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# Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 9)*

## —A Fast Scheduling Method—

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**Abstract**

A scheduling method for electric power systems is reported. The schedule is made to satisfy the limits of both system security and environmental pollution. The limit of system security is considered as the probability of the power supply. And power supply is considered possible only when every power flow satisfies each transmission line's capacity in this report. Each thermal unit and line are considered to suffer a failure probabilistically. The nitrogen oxide emission from thermal stations is considered as a typical example of environmental pollution. Its limit is considered as a total value for all thermal units.

Two methods of approximation are introduced so that the schedule can be estimated rapidly. The first is a method of adjustment of the output power of thermal units to satisfy the line capacities. Only two units adjust their power to satisfy them approximately. The second is the control of the emission. It is controlled by only one system state whose elements are all healthy, which may not introduce a strictly economical schedule.

The proposed method is applied to a model system. Results of simulations are shown by some kinds of limits. That is, the probability of power supply is improved by about 15 [%] with the proposed method. The emission can be reduced by about 10 [%].

## 1. Introduction

When thermal power stations are operated, many conditions are required. Electricity must be cheap, its quality must be good, and the environment must be clean. But huge calculations are necessary for even only security estimation. In this report, a fast scheduling method for electric power systems is described to satisfy both the constraint of the environmental pollution and the probability of the power supply.

Each element in a power system is considered to fail probabilistically. This gives us each probabilistic state of the power system. In this report, each transmission line and each thermal unit are considered to fail. The power supply is considered possible only when every line capacity is satisfied. When general economic power dispatch introduces over flows in some transmission lines, then the output power of the thermal units must be adjusted to satisfy each line's

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capacity. The proposed method wishes to rapidly decide this adjustment. Output power is adjusted in two thermal units and the adjusted units are chosen so as to introduce approximately economic operation.

The limit of nitrogen oxide ($NO_x$) emission from thermal units is considered as a typical example of an evironmental constraint in this report. The constraint is considered as the expected value of total emission of every thermal unit. To rapidly find out a scheduled operation which can satisfy the emission constraint, the emission is controlled only by a system state in which every unit and line are healthy. Other system states are operated considering only the line capacities without the emission constraint.

The proposed method is simulated by a model power system. The probabilities of the power supply are shown with many kinds of load level and the line capacities. The expected values of the operating cost are shown when the emission constraints and the line capacities are satisfied. The appropriateness of the proposed method is described by these results.

## 2. Economic Load Dispatch[1]

As we know, the most economic load dispatch is obtained by a minimization of eq. (1) when we can neglect transmission losses.

$$\phi = \sum_m f_m + \lambda \cdot (Ps - \sum_m g_m) \qquad (1)$$

Where, $f_m$ is the fuel cost of the $m$-th thermal unit, and it is estimated by eq. (2) which is later shown. $\lambda$ is the LaGrange's multiplier which is concerned with the demand supply balance. $Ps$ is the system load. $g_m$ is the output power of the $m$-th thermal unit.

$$f_m = a_m + b_m \cdot g_m + c_m \cdot g_m^2 \qquad (\underline{g_m} \leqq g_m \leqq \overline{g_m}) \qquad (2)$$

Where, $a_m$, $b_m$ and $c_m$ are the characteristic constants of the $m$-th thermal unit. $\underline{g_m}$ and $\overline{g_m}$ are the lower and upper limits respectively of the output power of the $m$-th unit.

When eq. (1) is minimized, the output power becomes eq. (3) because of $\partial\phi/\partial g_m = 0$.

$$g_m = \frac{\lambda - b_m}{2 \cdot c_m} \qquad (3)$$

## 3. Load Flow

Power flow is estimated by the DC method, because of its speed and convenience[2]. Then, power flow $[i]$ becomes eq. (4)[3].

$$[i] = [e]\,[Pb] \qquad (4)$$

Where, $[i]$ is the column vector whose element $i_l$ is the power flow of the $l$-th transmission line. $[e]$ is a sensitivity matrix whose element becomes eq. (5). $[Pb]$ is the column vector whose element $Pb_n$ represents the power of the $n$-th bus.

$$e_{ln}=(b_{jn}^{-1}-b_{kn}^{-1})/x_l \tag{5}$$

Where, $b_{jn}^{-1}$ and $b_{kn}^{-1}$ represent the elements of an inverse matrix of the susceptance matrix, and the direction of the $l$-th power flow is from the $j$-th bus to the $k$-th bus. $x_l$ of eq. (5) represents the reactance.

Eq. (4) is modified to eq. (6).

$$i_l=\sum_n e_{ln}\cdot Pl_n+\sum_m e_{lm}\cdot g_m \tag{6}$$

Where, $Pl_n$ represents the load of the $n$-th bus.

## 4. A Satisfaction of Line Capacities

Economic power flows are estimated by eqs. (3) and (6). If some flows among them exceed the line capacities then the output powers of the thermal units must be modified to satisfy the line capacities. A proposed method for this is as follows.

When we desire the strictly economical satisfaction of every line's capacity, we must adjust the output of every thermal power unit to achieve it[4]. But this strictness requires many repeated calculations. In the proposed method, to achieve speedy satisfaction rather than strict economy, the outputs of two thermal units are adjusted to satisfy the line capacities.

### 4.1 In the Case that Only One Line Capacity Should be Considered

When the $m'$-th unit increases power by $\Delta g$ and the $m''$-th unit decreases power by the same value, then the increase of the $l$-th power flow becomes eq. (7) from eq. (6).

$$\Delta i_l=(e_{lm'}-e_{lm''})\cdot\Delta g \tag{7}$$

When the $l$-th power flow must be increased as much as $\Delta i_l$ to satisfy the capacity, then a pair of thermal units is chosen whose $\Delta g$ is the lowest, because, the output's modification by this pair introduces an operating state nearer to the original economic state.

When the sign of $\Delta g$ is made the same as the sign of $\Delta i_l$, then the absolute value of $\Delta g$ should be the lowest to achieve the state nearer to economic one. The lowest absolute value of $\Delta g$ is introduced by the greatest $e_{lm'}$ and the least $e_{lm''}$. This means that the unit pair can be chosen independently of the value and the sign of $\Delta i_l$.

### 4.2 In the Case that many Line Capacities Should be Considered

From the previous description, it is obvious that the unit pair should be chosen by greater $e_{lm'}$ and less $e_{lm''}$. However, a certain thermal unit may have less $e_{l'm}$ and greater $e_{l''m}$ among the considered line capacities (for example $l'$ and $l''$) because eq. (7) becomes simultaneous equations according to the number of considered line capacities. This means that the unit pair can not be chosen simply when many line capacities should be considered.

In the proposed method, the unit pair is chosen by weighted $e_{lm}$, and $\Delta i_l$

is used as the weight of $e_{lm}$. In other words, the unit pair is composed of the greatest unit and the least unit as to $E$ of eq. (8).

$$E=\sum_{l} \Delta i_l \cdot e_{lm} \tag{8}$$

Where, $\sum$ of the right hand of eq. (8) means a summation of the over flow lines. Reasons are as follows why the proposed method chooses $\Delta i_l$ as the weight of $e_{lm}$. The thermal output varies linearly depending on $i_l$ from eq. (6), and the greater absolute value of $\Delta i_l$ tends to introduce the less economic operation. That is, the proposed method gives priority to the greater absolute value of $\Delta i_l$ when many power flows are over capacity and the values of their $e_{lm}$ are the same. Another advantage of the weight of $\Delta i_l$ is that the sign of $\Delta i_l \cdot e_{lm}$ shows directly whether the $m$-th thermal unit should increase power or reduce power to satisfy the $l$-th line capacity. This is convenient when the $m$-th unit must increase power to satisfy a line capacity and it must decrease power to satisfy another capacity and a decision is required as to whether the output of the $m$-th unit should be increased or decreased to satisfy the capacities. The proposed method gives priority to the greater absolute value of $\Delta i_l \cdot e_{lm}$ in the end.

### 4.3 Repetition to Satisfy Every Line's Capacity

From eq. (8), the power of the thermal unit is increased whose $E$ is the greatest. The power of the least one is decreased. The adjusted value of the output power is estimated by eq. (7) using the chosen unit pair. However, each thermal unit has lower and upper limits of output power. The adjusted value by eq. (7) may introduce thermal output outside these limits. Futhermore, when the unit pair which is chosen by eq. (8) changes its output by the value which is estimated by eq. (7), then the power flow of a few lines may exceed the line capacities more than before the output change.

In these cases, adjustment must be repeated. To avoid an oscillatory or divergent repetition, each thermal unit is given an adjustable direction of output power. In other words, when each thermal unit is chosen for the first time by eq. (8) as a member of the proper unit pair, then the direction of output adjustment is fixed according to whether the value of $E$ is the greatest or least. This direction is not changed even after the state of overflow is changed. This means that the greatest $E$ is chosen except for units whose adjustable directions are fixed to decrease power. The least $E$ is done except for units whose directions are up. The concrete steps of the estimation are shown in the next section.

### 4.4 Estimation Steps to Satisfy All Line Capacities

Step 1 : Economic load dispatch by eq. (3).

Step 2 : Estimation of load flow by eq. (6).

Step 3 : Check of every line's capacity. If none of the lines exceeds the capacity then satisfaction is completed.

Step 4 : Find out the greatest and the least $E$ of eq. (8). The greatest $E$ must be discovered among the thermal units whose power is not at the upper limit and whose adjustable direction is not down and whose condition is not failure. The least $E$ is also similarly found out. If either the greatest or the least $E$ does not exist, then the satisfaction of line capacities is impossible. When the greatest or the least $E$ is introduced by a thermal unit whose adjustable direction has not yet been fixed, the direction is fixed according to the value of $E$.

Step 5 : Decision of the adjusted value of output power by eq. (7). Among over flow lines, the greatest $\Delta g$ must be discovered. If the greatest $\Delta g$ is not positive, then the members of the unit pair should be exchanged. The value by which the unit pair should have their output changed is the lower one of the greatest $\Delta g$ of eq. (7) and the adjustable value which is the difference between the present output and the upper or lower output limit.

Step 6 : Output power adjustment of the chosen unit pair, and repeat estimation from Step 2.

## 5. Satisfaction of both The Environmental Constraint and The Line Capacities

In the proposed method, the $NO_x$ emission constraint is considered as being concerned with the expected value when failure states happen probabilistically in the electrical power system. If an operating schedule is desired to satisfy the emission constraint exactly economically, then the emission must be controlled properly for every kind of system state. But there are too many kinds of state, and huge estimations are necessary to match the expected value of the emission with the constraint.

In the proposed method, because the probability of a system state whose elements are all healthy is greater than that of other states, the $NO_x$ emission is controlled only by this all healthy state to satisfy the emission constraint of the final expected value.

### 5.1 Objective Function for the Emission Constraint

When the emission constraint is also considered, the objective function is expanded to eq. (1)′.

$$\phi' = \sum_m f_m + \lambda\cdot(Ps - \sum_m g_m) + \mu\cdot(Y - \sum_m y_m) \qquad (1)'$$

Where, $\mu$ is the LaGrange's multiplier which is concerned with the emission constraint. $y_m$ is $NO_x$ emission from the $m$-th thermal unit and is estimated by eq. (9) which is described later. $Y$ is the deterministic emission limit of $NO_x$. The concrete value of $Y$ can be estimated easily, because the probability of the all healthy state is known and it can also be known by about how much expected emission should be reduced. The reason why the reduced emission can be known is that the emission control is considered after the economic operation is obtained which considers only line capacities.

$$y_m = d_m \cdot f_m \tag{9}$$

Where, $d_m$ is the characteristic constant of the $m$-th thermal unit. A minimum condition of eq (1)′ is eq (3)′ because of $\partial\phi'/\partial g_m = 0$.

$$g_m = \frac{\lambda}{2 \cdot c_m \cdot (1 - \mu \cdot d_m)} - \frac{b_m}{2 \cdot c_m} \tag{3'}$$

### 5.2 Modification of Eq. (8) to Consider Both the Emission Constraint and the Line Capacities

When the thermal unit of the maximum $E$ increases output power and the emission constraint must be considered, then the unit is desired to have a good characteristic for the emission quantity. The harder emission constraint should be considered as the more priority constraint. From the Kuhn- Tucker conditions and a convexity for the objective function, the harder emission constraint introduces the negative value of $\mu$. Then eq. (8) is modified to eq. (8)′ using the weight of $-\mu$ to consider the emission constraint.

$$E' = \sum_l \Delta i_l \cdot e_{lm} - \mu \cdot \frac{dy_m}{dg_m} \tag{8'}$$

When the emission constraint must be considered, eq. (8)′ is used to decide the proper unit pair to adjust output power together considering the line capacities. The two kinds of constraint of the emission and the line capacities are only considered with the system state whose elements are all healthy in the proposed method, as described previously.

## 6. Simulations by a Model Power System

Table 1 shows the characteristic constants of each thermal unit of a model system[5]. Line data is shown in Table 2. Table 3 shows the load data of each node of the model system. In the following simulations, each node load varies depending on the rates of Table 3 and the systen load. The model system is also shown in Fig. 1.

The economic power flow of each line was estimated by eqs. (3) and (6) at 485 [MW] without the considerations for the line capacities and the emission constraint. The Capacity value of each line was defined as 150 [%] of this economic flow.

A power supply probability is shown in Fig. 2 when line capacities were considered only for lines No. 1 to No. 13 and every thermal unit was considered as never failing and the emission constraint was not considered. Line faults were considered only when a fault happens on only one line. In other words, power supply was considered impossible when faults happen on two or more lines simultaneously. In the figure, economic operation indicates results only by eq. (3) without consideration of the line capacities. The results of the proposed method were obtained by the output adjustment of eq. (8). Because each line capacity was fixed independently of the system load change in these simulations,

**Table 1.** Characteristic constants of thermal units

| No. | Node | $f_m = a_m + b_m \cdot g_m + c_m \cdot g_m^2$ [$] $a_m$ | $b_m$ | $c_m \times 1000$ | $d_m$ $\frac{kg}{\$}$ | $\underline{g_m}$ [MW] | $\overline{g_m}$ [MW] | error rate [%] |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 40 | 3.6 | 5.0 | — | 30 | 120 | 1.478 |
| 2 | 2 | 60 | 3.4 | 4.0 | 0.258 | 30 | 120 | 1.478 |
| 3 | 3 | 60 | 3.4 | 4.0 | 0.266 | 30 | 120 | 2.439 |
| 4 | 4 | 50 | 3.5 | 4.5 | 0.241 | 30 | 120 | 2.439 |
| 5 | 5 | 40 | 3.5 | 4.5 | 0.250 | 30 | 120 | 2.913 |

Note: No. 1 unit is not constrained for emission because it is constructed in a remote area.

**Table 2.** Line data

| Line | Node | $X$ | Error rate | Line | Node | $X$ | Error rate |
|---|---|---|---|---|---|---|---|
| 1 | 1– 9 | 0.50 | 0.030 | 10 | 5– 6 | 0.36 | 0.024 |
| 2 | 1–11 | 0.16 | 0.010 | 11 | 5– 9 | 0.16 | 0.010 |
| 3 | 2– 3 | 0.50 | 0.030 | 12 | 7– 8 | 0.16 | 0.010 |
| 4 | 2– 7 | 0.28 | 0.020 | 13 | 7–10 | 0.24 | 0.016 |
| 5 | 2–10 | 0.16 | 0.010 | 14 | 8– 9 | 0.36 | 0.024 |
| 6 | 3– 4 | 0.24 | 0.016 | 15 | 8–10 | 0.24 | 0.016 |
| 7 | 4– 6 | 0.28 | 0.020 | 16 | 8–11 | 0.28 | 0.020 |
| 8 | 4– 8 | 0.28 | 0.020 | 17 | 10–11 | 0.36 | 0.024 |
| 9 | 4– 9 | 0.50 | 0.030 | | | | |

Base: 100 MVA

**Table 3.** Load data

| Node | Load [MW] | Node | Load [MW] |
|---|---|---|---|
| 1 | 0 | 7 | 69.23 |
| 2 | 0 | 8 | 155.77 |
| 3 | 0 | 9 | 121.15 |
| 4 | 0 | 10 | 43.27 |
| 5 | 0 | 11 | 43.27 |
| 6 | 17.31 | | |

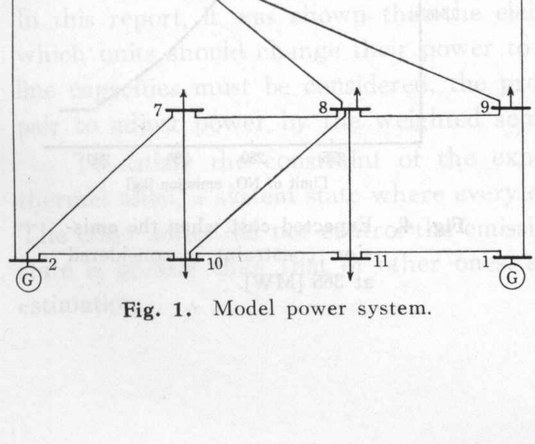


**Fig. 1.** Model power system.

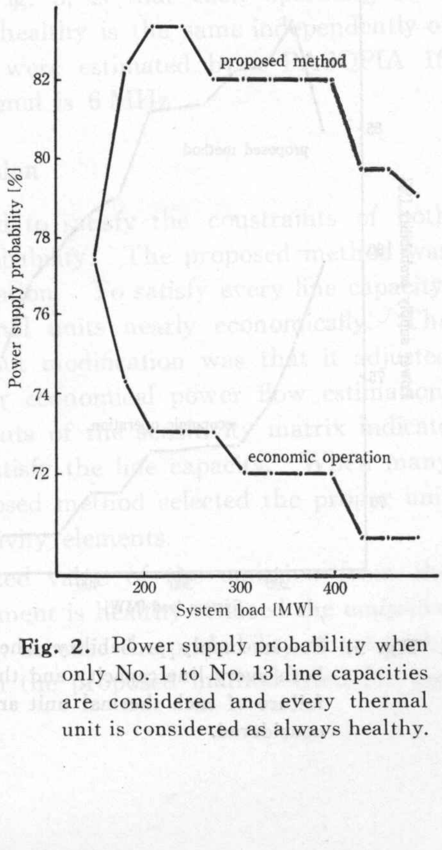


**Fig. 2.** Power supply probability when only No. 1 to No. 13 line capacities are considered and every thermal unit is considered as always healthy.

power supply probability is reasonably increasing depending on the load decrease in Fig. 2. But exceptions are below 200 [MW]. The reason is that load level is too low. The least load of Fig. 2 is 150 [MW] and this level is the same as the total of $\underline{g_m}$. At this load level, every thermal unit must be operated at $\underline{g_m}$ to satisfy the supply and demand balance. Then each unit has no adjustable output region to satisfy the line capacities and the proposed method could not conduct an artificial operation to satisfy them and the power supply probabilities of both economic operation and operation by the proposed method became the same. If a unit commitment is optimized at these load levels, then the power supply probability will be futher improved. But even by Fig. 2. the improvement of the power supply probability is known by more than 10 [%] at many load levels using for the proposed method. The computing time was 15.16 seconds as an averaged value for each load level.

Results are shown in Fig. 3 when every line capacity and each unit fault were considered. For faults, the power supply was considered impossible when line faults happen on two or more lines at the same time as previously, but it was considered that the fault of a thermal unit may happen singly or may happen simultaneously with a line fault. At high load levels, the probability of power supply is less than Fig. 2 because Fig. 3 considered every line capacity. To the contrary, the probability of the power supply is greater than Fig. 2 at low load levels. A reason for this is the consideration of the thermal unit faults.

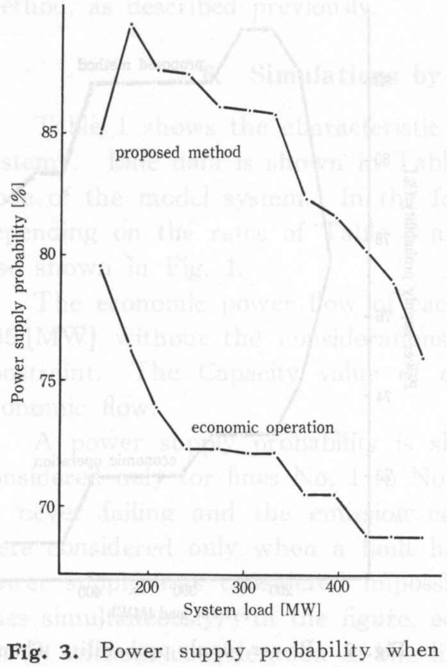


**Fig. 3.** Power supply probability when both every line capacity and the failure of each thermal unit are considered.

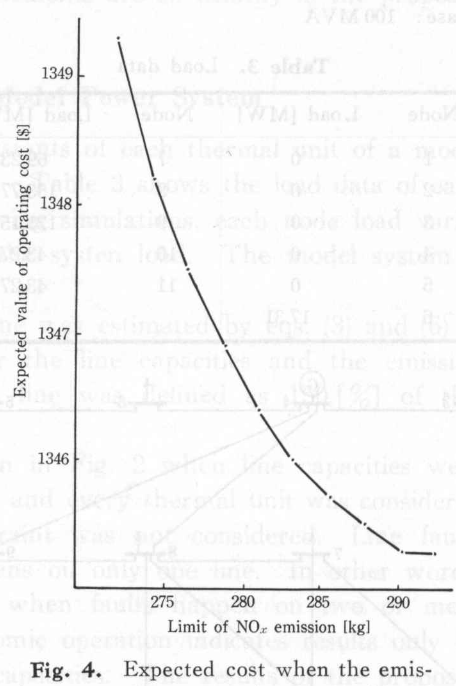


**Fig. 4.** Expected cost when the emission constraint is considered at 365 [MW].

From Fig. 3, it is known that the proposed method can improve the power supply probability even when every line capacity is constrained. The computing time was 2 minutes 0.00 seconds which is average for a load level.

Fig. 4 shows the expected cost when $NO_x$ emission was constrained at each value. Three kinds of fault states were considered as Fig. 3. That is, the first was the line fault on only one line, the second was the fault of only one thermal unit, and the third was simultaneous faults on one line and one unit. Every line capacity was considered similarly to the previous simulation and the load level was fixed to 365 [MW]. The constraint values of the emission are the expected values. From Fig. 3. the power supply probability was 82.39 [%] and this probability does not vary depending on the emission constraint level. A reason for this is that the proposed method adjusts the emission only for a system state whose every element is healthy and the other states do not consider the emission constraint. If proper states give up the power supply according to hard emission constraint, both the power supply probability and the expected emission must be decreased. But the proposed method estimates approximately to actualize a fast estimation and then the probability of the power supply is not changed in Fig. 4. From the figure, it is known that the proposed method could reduce the emission by about 10 [%], simultaneously satisfying every line capacity.

The computing time was 19.11 seconds which was average for each emission constraint in Fig. 4. A reason why Fig. 4 was faster than Fig. 3 in spite of more kinds of constraint in Fig. 4 than Fig. 3, is that each operating state except for the state where all elements are healthy is the same independently of the emission constraint. These simulations were estimated by a PASOPIA 16 whose CPU is 8088+8087 and the clock signal is 6 MHz.

## 7. Conclusion

A fast scheduling method was described to satisfy the constraints of both the $NO_x$ emission and the power supply probability. The proposed method was an approximate one to realize a speedy estimation. To satisfy every line capacity, it modified the output power of the thermal units nearly economically. The reason why it could rapidly decide the output modification was that it adjusted the output power of two thermal units after economical power flow estimation. In this report, it was shown that the elements of the sensitivity matrix indicate which units should change their power to satisfy the line capacity. When many line capacities must be considered, the proposed method selected the proper unit pair to adjust power by the weighted sensitivity elements.

To satisfy the constraint of the expected value of the emission from the thermal units, a system state where every element is healthy reduced the emission. The other states did not control the emission, because the probability of a healthy state is greater than that of other ones, and the proposed method tried for fast estimation.

The proposed method was used for a model system. From results of simulations, many improvements by the proposed method were shown. The probability of power supply was improved by about 15 [%] by the proposed method compared with natural economic operations when the emission was not constrained. It was also shown that the expected value of the emission was reduced by about 10 [%] by the proposed method satisfying the probability of power supply. By these concrete results, the appropriateness of the proposed method became clear.

In future, the results of the proposed method will be compared with a strict method. Results of the comparison will also be reported. The proposed method may be expanded to decide the optimum unit commitment.

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## References

1) Koike, T.: Daigakukatei denryokuhasseikougaku, Ohm corp, Japan (February 1980).
2) Nakamura, Y. et al.: Memoirs of the Kitami Institute of Technology, Vol. 9, No. 2, p. 679 (March 1978).
3) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 16, No. 2 (March 1985).
4) Nakamura, Y. and Yamashiro, S.: Memoirs of the Kitami Institute of Technology, Vol. 17, No. 2 (March 1986).
5) Ueda, T. et al.: Trans. I.E.E. Japan, Vol. B-103, No. 5, p. 365 (May 1983).
6) Nakamura, Y. and Yamashiro, S.: National Convention Record of the I.E.E. of Japan, No. 958, p. 1215 (April 1986).