北見工業大学研究報告
第18巻 第2号 (昭和62年)

Memoirs of the Kitami Institute of Technology
Vol. 18 No. 2 (1987)

Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 10)*

—Comparison with Quadratic Programming Method—

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(Received September 29, 1986)

Abstract

We previously described a fast scheduling method to consider every line capacity and emission constraint when each line and thermal unit fail probabilistically. The method is an approximate one because only two thermal units control the output power to satisfy the lines capacity and only a healthy state controls the emission excepting faulty states.

The usefulness of the method is confirmed by comparing it with a strict method. The strict schedule is estimated by the quadratic programming method. This report describes the outlines of both the proposed method and the quadratic programming method. Both methods are applied to a model system and the results are shown concretely. It is also shown that the proposed method is useful because the maximum error is less than 2 [%], and the computing time of the proposed method is more than 420 times as fast as the quadratic programming method when the emission constraint is not considered and 1600 times as fast when it is considered.

1. Introduction

We previously reported a fast scheduling method[1] to consider every line capacity and the whole sum of NO_x emission from each thermal power station. The proposed method determined the optimum thermal output power approximately. There were two approximations in the method. The first was a thermal power modification by only two thermal units to satisfy each line capacity. The second was emission control to satisfy the limited expected value by only healthy system state excepting other faulty states.

To confirm the usefulness of the previously proposed method, the results are compared with a strict method. The strict method has already been reported[2] using quadratic programming. This comparison considers probabilistically not only the line fault but also the emission constraint. Calculation times are also compared between the two methods to check the speedy estimation of the proposed method.

2. Proposed Method

The fast method was described in the previous report[1] in detail. This chapter gives an outline of it.

* Presented at the Power Engineering Meeting of the I. E. E. of Japan (August 1986).
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2.1 Satisfaction of One Line's Capacity

When the power flow of each line is considered by the DC method[3], a change of power flow in the l-th line Δi_l becomes eq. (1)[4].

$$\Delta i_l = (e_{lm'} - e_{lm''}) \cdot \Delta g \quad (1)$$

Where, the transmission losses are neglected. $e_{lm'}$ and $e_{lm''}$ are the elements of the sensitivity matrix[3]. Δg is the output change of the thermal units. Eq. (1) means that the m'-th unit increases its power by Δg, the m''-th unit decreases power by the same value and the other units do not change their power.

If economic load dispatch introduces an overflow on one power line, then two thermal units modify their output power to satisfy the line capacity. This modification may not realize an ideally economical power change, but can satisfy the line capacity rapidly. The thermal unit pair which should change their power is selected so that they have the maximum and the minimum element respectively of the sensitivity matrix.

2.2 Satisfaction of Every Line Capacity

Even if overflow occurs on many lines, the proposed method modifies the output of only two units as in the previous case. The selection of the unit pair which should change power uses weighted elements of the sensitivity matrix, and the weights are the values by which each overflow line should modify its power flow to satisfy the capacity.

For two reasons, the power modification may necessitate some repeated estimations. The first reason is the upper and lower limits of each thermal output, and the second reason is a modification direction of each power line. The direction of modification means either increase or decrease of power flow, in here. The proposed weighted method can consider many capacities summarily, but every value of overflow does not always decrease by this method. In the repeated estimation process, each modification direction of a thermal unit is fixed to avoid an oscillation condition.

2.3 Satisfaction of an Emission Constraint

An objective function is eq. (2) when the NO_x emission constraint is considered.

$$\Phi = \sum_{m=1}^{M} f_m + \lambda \cdot (Ps - \sum_{m=1}^{M} g_m) + \mu \cdot (Y - \sum_{m=1}^{M} y_m) \quad (2)$$

Where, M is the number of the thermal units in a power system, and f_m, g_m and y_m are the fuel cost, output power and NO_x emission of the m-th thermal unit respectively. f_m and y_m are estimated by eqs. (3) and (4) which are described later. λ and μ are the LaGrange's multipliers which are concerned with the power supply and demand balance and the NO_x emission constraint respectively. Ps and Y respectively are the system load and the constraint value of NO_x. The emission constraint is satisfied economically when the objective function eq. (2) is minimized.

$$f_m = a_m + b_m \cdot g_m + c_m \cdot g_m^2 \qquad (\underline{g_m} \leq g_m \leq \overline{g_m}) \tag{3}$$

Where, a_m, b_m and c_m are the characteristic constants of the m-th thermal unit. $\underline{g_m}$ and $\overline{g_m}$ are the lower and the upper limit respectively of g_m.

$$y_m = d_m \cdot f_m \tag{4}$$

Whete, d_m is the characteristic constant of the m-th thermal unit.

We recognize that the emission is constrained by the expected value which is summed up for each system state. But the proposed method controls the value of NO_x of only one state whose elements are all healthy in order to achieve rapid estimation. The other system state is operated considering only line capacity, ignoring the emission constraint.

At a completely healthy state, the modification of thermal output must be estimated considering the emission constraint. The unit pair changes the power, which is selected by eq. (5) to have the maximum and minimum E.

$$E = \sum_l \Delta i_l \cdot e_m - \mu \cdot \frac{dy_m}{dg_m} \tag{5}$$

Where, the summation of eq. (5) means the sum of the overflow lines, and Δi_l means the capacity — power flow.

3. Quadratic Programming Method

We have already reported in detail the use of quadratic programming[2] as a strict method. It is summarized as follows.

The objective function becomes eq. (6) to apply the quadratic programming method.

$$\begin{aligned}\phi = \sum_{m=1}^{M} f_m + \lambda \cdot (Ps - \sum_{m=1}^{M} g_m) + \sum_{m=1}^{M} \{\nu_m \cdot (g_m - \overline{g_m})\} \\ - \sum_{l=1}^{L} \{\nu_{M+l} \cdot (I_l + i_l)\} + \sum_{l=1}^{L} \{\nu_{M+L+l} \cdot (i_l - I_l)\} \\ + \mu' \cdot (\sum_{m=1}^{M} y_m - Y)\end{aligned} \tag{6}$$

Where, ν and μ' are the LaGrange's multipliers. L is the number of transmission power lines, and I_l is the capacity of the l-th line.

Using Kuhn-Tucker conditions in eq. (6), the constraint equations for the quadratic programming become as follows.

$$\begin{aligned}2 \cdot c_m \cdot (1 + \mu' \cdot d_m) \cdot gx_m - \lambda + \nu_m - \sum_{l=1}^{L} (\nu_{M+l} \cdot e_{lm}) \\ + \sum_{l=1}^{L} (\nu_{M+L+l} \cdot e_{lm}) - Sl_m = -(b_m + 2 \cdot c_m \cdot \underline{g_m}) \cdot (1 + \mu' \cdot d_m)\end{aligned} \tag{7}$$

$$\sum_{m=1}^{M} gx_m + Ar_1 = Ps - \sum_{m=1}^{M} \underline{g_m} \tag{8}$$

$$gx_m + Sl_{M+m} = \overline{g_m} - \underline{g_m} \tag{9}$$

$$\sum_{m=1}^{M}(e_{lm}\cdot gx_m)-Sl_{2M+l}+Ar_{l+1}=-I_l-\sum_{n=1}^{N}(e_{ln}\cdot Pl_n)-\sum_{m=1}^{M}(e_{lm}\cdot g_m) \quad (10)$$

$$\sum_{m=1}^{M}(e_{lm}\cdot gx_m)+Sl_{2M+L+l}-Ar_{L+l+1}=I_l-\sum_{n=1}^{N}(e_{ln}\cdot Pl_n)-\sum_{m=1}^{M}(e_{lm}\cdot g_m) \quad (11)$$

Where, gx_m means $g_m-\underline{g}_m$. And Sl's are slack variables, Ar's are artificial variables. Pl_n is the load of the n-th bus and the number of busses is N.

A cost function for the quadratic programming is eq. (12).

$$\begin{aligned}F&=\sum_{m=1}^{M}gx_m\cdot Sl_m+\sum_{m=1}^{M}\nu_m\cdot Sl_{M+m}\\&+\sum_{l=1}^{L}\nu_{M+l}\cdot Sl_{2M+l}+\sum_{l=1}^{L}\nu_{M+L+l}\cdot Sl_{2M+L+l}+\sum_{m=1}^{2L+1}Ar_m\\&=\frac{1}{2}\cdot\left\{\sum_{m=1}^{M}gx_m\cdot\frac{\partial F}{\partial gx_m}+\sum_{m=1}^{M}Sl_m\cdot\frac{\partial F}{\partial Sl_m}+\sum_{m=1}^{M}\nu_m\cdot\frac{\partial F}{\partial\nu_m}\right.\\&+\sum_{m=1}^{M}Sl_{M+m}\cdot\frac{\partial F}{\partial Sl_{M+m}}+\sum_{l=1}^{L}\nu_{M+l}\cdot\frac{\partial F}{\partial\nu_{M+l}}+\sum_{l=1}^{L}Sl_{2M+l}\cdot\frac{\partial F}{\partial Sl_{2M+l}}\\&\left.+\sum_{l=1}^{L}\nu_{M+L+l}\cdot\frac{\partial F}{\partial\nu_{M+L+l}}+\sum_{l=1}^{L}Sl_{2M+L+l}\cdot\frac{\partial F}{\partial Sl_{2M+L+l}}\right\}+\sum_{m=1}^{2L+1}Ar_m \quad (12)\end{aligned}$$

where, $\frac{\partial F}{\partial gx_m}$, $\frac{\partial F}{\partial Sl_m}$, $\frac{\partial F}{\partial\nu_m}$, $\frac{\partial F}{\partial Sl_{M+m}}$, $\frac{\partial F}{\partial\nu_{M+l}}$, $\frac{\partial F}{\partial Sl_{2M+l}}$, $\frac{\partial F}{\partial\nu_{M+L+l}}$ and $\frac{\partial F}{\partial Sl_{2M+L+l}}$ are handled as constants. Their values are those used in the former stage in a repeated linear minimizing process. μ' is fixed during this linear process, and its value is estimated by trial and error.

4. Comparisons using a Model Power System

Table 1 shows the thermal characteristic constants of the model system[6]. Table 2 shows line data. The load of each bus was regarded as varying in proportion to Table 3.

The economic load dispatch was estimated by the law of equal incremental fuel cost[7] at 485 [MW] as in previous simulations[1]. Economic power flow was

Table 1. Characteristic constants of thermal units

No.	Bus	$f_m=a_m+b_m\cdot g_m+c_m\cdot g_m^2$ [\$] a_m	b_m	$c_m\times1000$	d_m [kg/\$]	$\underline{g}_m$ [MW]	$\overline{g}_m$ [MW]
1	1	40	3.6	5.0	—	30	120
2	2	60	3.4	4.0	0.258	30	120
3	3	60	3.4	4.0	0.266	30	120
4	4	50	3.5	4.5	0.241	30	120
5	5	40	3.5	4.5	0.250	30	120

Note: No. 1 unit is not constrained for emission because it is constructed in a remote area.

Table 2. Line data

Line	Bus	x	Error rate	Line	Bus	x	Error rate
1	1–9	0.50	0.030	10	5–6	0.36	0.024
2	1–11	0.16	0.010	11	5–9	0.16	0.010
3	2–3	0.50	0.030	12	7–8	0.16	0.010
4	2–7	0.28	0.020	13	7–10	0.24	0.016
5	2–10	0.16	0.010	14	8–9	0.36	0.024
6	3–4	0.24	0.016	15	8–10	0.24	0.016
7	4–6	0.28	0.020	16	8–11	0.28	0.020
8	4–8	0.28	0.020	17	10–11	0.36	0.024
9	4–9	0.50	0.030				

Base: 100 MVA

Table 3. Load data

Bus	Load [MW]	Bus	Load [MW]	Bus	Load [MW]
1	0	5	0	9	121.15
2	0	6	17.31	10	43.27
3	0	7	69.23	11	43.27
4	0	8	155.77		

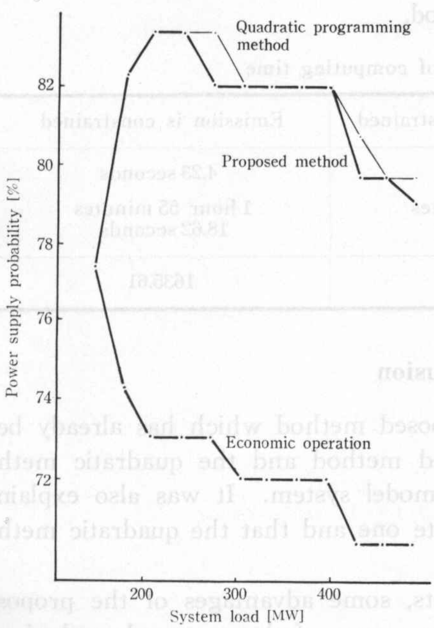

Fig. 1. Power supply probability when only No. 1 to No. 13 line capacities are considered.

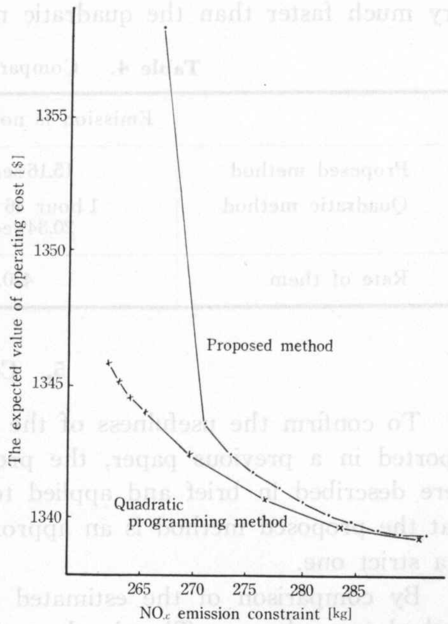

Fig. 2. The expected cost when the emission constraint is considered at 365 [MW].

obtained by the DC method. Each line capacity was also fixed at 150[%] of this economic flow as previously.

Fig. 1, the results of both methods are shown when line capacitys from the first to the 13th line are considered and the emission constraint is not. As in the previous report, economic operation shows the results of the simple law of equal incremental fuel cost. The curve of the proposed method is the same as the previous one. As Fig. 1 shows, the results of the proposed method were almost the same as those of the strict method except for some load levels. Even when the two results are different, the difference between them is very small. The biggest difference is about 1.74[%] of the power supply probability.

The load level near 365[MW] brings the same results for both methods in Fig. 1. The other results for this load level are shown in Fig. 2 when the capacities of Fig. 1 and the emission constraint are considered. The lowest costs of Fig. 2 are those at 365 [MW] of Fig. 1 which do not consider the emission constraint. The difference in these is very small in Fig. 2. Since the proposed method controls emission only during a completely healthy state and the quadratic method controls it in all states, the cost difference increases when the emission constraint decreases. But even the largest difference of cost is only about 1.11[%] of the operating cost. This shows that the estimation error of the proposed method is sufficiently small even when the emission constraint is satisfied.

Table 4 shows the computing times for each load level and each emission level when these were estimated by a PASOPIA 16 whose CPU is 8088+8087 and the clock signal is 6 MHz. The table indicates that the proposed method is very much faster than the quadratic method.

Table 4. Comparison of computing time

	Emission is not constrained	Emission is constrained
Proposed method	15.16 seconds	4.23 seconds
Quadratic method	1 hour 46 minutes 20.34 seconds	1 hour 55 minutes 18.62 seconds
Rate of them	420.87	1635.61

5. Conclusion

To confirm the usefulness of the proposed method which has already been reported in a previous paper, the proposed method and the quadratic method were described in brief and applied to a model system. It was also explained that the proposed method is an approximate one and that the quadratic method is a strict one.

By comparison of the estimated results, some advantages of the proposed method were shown. That is, the estimation error of the proposed method was lower than 2[%], and the proposed method could estimate more than 420 times as rapidly as the quadratic method when the emission constraint was not considered. This difference in estimation speed was expanded to more than 1600 times when the emission constraint was considered.

We would like acknowledge various suggestions made by Dr. Toichiro Koike, the former President of Kitami Institute of Technology. We wish to express our sincere thanks to him.

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