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Estimation of Solar Radiation on a Tilted Surface with any Inclination and Direction Angle*

by Hiromu BABA** and Kimio KANAYAMA**

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Abstract

To predict the average total energy of solar radiation on a plate-type solar collector, the authors' procedure is explained which yields the monthly average solar radiation incident upon any tilted surface with various angles of inclination and direction. According to this method, the calculation from our measurements of the horizontal total and the normal direct solar radiations, the estimation from the sunshine hours obtained by Yoshidas' equation and the estimation using Berlage's equation were compared with each other, and with direct measurements of the solar radiation on a tilted suaface with a fixed inclination angle of 60° from horizontal at Kitami. Reasonably good agreement among them was generally obtained.

In terms of the inclination factor, therefore, the solar radiation on a tilted surface with various angles of inclination and direction at Kitami was estimated from the sunshine hours and the insolation map. The results of this estimation were then compared with those accurately calculated from our measurements of the horizontal total and normal direct solar radiations, and the direct measurements from the 60° tilted surface. Moreover, the solar radiation on a tilted surface at Sapporo, Sendai and Tokyo was estimated by means of this procedure using the sunshine hours and insolation map as a parameter of angles of inclination and direction.

1. Introduction

From the viewpoint of solar energy utilization, it is basically important to evaluate exactly the solar radiation incident upon the surface of a plate-type solar collector with any inclination and direction angle. However, the angle of the incident ray on a tilted surface changes hourly, and the intensity of solar radiation on a tilted surface also changes depending on the weather conditions at any location. Therefore, it is not easy to calculate exactly the solar radiation incident on a solar collector surface. For this purpose, global insolation must be separated into the direct and scattered components, and several investigations [1]~[9] on the subject of direct-scattered component separation of solar radiation have been published hitherto.

For the prediction of the average total energy of solar radiation upon a solar collector, the average solar radiation on a tilted surface over a long period, for instance, one day or one moth, is more effective than the hourly solar radiation on any day. With this point of view, using the observed global insolation incident on a horizontal surface, Liu and Jordan [2]~[4] have published a method to

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** Department of Mechanical Engineering, Kitami Institute of Technology.

calculate the daily average solar radiation on a tilted surface with various inclination angles toward due south only. Recently, Yoshida and Shinoki [10] have presented a procedure to estimate the monthly average solar radiation on a horizontal from the number of sunshine hours by treating statistically the data from weather observatories over a long period, and obtained to estimate an equation of correlation to separate the monthly average solar radiation obtained in this way into the two components of direct and scattered radiation. They have then made an insolation map indicating by contour lines the average solar radiation on a horizontal surface throughout the country of Japan [11], [12].

The authors [13] have also published a procedure for estimating the monthly average solar radiation on any tilted surface with various angles of inclination and direction, using the monthly average solar radiation on a horizontal surface and the correlation equation derived by Yoshida et al. Besides calculating the direct-scattered component separation of solar radiation by this procedure, it was also clarified that approximate values for the solar radiation on a tilted surface could also be obtained from the solar insolation map above mentioned.

In this paper, the authors first explain the procedure which yields the monthly average solar radiation on any tilted surface with various angles of inclination and direction and then by using the number of sunshine hours, the insolation map and Berlage's equation, the monthly average solar radiation on a tilted surface is estimated. Lastly the results calculated by the sunshine hours and the insolation map at special points for certain localities are presented for various angles of inclination and direction in terms of the inclination factor.

On the other hand, as the total radiation on a horizontal, the direct radiation on a normal and the total radiation on an inclined surface have all been measured at Kitami Institute of Technology since 1978 [14], [15], from the data of both the horizontal total radiation and the normal direct radiation, the energy of solar radiation on a tilted surface could be calculated with changing angles of inclination and direction. The monthly average solar radiation values on the tilted surface estimated from the number of sunshine hours and from the insolation map were not only compared with each other, but also compared with the results accurately calculated from our measurements of the horizontal total radiation and the normal direct radiation. They were also directly compared with one point of our measurements for the tilted surface with an inclination angle of 60° facing due south. Subsequently, the values estimated from the sunshine hours and the insolation map at Sapporo, Sendai and Tokyo were displayed, and the differences in the monthly average solar radiation on the tilted surface depending on locality were compared.

2. Calculation Method of the Inclination Factor

An outline of the authors' procedure for calculating an inclination factor to produce the monthly average solar radiation incident upon a tilted collector surface with any inclination and direction angle from the monthly average on a normal is as follows:

The total solar radiation received by a tilted collector surface is composed of three components; the direct solar radiation from the sun, the scattered solar radiation from the sky, and the reflected solar radiation from the ground. We can obtain the hourly total solar radiation incident upon the tilted surface, I_{IT}, by Eq. (1) in which the hourly direct solar radiation on a normal, I_D, the hourly scattered solar radiation, I_S, and the hourly total solar radiation on a horizontal, I_{HT}, are contained.

$$I_{IT}=I_D\cos i+I_S(1+\cos\theta)/2+\rho I_{HT}(1-\cos\theta)/2 \qquad (1)$$

Where, i is the solar incident angle on the tilted surface and is given by

$$\cos i=\sin h\cos\theta+\cos h\sin\theta\cos(|A-\alpha|) \qquad (2)$$

$$\sin h=\sin\phi\sin\delta+\cos\phi\cos\delta\cos t \qquad (3)$$

$$\sin A=\frac{\sin t\sin\delta}{\cos h} \qquad (4)$$

where, h: angle of solar altitude (= solar incident angle on a horizontal), θ: inclination angle of tilted surface from the horizontal, A: solar direction angle, α: direction angle of the tilted surface from due south (= zero for due south, negative for east and positive for west), ϕ: latitude, δ: solar declination, t: hour angle (= zero at solar noon, and one hour equals 15° of longitude with negative sign in the morning and positive sign in the afternoon), and ρ: albedo of the ground.

The relationship between the angles of tilted and horizontal surfaces, and the sun is shown in Fig. 1.

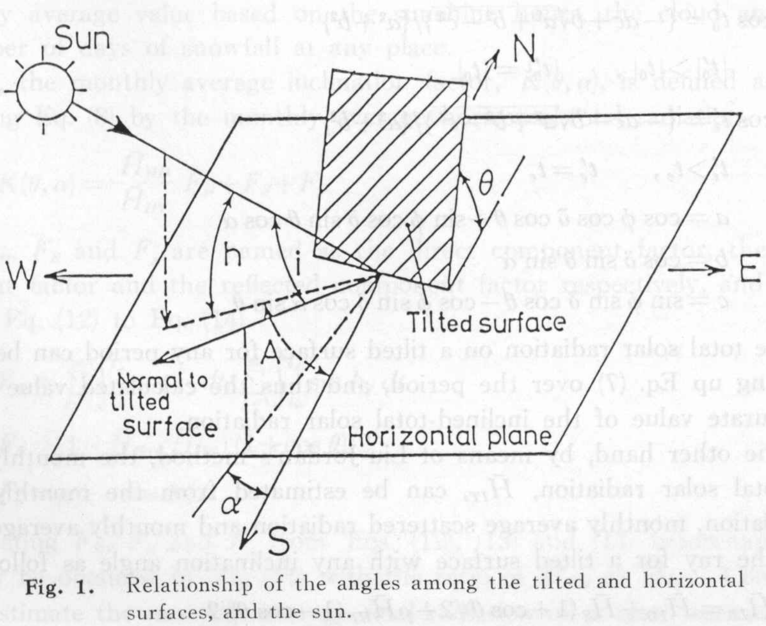

Fig. 1. Relationship of the angles among the tilted and horizontal surfaces, and the sun.

The solar declination δ is given by

$$\delta = 0.000504 + \cos\omega(-0.008562 + 0.011635\cos\omega - 0.001172\cos^2\omega) + \sin\omega(0.408858 + 0.001708\cos\omega - 0.012879\cos^2\omega) \quad (5)$$

ω is the angle of the earth on the revolution orbit measured from the vernal equinox and is given by Eq. (6) using the number of days n from January 1st.

$$\omega = 2\pi(n-80)/365 \qquad (n \geq 80)$$

or

$$2\pi(285+n)/365 \qquad (n < 80) \quad (6)$$

Next, the daily total solar radiation on a tilted surface is obtained by integrating Eq. (1) over the time from sunrise to sunset.

That is,

$$H_{IT} = \int_{t_0}^{t_e} I_{IT}\,dt = \int_{t_0'}^{t_e'} I_D \cos i\,dt + \int_{t_0}^{t_e} I_S(1+\cos\theta)/2\,dt + \rho\int_{t_0}^{t_e} I_{HT}(1-\cos\theta)/2\,dt \quad (7)$$

where, t_0: time angle of sunrise, t_e: time angle of sunset, t_0': time angle when a tilted surface is just irradiated by the direct radiation, and t_e': time angle when a tilted surface is just shaded from the direct radiation. These are obtained by the following equations.

$$\cos t_e = -\sin\phi\sin\delta/\cos\phi\cos\delta$$

$$t_0 = -t_e$$

$$\cos t_0' = \{-ac + b\sqrt{a^2+b^2-c^2}\}/\{a^2+b^2\}$$

$$|t_0'| > |t_0|, \qquad |t_0'| = |t_0|$$

$$\cos t_e' = \{-ac - b\sqrt{a^2+b^2-c^2}\}/\{a^2+b^2\}$$

$$t_e' > t_e, \qquad t_e' = t_e$$

$$a = \cos\phi\cos\delta\cos\theta + \sin\phi\cos\delta\sin\theta\cos\alpha$$

$$b = \cos\delta\sin\theta\sin\alpha$$

$$c = \sin\phi\sin\delta\cos\theta - \cos\phi\sin\delta\cos\alpha\sin\theta$$

Hence, the total solar radiation on a tilted surface for any period can be obtained by summing up Eq. (7) over the period, and thus the calculated value is named as an accurate value of the inclined-total solar radiation.

On the other hand, by means of Liu-Jordan's method, the monthly average inclined-total solar radiation, $\bar{H}_{IT}$, can be estimated from the monthly average direct radiation, monthly average scattered radiation and monthly average incident angle of the ray for a tilted surface with any inclination angle as follows:

$$\bar{H}_{IT} = \bar{H}_{ID} + \bar{H}_S(1+\cos\theta)/2 + \rho\bar{H}_{HT}(1-\cos\theta)/2 \quad (8)$$

$$\bar{H}_{ID}=\bar{H}_{HD}\sum_{j=1}^{m}\int_{t_0'}^{t_e'}\cos i_j\,dt\Big/\sum_{j=1}^{m}\int_{t_0}^{t_e}\sin h_j\,dt$$

where, $\bar{H}_{IT}$: monthly average inclined-total radiation, $\bar{H}_{ID}$: monthly average inclined-direct radiation, $\bar{H}_S$: monthly average scattered radiation, $\bar{H}_{HT}$: monthly average horizontal-total radiation, j: j-th day during measurement period, $\bar{H}_{HD}$: monthly average horizontal-direct radiation.

Besides, the estimation equation of the monthly average horizontal-total radiation and the direct-scattered component separation derived by Yoshida et al. is

$$\bar{K}_T=\bar{H}_{HT}/\bar{H}_{H0}=0.146+0.534\,n/N+0.036\sin h_{15}+0.047\,G_{10} \qquad (9)$$

where, $\bar{K}_T$: monthly average clear day index, $\bar{H}_{H0}$: monthly average extraterrestrial total radiation, n: monthly actual sunshine hours, N: monthly maximum possible sunshine hours, h_{15}: angle of the solar altitude at solar noon in the middle of the month, and G_{10}: snowfall index.

The equations for the direct-scattered component separation are

$$\bar{H}_{HD}=(1-\bar{K})\,\bar{H}_{HT}$$
$$\bar{H}_S=\bar{K}\bar{H}_{HT} \qquad (10)$$

where,

$$\bar{K}=\begin{cases}0.921-0.171\,\bar{K}_T-0.852\,n/N\\ 1.215-2.715\,\bar{K}_T+0.909\,\bar{K}_T^2+0.528\,C_i & \bar{K}_T\leqq 0.48\\ 1.402-3.207\,\bar{K}_T+2.224\,\bar{K}_T^2+0.528\,C_i & \bar{K}_T>0.48\end{cases}$$

$$C_i=n/N+C_m-1 \qquad C_m:\text{ Cloud amount}$$

From Eqs. (8), (9) and (10), the inclined-total radiation can be calculated as a monthly average value based on the sunshine hours, the cloud amount and the number of days of snowfall at any place.

Next, the monthly average inclination factor, $K(\theta,\alpha)$, is defined as Eq. (11) by dividing Eq. (8) by the monthly average horizontal-total radiation.

$$K(\theta,\alpha)=\frac{\bar{H}_{HD}}{\bar{H}_{HT}}F_D+F_S+F_\rho \qquad (11)$$

where, F_D, F_S and F_ρ are named as the direct component factor, the scattered component factor and the reflected component factor respectively, and these are given by Eq. (12) to Eq. (14).

$$F_D=\sum_{j=1}^{m}\int_{t_0'}^{t_e'}\cos i_j\,dt\Big/\sum_{j=1}^{m}\int_{t_0}^{t_e}\sin h_j\,dt \qquad (12)$$

$$F_S=(1-\bar{H}_{HD}/\bar{H}_{HT})(1+\cos\theta)/2 \qquad (13)$$

$$F_\rho=\rho(1-\cos\theta)/2 \qquad (14)$$

By calculating F_D, F_S and F_ρ from Eqs. (12), (13) and (14) beforehand, $K(\theta,\alpha)$ can easily be obtained by Eq. (11) with the ratio of $\bar{H}_{HD}$ to $\bar{H}_{HT}$. Consequently, we can estimate the monthly average total radiation on a tilted surface, $\bar{H}_{IT}$, by

Eq. (13). In this case, the monthly average total radiation and the monthly average direct radiation on the horizontal are obtained from either the number of sunshine hours by using the correlation equation or the insolation map presented by Yoshida et al..

Figure 2 shows an example of the calculated results of the factor of fraction

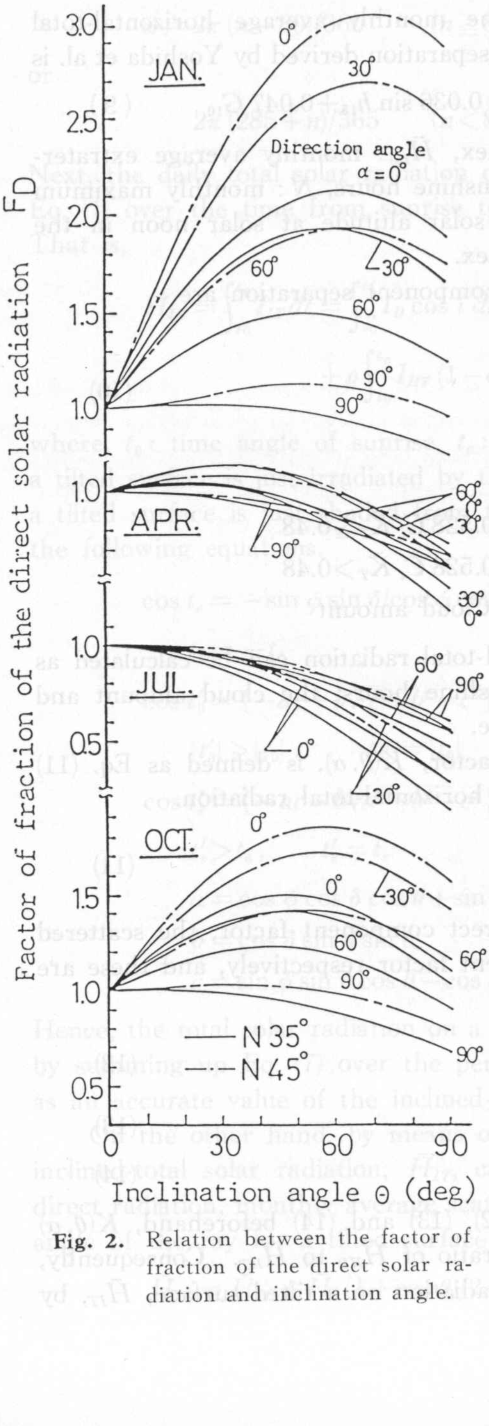

Fig. 2. Relation between the factor of fraction of the direct solar radiation and inclination angle.

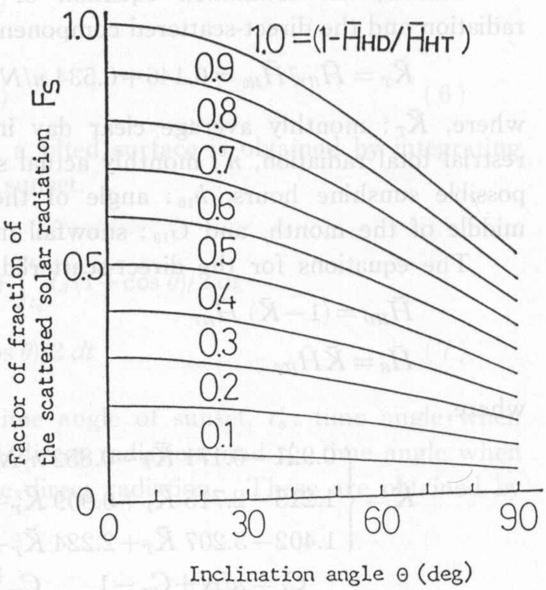

Fig. 3. Relation between the factor of fraction of the scattered solar radiation and inclination angle.

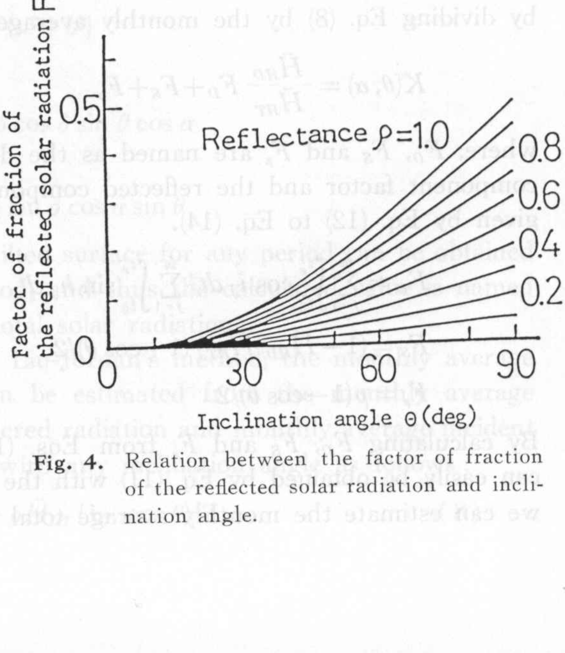

Fig. 4. Relation between the factor of fraction of the reflected solar radiation and inclination angle.

Table 1. Process in order to obtain the inclination factor from the insolation map at Kitami

Kitami 43°49′N⇒44°N JAN. $\alpha=0°$ $\rho=0.5$								
θ	$F_D(40°)$	$F_D(45°)$	$F_D(44°)$	M	$F_D \cdot M$	F_S	F_ρ	$K(\theta, 0)$
0°	1.00	1.00	1.00	$\bar{H}_{HT}=2.5\text{MJ/m}^2\text{d}$	0.45	0.55	0.0	1.00
30°	2.01	2.30	2.24	$\bar{H}_{HD}=5.6\text{MJ/m}^2\text{d}$	1.00	0.51	0.03	1.54
60°	2.48	2.98	2.88	$\frac{\bar{H}_{HD}}{\bar{H}_{HT}}=0.45$	1.29	0.41	0.13	1.83
90°	2.28	2.86	2.75		1.23	0.28	0.25	1.76

of the direct radiation component, F_D, at the latitudes of 35°N and 45°N with a parameter of the inclination angles from 0° to 90° and the direction angles of 0°, 30°, 60° and 90°, respectively. Figure 3 shows the factor of fraction of the scattered radiation component, F_S, with a parameter of factor $(1-\bar{H}_{HD}/\bar{H}_{HT})$. Figure 4 shows the factor of fraction of the reflected radiation component for K, F_ρ, with a parameter of albedo ρ. The process of obtaining the $K(\theta, \alpha)$ at Kitami City if using the insolation map is shown in Table 1.

3. Comparison between the Calculated and Measured Results, and Discussion thereof

Figure 5 shows the measurements of the monthly total solar radiation on the horizontal on the roof of our laboratory for the five years from 1978 to 1982, and the estimations from the number of sunshine hours at Kitami City (43°49′N, 143°55′E). Comparing the two values, the estimations agree very well with the measurements, except that the estimations of 540 MJ/(m²mon) in July 1978, are 10% smaller than the measurements of about 600 MJ/(m²mon), and in August 1979, and for the three months from March to May 1980, the estimated values are 6 to 11% larger than those measured. Considering that an error of a few percent may be contained within the measurement technique itself, it seems that the two values coincide with each other sufficiently accurately. Any influence on the measurement results because the pyranometer was changed in July, 1981, is scarcely noticeable.

Figure 6 shows values for monthly total solar radiation on a tilted surface of 60° from the horizontal produced by the following four methods: (1) our direct measurements of the monthly solar radiation, (2) calculations based on our measurements of the hourly horizontal total radiation and hourly normal direct radiation, (3) calculations obtained by means of Berlage's equation from the measurements of the hourly horizontal total radiation, and (4) estimation from the number of sunshine hours.

The values calculated by method (2) are larger than those measured for the four months from April to July (especially July) 1978, for July and September 1979, and for the four months from April to July 1982. For all periods of the

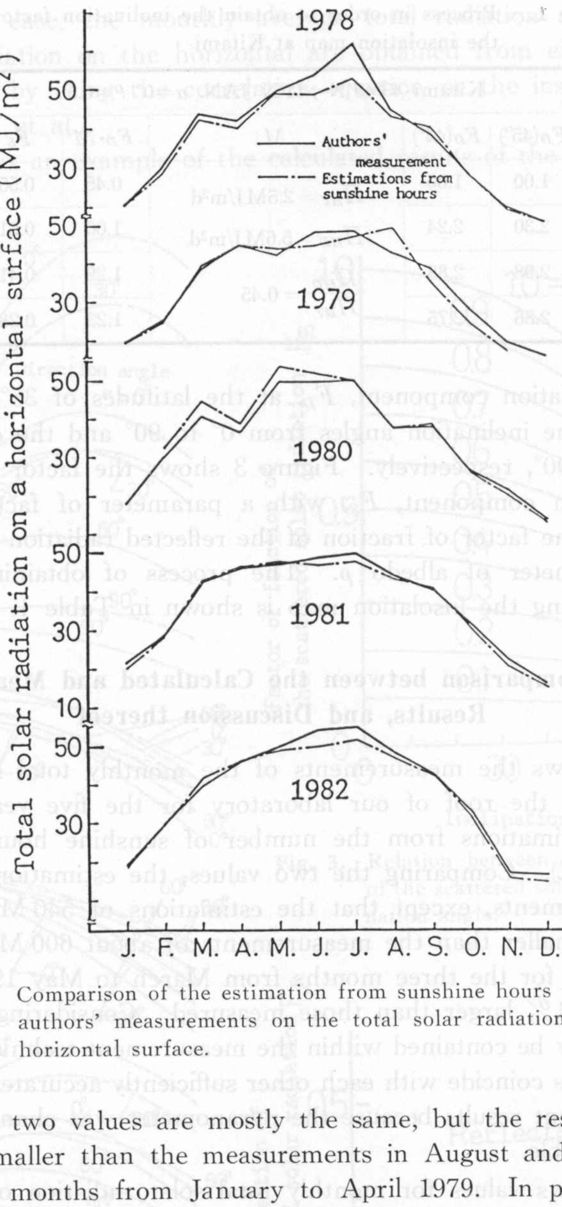

Fig. 5. Comparison of the estimation from sunshine hours with the authors' measurements on the total solar radiation on the horizontal surface.

year in 1980, the two values are mostly the same, but the results calculated by method (2) are smaller than the measurements in August and September 1978, and for the four months from January to April 1979. In particular the very small values for the monthly total solar radiation in August and September 1981 were caused by the fact that no measurements were made for 19 days over both months because the instruments were under repair. Apart from these values, however, the deviation between the two values is within about 7% and the maximum value of error is 13% in February 1979.

The calculations by method (3) are considerably larger than the measurements for the four months from January to April 1978, for the two periods from March to April and from September to December 1979, for the nine months excluding May to July of 1980, and for all seasons of the two years of 1981 and

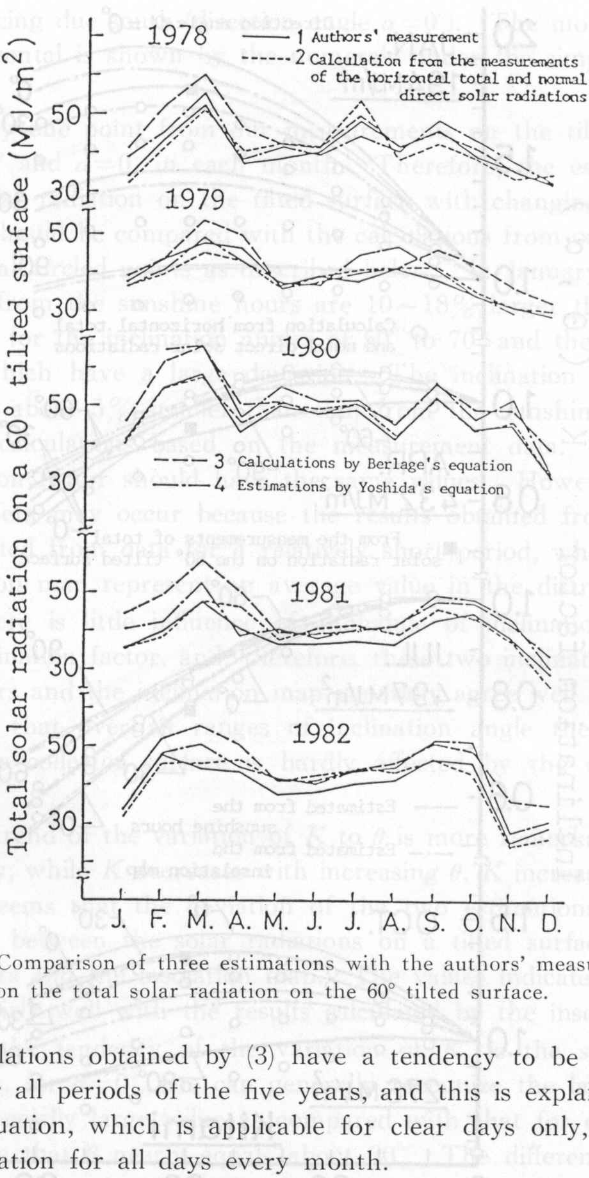

Fig. 6. Comparison of three estimations with the authors' measurements on the total solar radiation on the 60° tilted surface.

1982. The calculations obtained by (3) have a tendency to be larger than the measurements for all periods of the five years, and this is explained by the fact that Berlage's equation, which is applicable for clear days only, was applied to calculate the radiation for all days every month.

The estimations by method (4) are smaller than the measurements for the two months of July and August 1978; especially, for the five months of January, February, April, June and November 1979, and in April, 1981. All the values of estimation for months other than those mentioned above in each year are larger than the measurements. There are rare cases that the estimations are smaller than the measurements, but the values obtained by methods (3) and (4) tend to be larger than the measurements on the whole.

Figure 7 shows the relationship of the inclination factor, $K(\theta, \alpha)$, which is related to evaluate the monthly total radiation on a tilted surface, versus the inclination angle θ, with a parameter of the direction angle α in January, April,

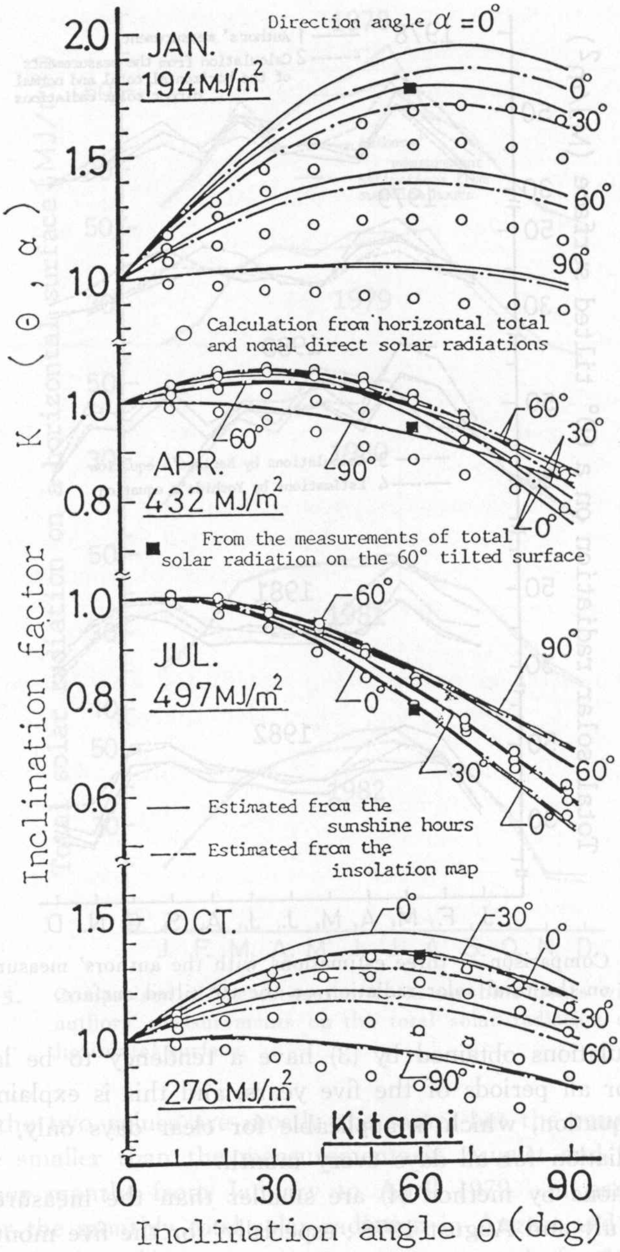

Fig. 7. Relation between the inclination factor and inclination angle at Kitami.

July and October at Kitami City. The solid line in the figure shows the estimations obtained from the number of sunshine hours of the observation data for the five years from 1978 to 1982, and the chain line shows the estimations from the insolation map. The circled points indicate K calculated by means of method (2), and the black squared points indicate K from our measurements on the tilted surface of 60° facing due south (direction angle $\alpha=0°$). The monthly total radiation on the horizontal is shown by the numerals under the simplified expression of the month.

There is only one point from our measurements on the tilted surface with the angles $\theta=60°$ and $\alpha=0°$ in each month. Therefore, the estimations of the monthly total solar radiation on the tilted surface with changing inclination and direction angles should be compared with the calculations from our measurements denoted by several circled points as described below. In January, the inclination factors obtained from the sunshine hours are 10～18% larger than those circled points, especially, for the inclination angles of 50° to 70° and the direction angles of 30° to 60°, which have a large deviation. The inclination factor from the insolation map is about 5% smaller than that from the sunshine hours, so this is closer to the calculations based on the measurement data. Generally, these kinds of inclination factor should have the same values. However, some differences between them may occur because the results obtained from the sunshine hours are calculated from data for a relatively short period, whereas the results from the insolation map represent an average value in the districts concerned.

In April, there is little influence of a change of inclination and direction angle on the inclination factor, and, therefore, these two inclination factors from the sunshine hours and the inclination map generally agree well with each other. This fact means that over all ranges of inclination angle the solar radiation incident upon the collector surface is hardly affected by the deviation of the direction angle.

In July, the trend of the variation of K to θ is more remerkable than shown in April. Namely, while K decreases with increasing θ, K increases with increasing α. Also it seems that the deviation of the two estimations of K depends on the difference between the solar radiations on a tilted surface calculated by the sunshine hours and the insolation map. The values indicated by the circled points coincide fairly well with the results calculated by the insolation map.

In October, the tendency of the variation of K, is the same as that in January. That is, for $\alpha=0°$, one can generally recognize the large value of K, and it has an especially large value as compared with that for other inclination angles in the case that θ nearly equals about 60°. The difference between the values by sunshine hours and the insolation map becomes larger than in the other three months, and the circled points at $\alpha=0°$ coincide well with the value by the insolation map.

Comparing these two estimations to the accurate calculations based on measurements, it is clear that those for all four months at Kitami City as described above mostly approximate to each other. In this case, however, one must note that the value of the measurements at the circled points may be somewhat affected by the shade of the surrounding buildings.

Figure 8 shows the estimations of K found by the sunshine hours and the insolation map for Sapporo (43°03′N, 141°20′E). In January, for the direction

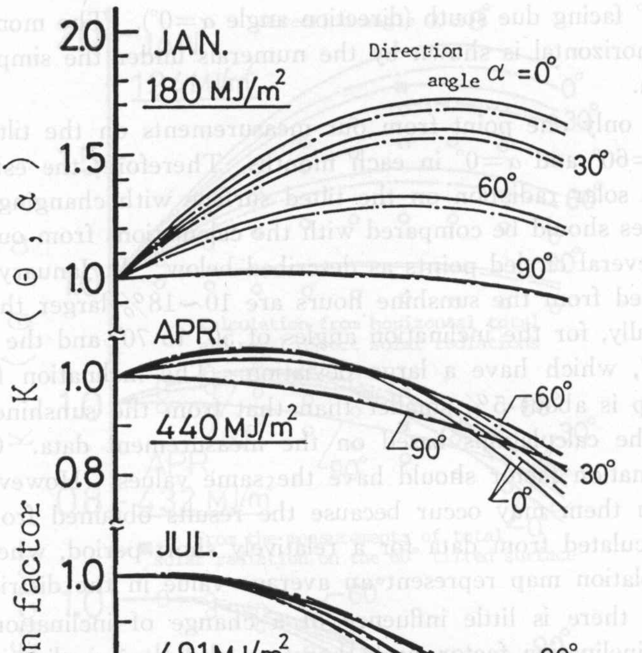

Fig. 8. Relation between the inclination factor and inclination angle at Sapporo.

angle $\alpha=0°$, the inclination factor increases with increasing inclination angle, and it has a maximum value of 1.7 at around $\theta=60°$, the increasing rate of K decreases with increasing α and when $\alpha=90°$, K decreases slightly with increasing θ. The influences of inclination and direction angles on the inclination factor at Sapporo are similar to those at Kitami through the four seasons, in January, April, July and October.

Figure 9 shows K at Sendai (38°16′N, 140°54′E) as an example of a locality

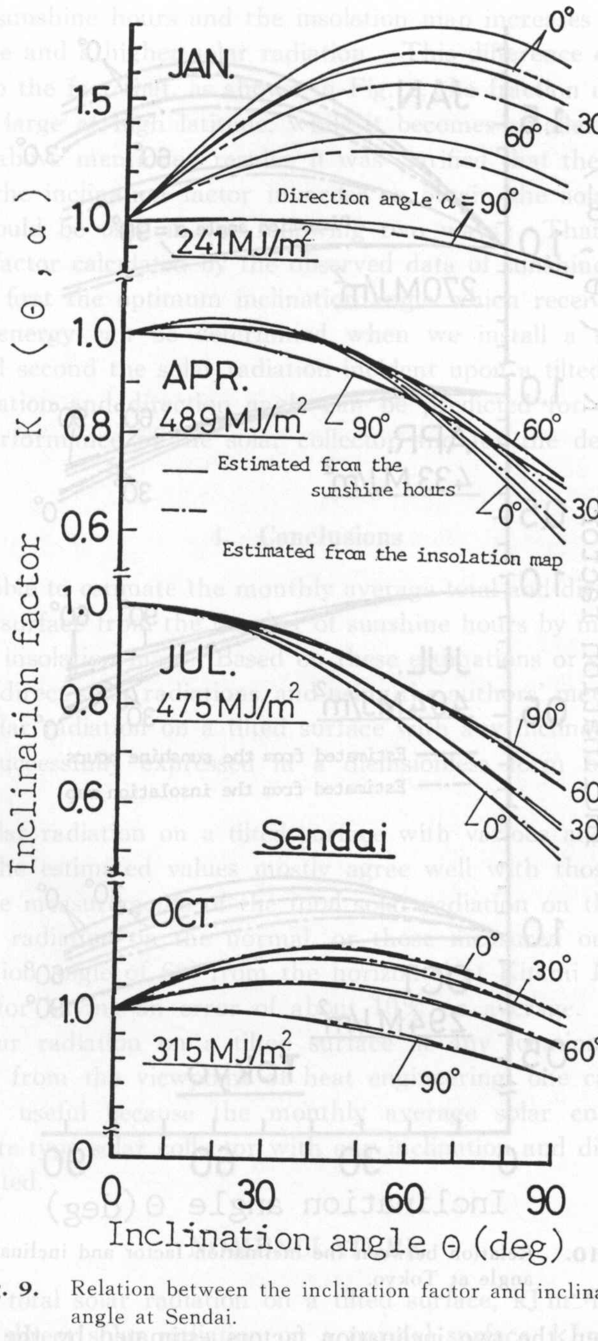

Fig. 9. Relation between the inclination factor and inclination angle at Sendai.

at a latitude of around 40°. Comparing the results at Kitami and Sendai, the difference of K between the two estimations decreases, so that the two values generally coincide with each other. Variation of the rate of the influence of α to K may be caused by the difference in the sunshine hours at Kitami and Sendai.

Figure 10 shows the results of K at Tokyo (35°41′N, 139°46′E). Better

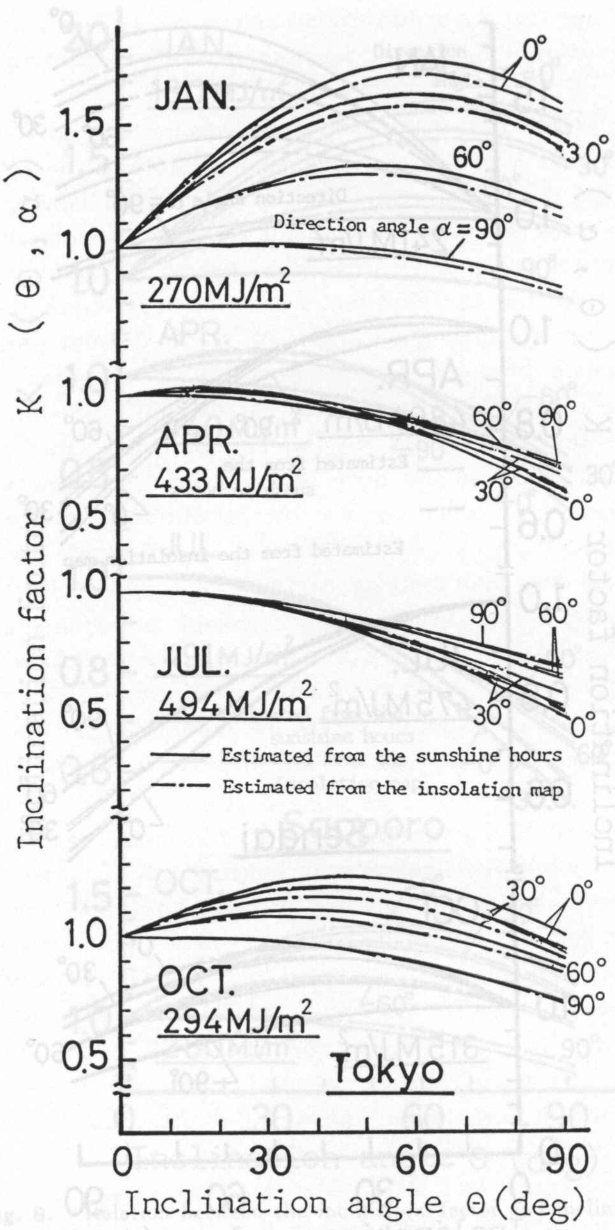

Fig. 10. Relation between the inclination factor and inclination angle at Tokyo.

agreement between the two inclination factors estimated by the sunshine hours and insolation map than those at Sendai is obtained, and, moreover, the influence of the inclination and direction angles on the inclination factor due to the change of seasons becomes smaller than at the other three localities.

By a comparison between the results of K at the four locations as can be seen in Figs. 7～10, it seems that the difference between the two inclination factors by the sunshine hours and the insolation map increases at a locality with a higher latitude and a higher solar radiation. This difference of K with locality might be due to the fact that, as shown in Fig. 2, the fraction of the direct solar radiation F_D is large at high latitude, while it becomes smaller at low latitude.

From the above mentioned results, it was clarified that the authors' method for estimating the inclination factor in order to obtain the solar radiation on a tilted surface could be used in the following two ways: That is, according to the inclination factor calculated by the observed data of sunshine hours or by the insolation map, first the optimum inclination angle which receives the maximum monthly solar energy can be determined when we install a solar collector at any locality, and second the solar radiation incident upon a tilted collector surface with any inclination and direction angle can be predicted for the evaluation of the monthly performance of the solar collector and for the design of the solar system.

4. Conclusions

It was possible to estimate the monthly average total and direct solar radiation on a horizontal surface from the number of sunshine hours by means of Yoshidas' equation or the insolation map. Based on these estimations or our measurements of the total and direct solar radiations, and using the authors' method, the monthly average total solar radiation on a tilted surface with any inclination and direction angle can be successfully expressed in a diemsionless form by the inclination factor.

For the solar radiation on a tilted surface with various angles of inclination and direction, the estimated values mostly agree well with those accurately calculated from the measurements of the total solar radiation on the horizontal and the direct solar radiation on the normal, or those measured on a tilted surface with an inclination angle of 60° from the horizontal at Kitami Institute of Technology, except for having an error of about 10% on average.

As the solar radiation on a tilted surface at any location in any country can be derived, from the viewpoint of heat engineering, one can recognize this method is very useful because the monthly average solar energy on a tilted surface of a plate-type solar collector with any inclination and direction angle can easily be estimated.

NOMENCLATURE

I_{IT}	hourly total solar radiation on a tilted surface, kJ m^{-2}h^{-1}
I_D	hourly direct solar radiation on a normal surface, kJ m^{-2}h^{-1}
I_{ID}	hourly direct solar radiation on a tilted surface, kJ m^{-2}h^{-1}
I_S	hourly scattered solar radiation, kJ m^{-2}h^{-1}
I_{HT}	hourly total solar radiation on a horizontal surface, kJ m^{-2}h^{-1}
I_{HD}	hourly direct solar radiation on a horizontal, kJ m^{-2}h^{-1}
H_{ID}	daily direct solar radiation on a tilted surface, MJ m^{-2}d^{-1}

H_{HD} daily direct solar radiation on a horizontal surface, MJ m^{-2}d^{-1}
H_S daily scattered solar radiation, MJ m^{-2}d^{-1}
H_{HT} daily total solar radiation on a horizontal, MJ m^{-2}d^{-1}
$\bar{I}_D$ daily average hourly direct solar radiation on a normal surface, MJ m^{-2}h^{-1}
$\bar{H}_{IT}$ monthly average daily total solar radiation on a tilted surface, MJ m^{-2}d^{-1}
$\bar{H}_{HD}$ monthly average daily direct solar radiation on a horizontal surface, MJ m^{-2}d^{-1}
$\bar{H}_S$ monthly average daily scattered solar radiation, MJ m^{-2}d^{-1}
$\bar{H}_{HT}$ monthly average daily total solar radiation on a horizontal surface, MJ m^{-2}d^{-1}
$K(\theta, \alpha)$ inclination factor
i angle of incidence of solar radiation upon tilted surface, deg
h angle of solar altitude (=incident angle of solar radiation on a horizontal surface), deg
θ inclination angle of tilted surface from horizontal, deg
α direction angle of tilted surface from due south (=zero for due south, negative for east and positive for west), deg
ϕ latitude, deg
δ solar declination, deg
A solar direction angle, deg
ρ albedo of the ground,
t hour angle (=zero at solar noon, and one hour $=15^\circ$ of longitude negative in the morning and positive in the afternoon), deg
t_0 sunrise hour angle on a horizontal surface, deg
t_e sunset hour angle on a horizontal surface, deg
t_0' sunrise hour angle on a tilted surface, deg
t_e' sunset hour angle on a tilted surface, deg
j date during a month
m number of days of the month
F_D factor for a fraction of the direct solar radiation to the total solar radiation
F_S factor for a fraction of the scattered solar radiation to the total solar radiation
F_ρ factor for a fraction of the reflected solar radiation from the ground to the total solar radiation

References

1) H. Berlage: Zur Theorie der Beleuchtung einer Horizontalen Flache durch Tageslicht, Meteorologische Zeitschrift, Vol. 45, No. 5 (1928), pp. 174-180.
2) B. Y. Liu and R. C. Jordan: The interrelationship and characteristic distribution of direct, diffuse and total solar radiation, Solar Energy, Vol. 4, No. 3, (1960), pp. 1-19.
3) B. Y. Liu and R. C. Jordan: Daily insolation on surface tilted toward the equator, ASHRAE Journal Vol. 3, No. 10, (1961), pp. 53-59.
4) B. Y. Liu and R. C. Jordan: Availability of solar energy for flat-plate solar heat col-

lectors, Apprication of solar energy for heating and cooling of building, ASHRAE GRP **170**, (1976), pp. V-1-26.

5) M. Udagawa and K. Kimura : The estimation of direct solar radiation from global radiation, Trans., of Archt. Inst. of Japan, **267**, (1978), pp. 83-89.

6) S. A. Klein and J. C. Theilacker : An algorithm for calculating monthly-average radiation on inclined surface, Trans., of ASME, J. Solar Energy Engineering, Vol. 103, (1981), pp. 29-33.

7) F. Butera, R. Festa and C. F. Ratto : Calculation of the monthly average of hourly and daily beam insolations on tilted surfaces, Solar Energy, Vol. 28, No. 6, (1982), pp. 547-550.

8) K. J. A. Revfeim : Simplified relationships for estimating solar radiation on any flat surface, Solar Energy, Vol. 28, No. 6, (1982), pp. 509-517.

9) K. Sharp : Calculation of monthly average insolation on a shaded surface at any tilt and azimuth, Solar Energy, Vol. 28, No. 6, (1982), pp. 531-538.

10) S. Yoshida and S. Shinoki : A method to separate monthly average total solar radiation into the diffuse and the direct components, Proc. of 6th Meeting of JSES, (1980), pp. 9-12.

11) S. Yoshida and S. Shinoki : Insolation map in all the country, Part 1, Monthly average horizontal total solar radiation and it's yearly variation, Solar Energy, J. of JSES, Vol. 7, No. 2, (1981), pp. 37-53.

12) S. Yoshida and S. Shinoki : Insolation map in all the country, Part 3, Monthly average horizontal direct solar radiation and it's yearly variation, ibid., Vol. 7, No. 4, (1981), pp. 51-66.

13) K. Kanayama and H. Baba : Decision of the optimum tilting angle and inclination ratio of solar radiation on a flat-plate solar collector, Solar Energy, J. of JSES, Vol. 8, No. 2, (1982), pp. 13-19.

14) H. Baba and K. Kanayama : Studies of measuring and estimating of solar radiation (1st Report, Adjustment and analysis on the measurements of solar radiations at Kitami Institute of Technology), Trans. of JSME, Vol. 51, No. 467, (1985), pp. 2193-2201.

15) H. Baba and K. Kanayama : Studies of measuring and estimating of Solar radiation (2nd Report, Analysis of the measurements of solar radiation at Kitami Institute of Technology and calculation of solar radiation on tilted surface from that), Trans. of JSME, Vol. 51, No. 467, (1985), pp. 2451-2456.