}})\cdot t_2})\Big]\cdot e^{-(p_{\mathrm{on}}+q_{\mathrm{on}})\cdot t_3}\\&\quad+\frac{q_{\mathrm{on}}}{p_{\mathrm{on}}+q_{\mathrm{on}}}\cdot(1-e^{-(p_{\mathrm{on}}+q_{\mathrm{on}})\cdot t_3})\end{aligned} \tag{7}$$

Where, p_{up} means the frequency of outage in the period of transition leading re-startup.

3.2 A variation of state probability after re-startup

The operation of re-startup is considered as being almost always successful ($p_{\mathrm{up}} \ll 1$), and it can also be recognized that the mean time of the outage state is usually sufficiently shorter than the mean time of the outbreak interval of the outage during shutdown ($p_{\mathrm{off}} \ll q_{\mathrm{off}}$). When eq. (7) is differentiated by the shutdown period t_2 using the above investigations, then the result is obtained as eq. (8).

$$\frac{dS_3}{dt_2} = q_{\mathrm{off}} \cdot \left[1 - s(s_0, \mathrm{on}, t_1)\right] \cdot e^{-q_{\mathrm{off}} \cdot t_2} \cdot e^{-(p_{\mathrm{on}}+q_{\mathrm{on}}) \cdot t_3} \tag{8}$$

$0 \leqq s(s_0, \mathrm{on}, t_1) \leqq 1$ and it is usually near to 1, because it is the up probability at t_1. Since q_{off}, t_2, p_{on}, q_{on} and $t_3 \geqq 0$, $0 \leqq e^{-q_{\mathrm{off}} \cdot t_2} \leqq 1$ and $0 \leqq e^{-(p_{\mathrm{on}}+q_{\mathrm{on}}) \cdot t_3} \leqq 1$. q_{off} is the inverse of the mean time of the outage state during shutdown[9] and it is thought small because repairs to the thermal unit requiring more than about fifteen hours or so will influence the DSS schedule.

The value of eq. (8) can be thought sufficiently small because of the above considerations (the value of the model system which is described later was below 10^{-4}). Then the influence of a shutdown period is sufficiently small, and the Markov-like character can be recognized to be approximately kept.

4. A Decision of Optimum DSS Pattern

First of all, a proper DSS pattern is set and it is repeatedly improved as follows.

4.1 The effect rate of thermal station startup/shutdown

The output power of each thermal station at each time interval is decided so as to satisfy the constraints of both power supply probability and NO_x emission by the set DSS pattern. The decreasing operation cost is estimated at which the set DSS pattern is modified by the startup/shutdown of each thermal station at each time interval. At this time, the state probability after the $n+1$th time interval is regarded as invariant when the set DSS pattern is modified at the n-th time interval because of the consideration in the previous section. If the NO_x emission exceeds the constraint value in the modified pattern, then the operating cost is increased by the exceeded value times $-\mu$ which is the anticipated cost to satisfy the constraint[7]. When the set DSS pattern is modified by the m-th station at the n-th time interval, then the effect rate Ef_{mn} is estimated by dividing the new operating cost by the old one before the modification.

4.2 A search for a possible DSS pattern to satisfy all constraints

When the set DSS pattern can satisfy constraints of both power supply probability and NO_x emission, then the effect rate can be estimated as above. If either constraint can not be satisfied by the set DSS pattern, however, the effect rate is estimated as follows by a modified method.

A new expected value of NO_x emission and power supply probability is estimated by the modified pattern which is obtained by the startup/shutdown of the m-th thermal station at the n-th time interval. For improved values by the pattern modification, differences are respectively estimated between the new NO_x emission and old emission, the new power supply probability and old probability. The effect rate Ef_{mn} is re-defined as the sum of the divided these improved values by the corresponding constraint values respectively.

4.3 Modification of the DSS pattern

The maximum value is denoted by $Ef_{\max}$ among these Ef_{mn}. Since these effect rates are estimated approximately, the DSS pattern is modified according not only to $Ef_{\max}$ but also to near value to $Ef_{\max}$.

That is, using a proper scope $\varepsilon_{\text{patn}}$, when Ef_{mn} statisfy eq. (9), then the set DSS pattern is modified by starting-up/shutting-down the m-th station at the n-th time interval.

$$\left.\begin{array}{ll} Ef_{nm} \geqq Ef_{\max} - \varepsilon_{\text{patn}} & (Ef_{\max} \geqq 0) \\ Ef_{nm} \geqq -\varepsilon_{\text{patn}} & (Ef_{\max} < 0) \end{array}\right\} \tag{9}$$

4.4 The economical and practical satisfaction of power supply probability

Many new DSS patterns can be set using eq. (9). A selection method is considered for improving these new patterns in this section.

For example, two kinds of DSS pattern are considered, as in Table 1. Schedule A has a 70% probability of supplying power in a completely up state. The remaining 30% has some down elements in the system, and the power supply probability is 20%. The final probability of power supply becomes 90% collecting all the system states. The expected value of the operating cost is 150 at this time. On the other hand, the power supply probability of schedule B is 80% in a completely up state, and it becomes 95% when other down system states are collected. The final expected value of the operating cost is 160 by schedule B. Schedule A may be better if the pattern is selected simply ecoonomically.

The case of 88% is considered as the constraint value of the power supply probability. Since schedule A supplies power by 2% over the constraint, the

Table 1. Examples of schedule

		Schedule A			Schedule B		
Probabilistic constraint of the power supply			88	75		88	75
Completely up state	Probability of the power supply	70			80		
	Expected value of cost	100			120		
Some elements down	Probability of the power supply	20	18	5	15	8	0
	Expected value of cost	50	45	13	40	21	0
Total state	Probability of the power supply	90	88	75	95	88	80
	Expected value of cost	150	145	113	160	141	120

power supply must be cut by this value when the system is not completely up so that the system is operated as economically as possible considering the power supply constraint. This makes the expected value of the operating cost 145. For schedule B, the power supply is cut by 7% when some elements are down, and the expected value of the operating cost then becomes 141. Schedule B can then be recognized as better for this power supply constraint value.

The next value is 75% as the power supply constraint. The resulting expected value of the operating cost is 113 for schedule A because the power supply is cut by 15% in the down states. On the other hand, although the over supply of power is 20% for schedule B, the probability of power supply is 15% in the down states. Then, a further cut of power supply may be necessary in the up state for schedule B if economical operation is strictly mathematically required for this constraint. But this operation method means that the power supply is cut for no other reason than economy in spite of a completely up state. This method is not thought practical, and the proposed method cuts the power supply to satisfy the power supply constraint only when the system includes down elements. Schedule A is recognized as better for the constraint of 75% according to the proposed method.

5. Simulations by a Model Power System

Tables 2 and 3 show the thermal and probabilistic characteristic constants of the model system[10]. Table 4 shows the line data. The load of each bus was

Table 2. Characteristic constants of thermal stations

No.	Bus	$f_m=a_m+b_m\cdot g_m+c_m\cdot g_m^2$ [\$]			d_m	$\underline{g}_m$	$\overline{g}_m$
		a_m	b_m	$c_m\times 1000$	$\left[\frac{kg}{\$}\right]$	[MW]	[MW]
1	1	40	3.6	5.0	—	30	120
2	2	60	3.4	4.0	0.258	30	120
3	3	60	3.4	4.0	0.266	30	120
4	4	50	3.5	4.5	0.241	30	120
5	5	40	3.5	4.5	0.250	30	120

Note: No. 1 unit is not constrained for emission because it is constructed in a remote area.

Table 3. Failure rate and repair rate of thermal stations

Thermal unit	Scheduled operation		Scheduled shutdown		Failure frequency for start up
	failure	repair	failure	repair	
1	0.0006	0.02	0.0005	0.02	0.01
2	0.0005	0.02	0.0004	0.02	0.01
3	0.0004	0.02	0.0003	0.02	0.01
4	0.0004	0.02	0.0003	0.02	0.01
5	0.0003	0.02	0.0002	0.02	0.01

Table 4. Line data

Line	Node	X	Error rate	Line	Node	X	Error rate
1	1– 9	0.50	0.030	10	5– 6	0.36	0.024
2	1–11	0.16	0.010	11	5– 9	0.16	0.010
3	2– 3	0.50	0.030	12	7– 8	0.16	0.010
4	2– 7	0.28	0.020	13	7–10	0.24	0.016
5	2–10	0.16	0.010	14	8– 9	0.36	0.024
6	3– 4	0.24	0.016	15	8–10	0.24	0.016
7	4– 6	0.28	0.020	16	8–11	0.28	0.020
8	4– 8	0.28	0.020	17	10–11	0.36	0.024
9	4– 9	0.50	0.030				

Base : 100 MVA

Table 5. Load data

Bus	Load [MW]	Bus	Load [MW]
1	0	7	40
2	0	8	90
3	0	9	70
4	0	10	25
5	0	11	25
6	10		

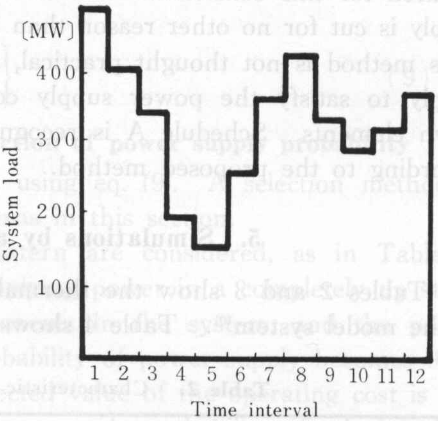

Fig. 1. Load curve.

Table 6. Economic operation

Time interval	Thermal stations 1	2	3	4	5	Probability of power supply	Expected value of cost	Expected value of emission
1						75.97	1625	351
2						80.26	1427	310
3				×		74.12	1103	231
4		×		×		79.92	709	139
5	×	×		×		82.90	561	146
6	×	×				80.26	893	226
7		×				68.68	1083	234
8						78.15	1439	312
9		×				76.59	1109	237
10	×	×				76.25	945	239
11	×	×				74.91	1025	259
12			×			77.11	1232	247

The probability of power supply was 77.09% in a whole day.
The expected value of the operating cost was 26,300 $, and the expected value of the emission was 5,862 kg.

Table 7. Optimum operation (probabilistic constraint of the power supply is 78%, emission constraint is 5,600 kg, ε_{patn} is 0.0014)

Time interval	Thermal stations 1	2	3	4	5	Probability of power supply	Expected value of cost	Expected value of emission
1						75.97	1569	327
2						80.26	1364	285
3				×		74.12	1074	220
4			×			86.63	749	146
5						84.92	630	130
6			×			83.83	905	174
7						83.49	1254	263
8						78.25	1376	288
9						85.64	1182	248
10						85.75	1038	218
11						85.69	1134	238
12						81.92	1252	262

The expected value of the cost was 27,067 $ in whole day.

regarded as varying in proportion to Table 5 and Fig. 1.

The ordinal economic load dispatch was estimated at the first time interval in every station service. Its power flow was also estimated, and each line's capacity was fixed at 150% of the economic flow. As an initial DSS pattern which was used in the beginning of the proposed method, every thermal station was assumed to be always in service.

First of all, an economic operation schedule was estimated by using the proposed method. The result is shown in Table 6. To obtain this result, the output power of each thermal station was modified with each DSS pattern so that each of the power supply probability becomes maximum, and the most economical schedule was selected. Although the load level is not so different between the third time interval and the 7th interval by Fig. 1, the corresponding unit commitments are different in Table 6. The reason for this is as follows. The unit commitment of the third time interval could not operate at the 7th interval because of the power capacity. On the contrary, when the unit commitment of the 7th time interval was operated at the third interval, then the possibility of the power supply increased and the expected value of the operating cost increased too. This means that a lower probability of power supply was selected because only economy was considered in Table 6.

Secondly, the optimum schedule was estimated by the proposed method considering constraints of both power supply probability

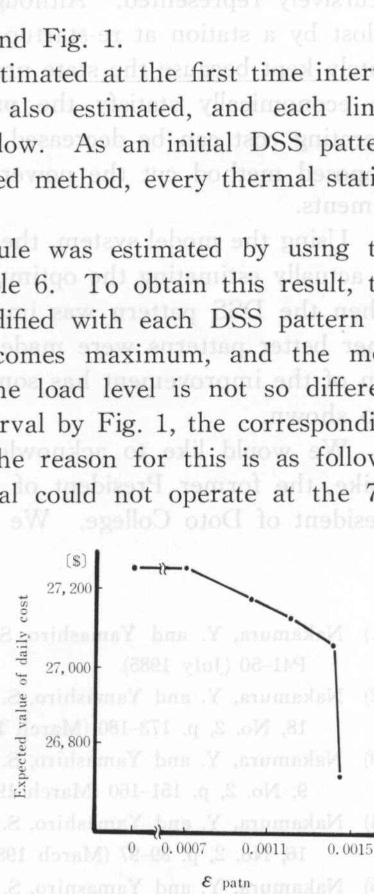

Fig. 2. Optimization by ε_{patn}.

and the expected value of the NO_x emission. The result is shown in Table 7. To satisfy the constraints in Table 7, almost all thermal stations were in service which were previously out of service in Table 6.

Many optimum operations were estimated by using the constraints of Table 7 and various ε_{patn}. The results are shown in Fig. 2. The usefulness of the proposed ε_{patn} can be seen.

6. Conclusion

A scheduling method was described which can economically satisfy the probability constraint of the power supply and the environmental constraint simultaneously. The estimation method of the optimum start-stop pattern was also described for the economical satisfaction of these constraints. To realize these, a rapid method was shown which can decide the modification of the output power of thermal stations so as to supply power while considering the environmental constraint.

It was shown that the state probability of the thermal station can be briefly recursively represented. Although, strictly speaking, the Markov-like character is lost by a station at re-startup, it was shown that it was nevertheless approximately kept because the state probability depends on the length of the off service. To economically statisfy the probability constraint of the power supply, the operating cost can be decreased by cutting power supply, but to be practical the proposed method cut the power supply only when the system had some down elements.

Using the model system, the usefulness of the proposed method was inspected by actually estimating the optimum operation which could satisfy each constraint. When the DSS pattern was improved, not only one new best pattern but also other better patterns were made by using the certain scope, because the estimation of the improvement has some errors. And the usefulness of this scope was also shown.

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