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A Fast Approximate Fault Simulation Method for Power Systems Contingency Selection*

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Abstract

A new contingency analysis method for contingency selection is proposed in this paper. In the method proposed, a fault is simulated by power injections to adjacent buses of the fault devices in a pre-fault system, and by essentially a non-iterative linear calculation to avoid re-calculation of a *Z*-matrix or refactor of a *Y*-matrix. As more accurate load flow calculation is available by the method proposed than by the sensitivity approach, more reliable contingency selection is available, even though the computational speed of the method is a little slower than that of the method using the sensitivity approach.

From the IEEE 30 bus example, it is found that the method is satisfactorily applied to the contingency selection from the view of both the accuracy and the computational speed. The error of the method is less than 1% in voltage and less than 1 degree in phase angle for most fault cases of the IEEE 30 bus system. The computational burden is less than 10% of that of *P-Q* decoupled load flow method in most fault cases.

1. Introduction

It is impossible to complete contingency analysis for all contingency cases, even with the newly developed digital computer, and even though contingency analysis is essential to security control in a power system. The sensitivity analysis methods or the contingency simulation method for voltage and overload contingency analysis[1~4] have been developed. However, the error of such approaches is not satisfactory for deciding security control strategies. Another feasible approach for contingency analysis is the "contingency selection" technique : several severe contingency cases are selected by using a so called "performance index" and only the selected cases are analyzed precisely by accurate calculation, for example the "Newton-Raphson" method. As the performance index has usually been calculated by the sensitivity approach, mis-ranking or mis-selection have been a major problem.

The purpose of this paper is to develop a new contingency analysis method for contingency selection. In the method proposed, a fault is simulated by power injection ($P+jQ$) to adjacent buses of the fault devices in the pre-fault power system and by essentially non-iterative linear calculation to avoid re-lcalculation of the *Z* matrix or refactor of the *Y* matrix. As more accurate load flow cal-

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culation is possible by the method proposed than by the sensitivity approach, more reliable contingency ranking or contingency selection is possible, even though the computational speed of the method is a little slower than that of the method using the sensitivity approach. Recently, computational speed is not a major problem for the contingency selection algorithm if it is restricted to several to ten times of that of the existing method, because the hardware speed is becoming fast ten times in every 2～3 years.

The fault simulation algorithm is developed in Chapter 2 and the numerical example for the IEEE 30 bus system is demonstrated in Chapter 3.

2. Fault Simulation Algorithm

2.1 Simulation of Line outage

The power system model can be described as shown in Fig. 1(a), if the fault line ij (line from bus i to bus j) is placed in the center of the figure, and the following equations hold in Fig. 1(a).

$$\left.\begin{aligned} P_i+jQ_i&=(P_{i\alpha}+P_{ij})+j(Q_{i\alpha}+Q_{ij}) \\ P_j+jQ_j&=(P_{j\beta}+P_{ji})+j(Q_{j\beta}+Q_{ji}) \end{aligned}\right\} \qquad (1)$$

Where, $P_{i\alpha}$, $Q_{i\alpha}$ are the summation of the power flows of the lines connected to the bus i except fault line ij, $P_{j\beta}$, $Q_{j\beta}$ are also the summation of the power flows of the lines connected to the bus j except line ij. P_i+jQ_i or P_j+jQ_j represents the summation of generating power and the load of bus i or bus j respectively.

The power system shown in Fig. 1(a) changes to that shown in Fig. 1(b) just after the fault of line ij. The line power flows and the bus voltages of the system in Fig. 1(b) are simulated by power injections to bus i and bus j as shown in Fig. 1(c).

Let us assume the power injected to bus i and bus j are $\Delta P_i+j\Delta Q_i$ and $\Delta P_j+j\Delta Q_j$ respectively. Then,

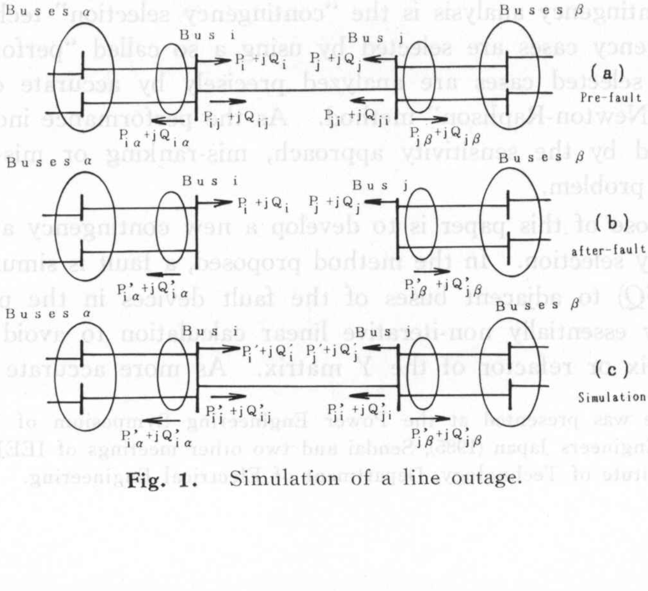

Fig. 1. Simulation of a line outage.

$$\left.\begin{aligned} P'_i+jQ'_i &= (P_i+\Delta P_i)+j(Q_i+\Delta Q_i) \\ P'_j+jQ'_j &= (P_j+\Delta P_j)+j(Q_j+\Delta Q_j) \end{aligned}\right\} \quad (2)$$

$$\left.\begin{aligned} P'_{i\alpha}+jQ'_{i\alpha} &= (P_{i\alpha}+\Delta P_{i\alpha})+j(Q_{i\alpha}+\Delta Q_{i\alpha}) \\ P'_{j\beta}+jQ'_{j\beta} &= (P_{j\beta}+\Delta P_{j\beta})+j(Q_{j\beta}+\Delta Q_{j\beta}) \end{aligned}\right\} \quad (3)$$

Where, superscript dash (′) indicates the after-fault value. If $\Delta P_i+j\Delta Q_i$ and $\Delta P_j+j\Delta Q_j$ are known, the line power flows and the bus voltages of the after-fault system are simulated by solving the following equations just once.

$$\begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta\mathbf{V} \end{bmatrix} \doteqdot \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{0} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta\mathbf{V} \end{bmatrix} \quad (4)$$

or

$$\Delta\theta = [\mathbf{Y}_{11}]^{-1}\Delta\mathbf{P} \quad (5)$$

$$\Delta\mathbf{V} = [\mathbf{Y}_{22}]^{-1}\{\Delta\mathbf{Q}-\mathbf{Y}_{21}\Delta\mathbf{Q}\} \quad (6)$$

Where,

$$\left.\begin{aligned} \Delta\mathbf{P} &= [0, \cdots, 0, \Delta P_i, 0, \cdots, 0, \Delta P_j, 0, \cdots, 0]^{\mathrm{T}} \\ \Delta\mathbf{Q} &= [0, \cdots, 0, \Delta Q_i, 0, \cdots, 0, \Delta Q_j, 0, \cdots, 0]^{\mathrm{T}} \end{aligned}\right\} \quad (7)$$

In eq, (5), for line outage, the direct current power flow method is applied ($\mathbf{Y}_{21}=0$). However, $\mathbf{Y}_{21}$ is included in the calculation when such a large power change occurs as a load outage or a generator outage. $[\mathbf{Y}_{11}]^{-1}$ and $[\mathbf{Y}_{22}]^{-1}$ are solved by using triangular factorization and the inverse of the matrix need not be calculated directly.

From Fig. 1 (a) and (c), the change in power flow is related to the change in the power injections at end bus i and j as follows[3].

$$\begin{bmatrix} \Delta P_{i\alpha} \\ \Delta P_{j\beta} \\ \Delta Q_{i\alpha} \\ \Delta Q_{i\beta} \end{bmatrix} = \begin{bmatrix} \partial P_{i\alpha}/\partial P_i & \partial P_{i\alpha}/\partial P_j & \partial P_{i\alpha}/\partial Q_i & \partial P_{i\alpha}/\partial Q_j \\ \partial P_{j\beta}/\partial P_i & \partial P_{j\beta}/\partial P_j & \partial P_{j\beta}/\partial Q_i & \partial P_{j\beta}/\partial Q_j \\ \partial Q_{i\alpha}/\partial P_i & \partial Q_{i\alpha}/\partial P_j & \partial Q_{i\alpha}/\partial Q_i & \partial Q_{i\alpha}/\partial Q_j \\ \partial Q_{j\beta}/\partial P_i & \partial Q_{j\beta}/\partial P_j & \partial Q_{j\beta}/\partial Q_i & \partial Q_{j\beta}/\partial Q_j \end{bmatrix} \begin{bmatrix} \Delta P_i \\ \Delta P_j \\ \Delta Q_i \\ \Delta Q_j \end{bmatrix} \quad (8)$$

Where,

$$\left.\begin{aligned} \Delta P_{i\alpha} &= P'_{i\alpha}-P_{i\alpha} = P_i-P_{i\alpha} = P_{ij} \\ \Delta P_{j\beta} &= P'_{j\beta}-P_{j\beta} = P_j-P_{j\beta} = P_{ji} \\ \Delta Q_{i\alpha} &= Q'_{i\alpha}-Q_{i\alpha} = Q_i-Q_{i\alpha} = Q_{ij} \\ \Delta Q_{j\beta} &= Q'_{j\beta}-Q_{j\beta} = Q_j-Q_{j\beta} = Q_{ji} \end{aligned}\right\} \quad (9)$$

Eq. (8) is rewritten by linear approximation as follows.

$$\begin{bmatrix} \Delta P_{i\alpha} \\ \Delta P_{j\beta} \\ \Delta Q_{i\alpha} \\ \Delta Q_{j\beta} \end{bmatrix} = \begin{bmatrix} P_{ij} \\ P_{ji} \\ Q_{ij} \\ Q_{ji} \end{bmatrix} = \begin{bmatrix} \partial P_{i\alpha}/\partial\theta & \partial P_{i\alpha}/\partial V \\ \partial P_{j\beta}/\partial\theta & \partial P_{j\beta}/\partial V \\ \partial Q_{i\alpha}/\partial\theta & \partial Q_{i\alpha}/\partial V \\ \partial Q_{j\beta}/\partial\theta & \partial Q_{j\beta}/\partial V \end{bmatrix}$$

$$\times\begin{bmatrix}\partial\theta/\partial P_i & \partial\theta/\partial P_j & \partial\theta/\partial Q_i & \partial\theta/\partial Q_j\\ \partial V/\partial P_i & \partial V/\partial P_j & \partial V/\partial Q_i & \partial V/\partial Q_j\end{bmatrix}\begin{bmatrix}\Delta P_i\\ \Delta P_j\\ \Delta Q_i\\ \Delta Q_j\end{bmatrix} \tag{10}$$

where, $\partial P_{i\alpha}/\partial\theta\cdots\partial Q_{j\beta}/\partial V$ and $\partial\theta/\partial P_i\cdots\partial V/\partial Q_j$ are shown more precisely in the Appendix.

Usually, as $\partial P_{i\alpha}/\partial V$, $\partial P_{j\beta}/\partial V$, $\partial V/\partial P_i$, $\partial V/\partial P_j$, $\partial\theta/\partial Q_i$, $\partial\theta/\partial Q_j$ are sufficiently small compared with other terms, eq. (10) is rewritten as follows without any remarkable error.

$$\begin{bmatrix}\Delta P_{i\alpha}\\ \Delta P_{j\beta}\\ \Delta Q_{i\alpha}\\ \Delta Q_{j\beta}\end{bmatrix}=\begin{bmatrix}P_{ij}\\ P_{ji}\\ Q_{ij}\\ Q_{ji}\end{bmatrix}\doteqdot\begin{bmatrix}\partial P_{i\alpha}/\partial\theta & 0\\ \partial P_{j\beta}/\partial\theta & 0\\ \partial Q_{i\alpha}/\partial\theta & \partial Q_{i\alpha}/\partial V\\ \partial Q_{j\beta}/\partial\theta & \partial Q_{j\beta}/\partial V\end{bmatrix}$$

$$\times\begin{bmatrix}\partial\theta/\partial P_i & \partial\theta/\partial P_j & 0 & 0\\ 0 & 0 & \partial V/\partial Q_i & \partial V/\partial Q_j\end{bmatrix}\begin{bmatrix}\Delta P_i\\ \Delta P_j\\ \Delta Q_i\\ \Delta Q_j\end{bmatrix}=\begin{bmatrix}\mathbf{H}_{11} & 0\\ \mathbf{H}_{21} & \mathbf{H}_{22}\end{bmatrix}\begin{bmatrix}\Delta P_i\\ \Delta P_j\\ \Delta Q_i\\ \Delta Q_j\end{bmatrix} \tag{11}$$

Where,

$$\left.\begin{aligned}\mathbf{H}_{11}&\doteqdot\begin{bmatrix}\partial P_{i\alpha}/\partial P_i & \partial P_{i\alpha}/\partial P_j\\ \partial P_{j\beta}/\partial P_i & \partial P_{j\beta}/\partial P_j\end{bmatrix}\\ \mathbf{H}_{21}&\doteqdot\begin{bmatrix}\partial Q_{i\alpha}/\partial P_i & \partial Q_{i\alpha}/\partial P_j\\ \partial Q_{j\beta}/\partial P_i & \partial Q_{j\beta}/\partial P_j\end{bmatrix}\\ \mathbf{H}_{22}&\doteqdot\begin{bmatrix}\partial Q_{i\alpha}/\partial Q_i & \partial Q_{i\alpha}/\partial Q_j\\ \partial Q_{j\beta}/\partial Q_i & \partial Q_{j\beta}/\partial Q_j\end{bmatrix}\end{aligned}\right\} \tag{12}$$

Eq. (11) can be solved as follows.

$$\begin{bmatrix}\Delta P_i\\ \Delta P_j\end{bmatrix}=[\mathbf{H}_{11}]^{-1}\begin{bmatrix}P_{ij}\\ P_{ji}\end{bmatrix} \tag{13}$$

$$\begin{bmatrix}\Delta Q_i\\ \Delta Q_j\end{bmatrix}=[\mathbf{H}_{22}]^{-1}\left\{\begin{bmatrix}Q_{ij}\\ Q_{ji}\end{bmatrix}-[\mathbf{H}_{21}]\begin{bmatrix}\Delta P_i\\ \Delta P_j\end{bmatrix}\right\} \tag{14}$$

The corrections to the phase angles and the magnitudes of the voltages after the fault can be determined by substituting ΔP_i, ΔP_j, ΔQ_i, ΔQ_j to eq. (4). ΔP_i, ΔP_j, ΔQ_i, ΔQ_j are calculated by eq. (13) and eq. (14).

2.2 Simulation of load outage

Load outage is simulated by a power injection to the load bus which creates the same changes of power flow as load outage does to the lines connected to the load bus. This relation is shown in Fig. 2 for the load outage at bus j. From Fig. 2, the following equation must hold.

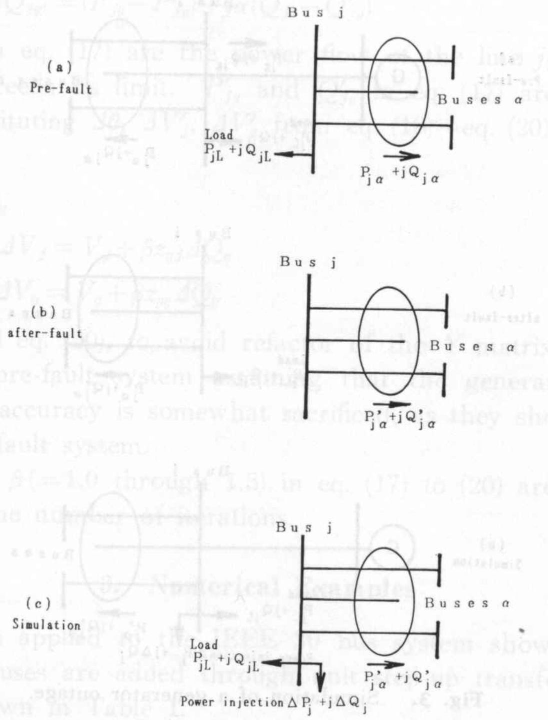

Fig. 2. Simulation of a load outage.

$$\begin{bmatrix}\Delta P_{j\alpha}\\ \Delta Q_{j\alpha}\end{bmatrix}=\begin{bmatrix}P_{jL}\\ Q_{jL}\end{bmatrix}=\begin{bmatrix}\partial P_{j\alpha}/\partial P_j & \partial P_{j\alpha}/\partial Q_j\\ \partial Q_{j\alpha}/\partial P_j & \partial Q_{j\alpha}/\partial Q_j\end{bmatrix}\begin{bmatrix}\Delta P_j\\ \Delta Q_j\end{bmatrix}$$
$$=\begin{bmatrix}\partial P_{j\alpha}/\partial\theta\cdot\partial\theta/\partial P_j & \partial P_{j\alpha}/\partial V\cdot\partial V/\partial Q_j\\ \partial Q_{j\alpha}/\partial\theta\cdot\partial\theta/\partial P_j & \partial Q_{j\alpha}/\partial V\cdot\partial V/\partial Q_j\end{bmatrix}\begin{bmatrix}\Delta P_j\\ \Delta Q_j\end{bmatrix} \quad (15)$$

Where, P_{jL} and Q_{jL} are the real and the reactive power of the load of bus j. The correction to the phase angle and the magnitude of the voltages can be determined by substituting ΔP_j, ΔQ_j calculated by eq. (15) to eq. (4).

2.3 Simulation of Generator Outage

Generator outage is also simulated elementally by a power injection to the generator bus. However, the technique developed above can not be directly applied to the generator outage case, because the generator bus is normally $P-V$ specified and does not explicitly appear in the $V-Q$ part of the load flow equation. Therefore, the power injection for simulating generator outage is injected to the bus adjacent to the generator bus. The total power flow change of the power system side lines connected to the bus j (adjacent to the bus g) must be equal to the power flow of the line ij just before the fault. From Fig. 3, the following equation must hold.

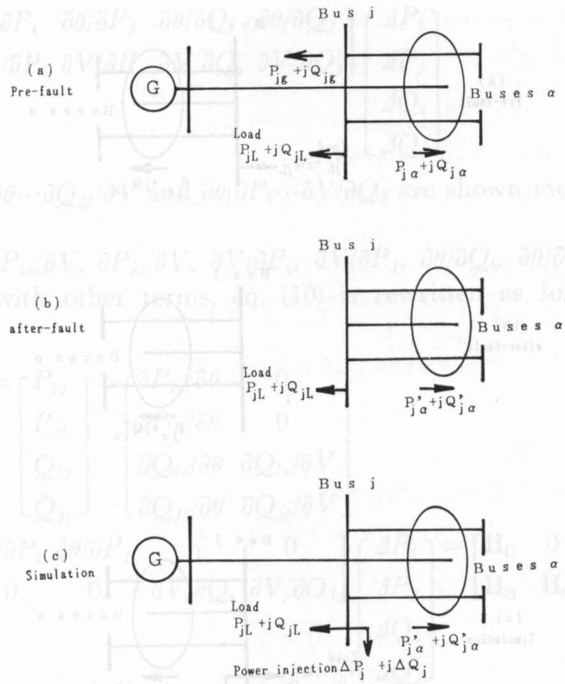

Fig. 3. Simulation of a generator outage.

$$\begin{bmatrix} \Delta P_{j\alpha} \\ \Delta Q_{j\alpha} \end{bmatrix} = \begin{bmatrix} P_{jg} \\ Q_{jg} \end{bmatrix} = \begin{bmatrix} \partial P_{j\alpha}/\partial P_j & \partial P_{i\alpha}/\partial Q_j \\ \partial Q_{j\alpha}/\partial P_j & \partial Q_{j\alpha}/\partial Q_j \end{bmatrix} \begin{bmatrix} \Delta P_j \\ \Delta Q_j \end{bmatrix}$$
$$= \begin{bmatrix} \partial P_{j\alpha}/\partial\theta \cdot \partial\theta/\partial P_j & \partial P_{j\alpha}/\partial V \cdot \partial V/\partial Q_j \\ \partial Q_{j\alpha}/\partial\theta \cdot \partial\theta/\partial P_j & \partial Q_{j\alpha}/\partial V \cdot \partial V/\partial Q_j \end{bmatrix} \begin{bmatrix} \Delta P_j \\ \Delta Q_j \end{bmatrix} \qquad (16)$$

The line power flows and the magnitude of bus voltages just after the fault are calculated by eq. (4) and ΔP_j, ΔQ_j calculated from eq. (16). In this paper, it is assumed that bus g is connected to bus j only through a unit step-up transformer impedance as shown in Fig. 3.

2.4 Treatment of Generator Q-Limits

In some fault conditions, generator reactive power exceeds its upper or lower limit in order to hold its desired terminal voltage as a consequence of the simulation above. However, in a real system, the generator changes its terminal voltage and fixes its reactive power output to its limit in such a situation. This means that the generator bus type should be changed from $P-V$ specified to $P-Q$ specified in the simulation. This situation can be simulated as follows avoiding the refactor of the Y matrix.

Bus injections ΔP_j and ΔQ_j are calculated by substituting ΔP_{jg}, $\alpha\Delta Q_{jg}$ to P_{jg} and Q_{jg} in eq. (16) respectively. The bus injections calculated by eq. (16) are iteratively injected to the bus j adjacent to generator bus g until $\Delta Q_g=0$ is reached. Where, ΔP_{jg} and $\alpha\Delta Q_{jg}$ are calculated from eq. (17).

$$(\Delta P_{jg}+j\alpha\Delta Q_{jg})=(P_{jg}-P'_{jg})+j\alpha(Q_{jg}-Q'_{jg}) \tag{17}$$

P_{jg} and Q_{jg} in eq. (17) are the power flow of the line jg when generator reactive power exceeds its limit. P'_{jg} and Q'_{jg} in eq. (17) are the power flow calculated by substituting $\Delta\theta$, $\Delta V'_j$, $\Delta V'_g$ from eq. (18)~eq. (20) to the load flow equations.

$$\Delta\theta=\theta_j-\theta_g \tag{18}$$

$$V'_j=V_j+\Delta V_j=V_j+\beta z_{gj}\Delta Q_g \tag{19}$$

$$V'_g=V_g+\Delta V_g=V_g+\beta z_{gg}\Delta Q_g \tag{20}$$

In eq. (19) and eq. (20), to avoid refactor of the Y matrix, Z_{gj} and Z_{gg} are calculated in the pre-fault system assuming that the generator bus is $P-Q$ specified, although accuracy is somewhat sacrificed, as they should be calculated by using the after-fault system.

$\alpha=Q_g/Q'_{jg}$ and $\beta\,(=1.0$ through $1.5)$ in eq. (17) to (20) are the acceleration factors to reduce the number of iterations.

3. Numerical Examples

The method is applied to the IEEE 30 bus system shown in Fig. 4. In Fig. 4, generator buses are added through unit step-up transformers and their impedances are shown in Table 1.

The worst 10 cases selected from the simulated load flow result are shown in Table 2. In the simulation, $Y_{21}=0$ when it is assumed that the change of real power by the fault is less than 0.05 (P.U.) and $\beta=1.3$. From Table 2, it can be found that the absolute error compared to precise calculation is less than 1% in voltage and less than 1 degree in phase angle for almost all cases. The cases in which the error exceeds the

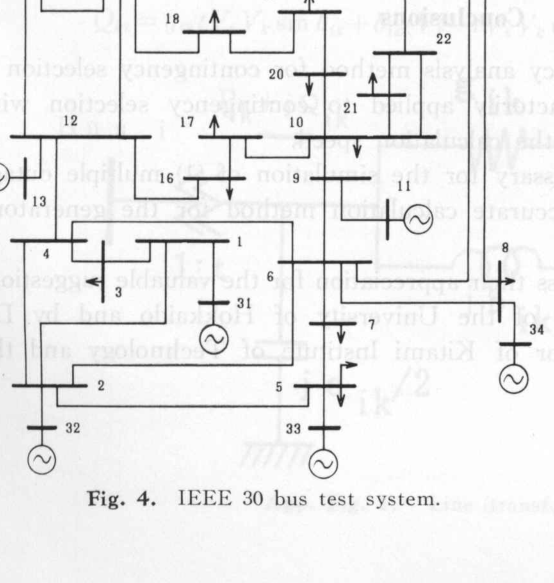

Fig. 4. IEEE 30 bus test system.

Table 1. Impedances for Aditional Lines

LINES	$r+jx$
42 (L 31 1)	$0.005+j0.05$
43 (L 32 2)	$0.01+j0.1$
44 (L 33 5)	$0.01+j0.1$
45 (L 34 8)	$0.01+j0.1$

Table 2. Computational Result IEEE 30 Bus Example

Fault cases	Absolute Error (Bus voltage) [P. U.]		Absolute Error (Phase Angle) [degree]	
	Average	Maximum	Average	Maximum
G 32	0.0234	0.0281	2.2302	3.1890
G 33	0.0081	0.0101	0.7592	0.9438
G 34	0.0180	0.0217	0.8923	1.1575
D 5	0.0050	0.0263	0.9805	1.0956
D 21	0.0014	0.0058	0.9595	1.1989
D 30	0.0048	0.0389	0.4163	0.6842
L 4 12	0.0191	0.0482	0.3857	0.7068
L 6 7	0.0042	0.0307	0.3225	0.8932
L 15 23	0.0056	0.0141	0.0795	0.8465
L 16 17	0.0055	0.0177	0.0346	0.2430

Fault cases: **G***i*: generator outage, **D***i*: load outage, **L***ij*: line *ij* outage

allowable limits (1% or 1 degree) are such cases as generator outage cases, line-or-load fault cases near the generator bus or in cases where loops are forced to open by the fault. As the reactive power of each remaining generator hits its limit in almost all the generator outage cases, it is easily inferred that the treatment of the generator Q-limit may be the cause of the error. This is true but it is difficult to improve the accuracy, because it is caused by heavy non-linearity of the $V-Q$ relation when generator reactive power exceeds its limit. However such special cases are restricted, and easily predicted if the system is once determined. Then the method can be satisfactorily applied to contingency selection. The computational burden is less than 10% of that of the $P-Q$ decoupled load flow method in almost all fault cases.

4. Conclusions

A fast approximate contingency analysis method for contingency selection is proposed. The method is satisfactorily applied to contingency selection with regard to both the accuracy and the calculation speed.

Further consideration is necessary for the simulation of (1) multiple outage (2) bus outage and (3) a more accurate calculation method for the generator's Q-limit violation.

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Appendix

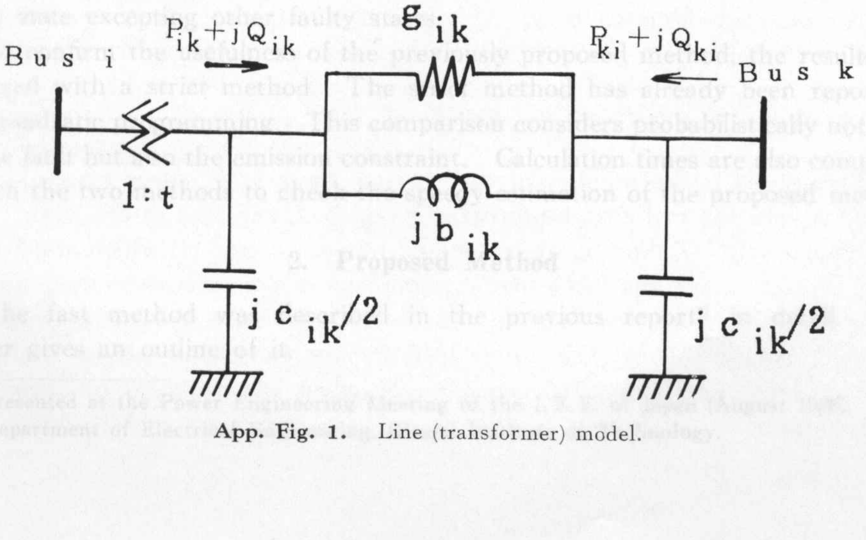
Calculation of $\partial P_{i\alpha}/\partial\theta, \cdots, \partial Q_{j\beta}/\partial V, \partial\theta/\partial P_i, \cdots, \partial V/\partial Q_j$ $\partial P_{i\alpha}/\partial\theta, \cdots, \partial Q_{j\beta}/\partial V$ are vectors and are shown below.

$$\left.\begin{aligned}
\partial P_{i\alpha}/\partial\theta &= [\partial P_{i\alpha}/\partial\theta_1,\ \partial P_{i\alpha}/\partial\theta_2,\ \cdots, \partial P_{i\alpha}/\partial\theta_l,\ \cdots, \partial P_{i\alpha}/\partial\theta_n] \\
\partial P_{j\beta}/\partial\theta &= [\partial P_{j\beta}/\partial\theta_1,\ \partial P_{j\beta}/\partial\theta_2,\ \cdots, \partial P_{j\beta}/\partial\theta_l,\ \cdots, \partial P_{j\beta}/\partial\theta_n] \\
&\ \vdots \\
\partial P_{j\beta}/\partial V &= [\partial Q_{j\beta}/\partial V_1, \partial Q_{j\beta}/\partial V_2, \cdots, \partial Q_{j\beta}/\partial V_l, \cdots, \partial Q_{j\beta}/\partial V_n]
\end{aligned}\right\} \quad \text{(App. 1)}$$

$$P_{i\alpha} = \sum_{k\in\alpha} P_{ik},\ P_{j\beta} = \sum_{k\in\beta} P_{jk},\ Q_{i\alpha} = \sum_{k\in\alpha} Q_{ik},\ Q_{j\beta} = \sum_{k\in\beta} Q_{jk} \quad \text{(App. 2)}$$

$\partial P_{i\alpha}/\partial\theta_l$ the elements of the vector $\partial P_{i\alpha}/\partial\theta$, have a value when $l=k$ or $l=i$. These values can be calculated as eq. (App. 4) and eq. (App. 5) by referencing App. Fig. 1 and eq. (App. 3).

$$\left.\begin{aligned}
P_{ik} &= b_{ik} t V_i V_k \sin\theta_{ik} + g_{ik}(t^2 V_i^2 - t V_i V_k \cos\theta_{ik}) \\
P_{ki} &= -b_{ik} t V_i V_k \sin\theta_{ik} + g_{ik}(V_k^2 - t V_i V_k \cos\theta_{ik}) \\
Q_{ik} &= -g_{ik} t V_i V_k \sin\theta_{ik} + b_{ik}(t^2 V_i^2 - t V_i V_k \cos\theta_{ik}) - c_{ik} t^2 V_i^2 \\
Q_{ki} &= g_{ik} t V_i V_k \sin\theta_{ik} + b_{ik}(V_k^2 - t V_i V_k \cos\theta_{ik}) - C_{ik} V_k^2
\end{aligned}\right\} \quad \text{(App. 3)}$$



App. Fig. 1. Line (transformer) model.

$$\left.\begin{aligned}
\partial P_{ik}/\partial\theta_k &= -b_{ik}tV_iV_k\cos\theta_{ik}-g_{ik}tV_iV_k\sin\theta_{ik}\\
&\fallingdotseq -b_{ik}tV_k\cos\theta_{ik}\\
\partial P_{ik}/\partial\theta_i &= -\partial P_{ik}/\partial\theta_k\\
\partial P_{ki}/\partial\theta_i &= -b_{ik}tV_iV_k\cos\theta_{ik}+g_{ik}tV_iV_k\sin\theta_{ik}\\
&\fallingdotseq b_{ik}tV_iV_k\cos\theta_{ik}\\
\partial P_{ki}/\partial\theta_k &= -\partial P_{ki}/\partial\theta_i
\end{aligned}\right\}\qquad\text{(App. 4)}$$

$$\left.\begin{aligned}
\partial(Q_{ik}/V_i)/\partial V_k &= -g_{ik}t\sin\theta_{ik}-b_{ik}t\cos\theta_{ik}\\
\partial(Q_{ik}/V_i)/\partial V_i &= -g_{ik}t(V_k/V_i)\sin\theta_{ik}+b_{ik}\\
&\quad\times\{2t^2-t(V_k/V_i)\cos\theta_{ik}\}-2c_{ik}t\\
\partial(Q_{ki}/V_k)/\partial V_i &= g_{ik}t\sin\theta_{ik}+b_{ik}t\cos\theta_{ik}\\
\partial(Q_{ki}/V_k)/\partial V_k &= g_{ik}t(V_i/V_k)\sin\theta_{ik}+b_{ik}\\
&\quad\times\{2-t(V_i/V_k)\cos\theta_{ik}\}-2c_{ik}\\
\partial(Q_{ik}/V_i)/\partial\theta_k &= g_{ik}tV_k\sin\theta_{ik}-b_{ik}tV_k\cos\theta_{ik}\\
\partial(Q_{ik}/V_i)/\partial\theta_i &= -\partial(Q_{ik}/V_i)/\partial\theta_k\\
\partial(Q_{ki}/V_i)/\partial\theta_i &= g_{ik}tV_i\sin\theta_{ik}+b_{ik}tV_i\cos\theta_{ik}\\
\partial(Q_{ki}/V_k)/\partial\theta_k &= -\partial(Q_{ki}/V_k)/\partial\theta_i
\end{aligned}\right\}\qquad\text{(App. 5)}$$

On the other hand, $\partial\theta/\partial P_i, \cdots, \partial V/\partial Q_j$ are also vectors and shown as follows.

$$\left.\begin{aligned}
\partial\theta/\partial P_i &= [\partial\theta_1/\partial P_i,\ \partial\theta_2/\partial P_i,\ \cdots, \partial\theta_l/\partial P_i,\ \cdots, \partial\theta_n/\partial P_i]^T\\
&\ \ \vdots \\
\partial V/\partial Q_j &= [\partial V_1/\partial Q_j, \partial V_2/\partial Q_j, \cdots, \partial V_l/\partial Q_j, \cdots, \partial V_n/\partial Q_j]^T
\end{aligned}\right\}\qquad\text{(App. 6)}$$

Each vector of the eq. (App. 7) is corresponding to the ith column or the jth column of the $P-Q$ decoupled Z-bus matrix. The elements of the vectors $k=i(k=j)$, bus k is adjacent to bus $i(j)$ (bus k is referred to as next bus) or bus k is the bus adjacent to the next bus $i(j)$ are necessary for the calculation of eq. (8). These values can be calculated by the sparse Z-bus matrix method[5] without calculating the inverse of the Y matrix.