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Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 8)*

—A Consideration for Thermal Unit Security—

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Abstract

An optimum operation scheduling method is reported regarding a fault of the thermal units as probabilistic. Optimum operation is considered as which the reliability limit and the NO_x emission constraint can be economically satisfied. The reliability limit is satisfied by increasing kinds of the power supply states. The power supply is considered as possible only when every line's capacity is satisfied.

A strict method and an approximate method are introduced to estimate the change of thermal output power after a unit fault. The strict method is a simple application of the economic load dispatch. The approximate method is an economical dispatch of the output power of a faulty unit using the demand supply balance. From model system simulations where the number of thermal units is 5, the estimating speed of the approximate method is shown to be 15 times faster than the strict one. Estimating error of the approximate results is also shown to be small.

Line capacity is satisfied by modifying the economical power flow. The changed value of the thermal output power is estimated to satisfy the line capacity by using an exceeding flow value from the capacity. Many simulated results are shown applying the proposed method to a model system. It is concretely shown that the proposed method can estimate the optimum operation to satisfy the various reliability limits and the various emission constraints.

1. Introduction

Assuming the fault of thermal units as probabilistic, an optimum scheduling method is considered to satisfy both the reliability limit and the emission constraint. A strict estimation and an approximate estimation are introduced to evaluate the economical change of the thermal power after the fault of a thermal unit. The strict method dispatches a system load simply by using LaGrange's multiplier method. The approximate method uses the previous output power of the faulty unit. Making use of that, the power of the healthy units is always increased after a unit fault, and the approximate method is modified to improve precision.

The line capacity is satisfied as follows. First, the economic load dispatch is checked by whether there are overload lines. Secondly, using values of overflow, the LaGrange's multipliers are decided to absorb the overflow.

The proposed method is applied to a model system. Simulations are tried

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by using various constraints. The appropriateness of the proposed method is considered by the results of the simulations.

2. An Outline of Steps to estimate

The proposed method is shown in brief as follows.

Step 1: A unit commitment is composed assuming every thermal unit is healthy which is expected to make the best schedule. This means a scheduled unit commitment and it is checked whether proper or not as follows.

Step 2: Ignoring the line capacity and considering only the environmental constraint, economical load dispatch is estimated. The power flow of each line is checked whether overflow or not.

Step 3: If some lines are overloaded, then a new load dispatch is estimated to satisfy both the line capacity and the emission constraint.

Step 4: Assuming the fault of a thermal unit, the new unit commitment is composed by the remaining healthy units, and return to Step 2.

In step 4, when faults happen simultaneously in more than one thermal unit, then it is regarded as impossible to supply power in the proposed method, because the probability of this is usually negligibly small and it is generally difficult to satisfy every line's capacity and the emission constraint.

3. Objective Function for Step 2

Next, an objective function is considered to estimate the economical load dispatch in Step 2.

$$\phi = \sum_{m=1}^{M} f_m + \lambda \cdot \left(Ps - \sum_{m=1}^{M} g_m \right) + \mu \cdot \left(Y - \sum_{m=1}^{M} y_m \right) \tag{1}$$

Where, transmission losses are neglected, and M is the number of thermal generating units in a system. f_m, g_m and y_m are the fuel cost, output power and NO_x emission of the m-th thermal unit respectively. Eqs. (2) and (3) are later shown to estimate f_m and y_m. Ps and Y are the system load and the emission limit of NO_x respectively. λ of eq. (1) is the LaGrange's multiplier which is concerned with the demand supply balance, and μ is one concerned with the emission constraint.

$$f_m = a_m + b_m \cdot g_m + c_m \cdot g_m^2 \qquad (\underline{g_m} \leqq g_m \leqq \overline{g_m}) \tag{2}$$

$$y_m = d_m \cdot f_m \tag{3}$$

Where, a_m, b_m, c_m and d_m are the characteristic constants of the m-th thermal unit. $\underline{g_m}$ and $\overline{g_m}$ are the lower and upper limits respectively of the output power of the m-th unit.

A required load dispatch is obtained, when eq. (1) is minimized satisfying $Ps = \sum_{m=1}^{M} g_m$, $Y \geqq \sum_{m=1}^{M} y_m$.

4. A Strict Method and An Approximate Method for the Estimation of a Power Change

Two kinds of method are described to decide the new output power of thermal units in Step 2 when a fault occurs on a unit.

The strict method is easy to describe. It decides the output power of each of the thermal units operated simply by minimizing the objective function with the new unit commitment. An algorithm is easy, but it may take a long time to estimate because some repeated calculations are necessary to minimize the objective function.

The approximate method is as follows. The partial differentiation of eq. (1) becomes eq. (4) using eq. (3).

$$\frac{\partial \phi}{\partial g_m} = (1-\mu \cdot d_m) \cdot \frac{df_m}{dg_m} = \lambda \qquad (4)$$

Since the objective function is minimized at $\partial\phi/\partial g_m = 0$ because of its convexity, the output power becomes eq. (5) using eq. (2).

$$g_m = \frac{\lambda}{2 \cdot c_m \cdot (1-\mu \cdot d_m)} - \frac{b_m}{2 \cdot c_m} \qquad (5)$$

As the change of μ is small when a thermal unit shuts down and the emission constraint is satisfied by the remaining units[2], μ is considered as a constant independent of a unit fault. Eq. (6) shows the change of output power introduced by the fault of another unit when the change is denoted as Δg_m and the increment of λ is as $\Delta\lambda$.

$$\Delta g_m = \frac{\Delta\lambda}{2 \cdot c_m \cdot (1-\mu \cdot d_m)} \qquad (6)$$

Denoting the number of the faulty unit as k, eq. (7) is obtained by the demand supply balance after the fault.

$$g_k = \sum_{m \neq k} \Delta g_m \qquad (7)$$

Eliminating $\Delta\lambda$ from eqs. (6) and (7), the change of the output power of the healthy units becomes eq. (8).

$$\Delta g_m = \frac{g_k}{c_m \cdot (1-\mu \cdot d_m) \cdot \sum_{m' \neq k} \frac{1}{c_{m'} \cdot (1-\mu \cdot d_{m'})}} \qquad (8)$$

There are two reasons why eq. (8) is approximate. The first is the consideration of μ as a constant in eq. (6). The second is that the upper and lower limits of the output power are not regarded when eq. (6) was introduced. The approximate estimation may include some estimating errors, but it should be faster in calculation because it does not need repeated calculations. Moreover, the estimating errors become less significant when the most important fact is considered in Step 2 as to whether there are some overload lines or not.

5. A Modification of Eq. (8) to Improve the Precision

Because one of the reasons for the estimating errors is the upper and lower limit of thermal output, eq. (8) can be modified as follows. After a unit fault, the remaining healthy units must increase their output power because c_m and d_m are always positive and these introduce larger λ. This is known from eqs. (5)~(8). Then, the units whose output power was previously fixed to the upper limit should be removed from a summation of eq. (7). On the contrary for the units whose powers were previously fixed to the lower limit, the changes of power are unknown. Some of them may be increased or others may not depending on the value of g_k of the faulty unit. Moreover, the power change of the units whose previous power was near to the upper limit may be Δg_m of eq. (8) or not because the new output power $g_m+\Delta g_m$ may exceed the upper limit.

In these cases, a modified eq. (8)′ is still approximate, but it should introduce fewer estimating errors than eq. (8).

$$\Delta g_m=\frac{g_k}{c_m\cdot(1-\mu\cdot d_m)\cdot\sum\limits_{m'\in U}\dfrac{1}{c_{m'}\cdot(1-\mu\cdot d_{m'})}} \qquad (8)'$$

Where, U is a unit commitment whose units are healthfully operated at neither the upper limit nor zero.

6. A Method to Satisfy the Line Capacity and the Emission Constraint

To satisfy the line capacity and the emission constraint in Step 3 which was described in chapter 2, the objective function is subsequently modified.

$$\phi=\sum_{m=1}^{M}f_m+\lambda\cdot\left(Ps-\sum_{m=1}^{M}g_m\right)+\mu\cdot\left(Y-\sum_{m=1}^{M}y_m\right)$$
$$+\sum_{l=1}^{L}\nu_l\cdot(I_l-i_l)+\sum_{l=1}^{L}\nu_{L+l}\cdot(i_l-I_l) \qquad (1)'$$

Where, L is the number of lines, and I_l and i_l are the capacity and the power flow respectively of the l-th transmission line. Eq. (9)[1,3] is described later to estimate i_l. ν_l and ν_{L+l} are the LaGrange's multipliers which are concerned with the upper limit and the lower limit respectively of the power flow of the l-th line.

$$i_l=\sum_{n=1}^{N}e_{ln}\cdot Pl_n+\sum_{m=1}^{M}e_{lm}\cdot g_m \qquad (9)$$

Where, N denotes the number of buses. e_{ln} and e_{lm} are the elements of the sensitivity matrix. Pl_n represents the load of the n-th bus.

Because μ is constant which was assumed in chapter 4, eq. (10) is obtained denoting g'_m as the optimum power and λ' as the optimum λ of eq. (1)′.

$$g'_m = \frac{\lambda' + \sum_{l=1}^{L} e_{lm} \cdot (\nu_l - \nu_{L+l})}{2 \cdot c_m \cdot (1 - \mu \cdot d_m)} - \frac{b_m}{2 \cdot c_m} \tag{10}$$

Denoting i'_l as a power flow whose thermal power is g'_m, eq. (9) is modified to eq. (9)'.

$$i'_l = \sum_{n=1}^{N} e_{ln} \cdot Pl_n + \sum_{m=1}^{M} e_{ln} \cdot g'_m \tag{9 $'$}$$

Δi_l is defined as a power flow which should be increased to satisfy the capacity, it becomes eq. (11).

$$\Delta i_l = \begin{cases} I_l - i_l & (I_l < i_l) \\ -I_l - i_l & (-I_l > i_l) \end{cases} \tag{11}$$

New coefficient h_l is introduced, the value of it is set to 1 when the l-th power flow is greater than I_l, and it is set to -1 when $i_l < -I_l$. Then eq. (11) is modified to eq. (11)'.

$$\Delta i_l = h_l \cdot I_l - i_l \tag{11$'$}$$

On the other hand, because i'_l of the eq. (9)' satisfies the capacity, Δi_l becomes eq. (12), and it is substituted by eqs. (9) and (9)'.

$$\Delta i_l = i'_l - i_l = \sum_{m=1}^{M} e_{lm} \cdot (g'_m - g_m) \tag{12}$$

From the constraint of demand and supply balance, eq. (13) must be satisfied. Eqs. (5) and (10) are substituted to eqs. (12) and (13). From obtained equations, $\lambda' - \lambda$ is eliminated and eq. (11)' is substituted. The result is eq. (14).

$$\sum_{m=1}^{M} (g'_m - g_m) = 0 \tag{13}$$

$$h_l \cdot I_l - i_l = \sum_{l'=1}^{L} W_{ll'} \cdot \nu_{l'} \tag{14}$$

Where, $\nu_{l'}$ is exchanged by $\nu_{L-l'}$ when $i_{l'} < -I_{l'}$ in eq. (14). $W_{ll'}$ becomes eq. (15) and it is the same as the previous paper has described.

$$W_{ll'} = h_l \cdot h_{l'} \cdot \left\{ \sum_{m=1}^{M} \frac{e_{lm} \cdot e_{l'm}}{2 \cdot c_m \cdot (1 - \mu \cdot d_m)} - \frac{\left(\sum_{m=1}^{M} \frac{e_{l'm}}{c_m \cdot (1 - \mu \cdot d_m)} \right) \cdot \left(\sum_{m=1}^{M} \frac{e_{lm}}{2 \cdot c_m \cdot (1 - \mu \cdot d_m)} \right)}{\sum_{m=1}^{M} \frac{1}{c_m \cdot (1 - \mu \cdot d_m)}} \right\} \tag{15}$$

When the approximate method is used, the power flow is estimated by the approximate output of eq. (8)' and eq. (9). This flow is used at the solution of $\nu_{l'}$ in eq. (14). On the other hand, when the strict method is used, $\nu_{l'}$ is solved by a strict flow which is obtained by the strict thermal power.

7. Calculating Results of a Model Power System

The simulated model system is fundamentally the same as that previously used[1]. Most characteristics of the model system have been shown in detail on the previous report although the previous report showed even the failure rates of line and they were not used in this report. In this report, only the system figure is shown in Fig. 1. The characteristics of the thermal units are shown in Table 1 adding the failure rates of the thermal units to previous data. In this simulation, all thermal units were scheduled to compose the unit commitment which was described at Step 1 in chapter 2.

Table 1. Characteristic constants of thermal units

No.	Node	$f_m=a_m+b_m\cdot g_m+c_m\cdot g_m^2$ [\$]			d_m	$\underline{g_m}$	$\overline{g_m}$	error rate
		a_m	b_m	$c_m\times 1000$	$\left[\frac{kg}{\$}\right]$	[MW]	[MW]	[%]
1	1	40	3.6	5.0	—	30	120	1.478
2	2	60	3.4	4.0	0.258	30	120	1.478
3	3	60	3.4	4.0	0.266	30	120	2.439
4	4	50	3.5	4.5	0.241	30	120	2.439
5	5	40	3.5	4.5	0.250	30	120	2.913

Note: No. 1 unit is not constrained for emission because it is constructed in a remote area.

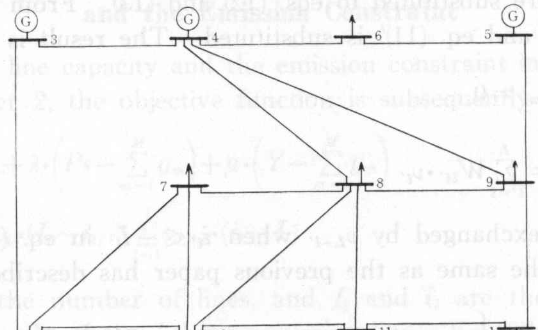

Fig. 1. Model power system.

When every thermal unit was assumed to be healthy and the economic operation was estimated regardless of the line capacities and the emission constraint, then the operating cost became \$1988.75 and the emission of NO_x became 430.24 kg. In later simulations, this economic flow was refered to as 100% capacity for each line.

First, setting the emission constraint to 410 kg, the operating cost was estimated satisfying the line capacity by the proposed method. The results are shown in Table 2. Blanks in the table mean that the economical operation could satisfy the line capacity when only emission constraint was satisfied, and the special considerations for line capacity are not necessary. — means it is impossible to satisfy all the constraints because they are too hard. ( ) means results of the approximate method and the others are strict results. From the table, it is known that the approximate method can estimate sufficiently precisely compared with the strict one. The computing time was 2.42 seconds by the strict method and 0.16 seconds by the approximate method in Step 2. These were cumputed by a PASOPIA 16 whose CPU is 8088+8087 and the clock signal is 6 MHz.

For each reliability limit, the expected cost is shown in Fig. 2 when the limit is economically achieved. The cost was almost the same for each line capacity. This is also understood from Table 2, and the reason for this, as has

Table 2. Operating cost to satisfy the line capacities by the proposed method at $Y=410$

Fault unit	Probability [%]	Line capacity [%] 200	210	220
None	89.70	 ( )	 ( )	 ( )
1	1.35	— (—)	— (—)	— (—)
2	1.35	— (—)	1993.6 (1993.5)	1993.6 (1993.5)
3	2.24	1993.7 (1992.9)	1993.6 (1992.6)	1993.6 (1992.6)
4	2.24	— (—)	1986.0 (—)	1985.6 (1984.1)
5	2.69	— (—)	— (—)	— (—)

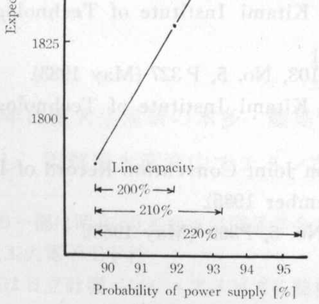

Fig. 2. Operating cost for each line capacity at $Y=410$.

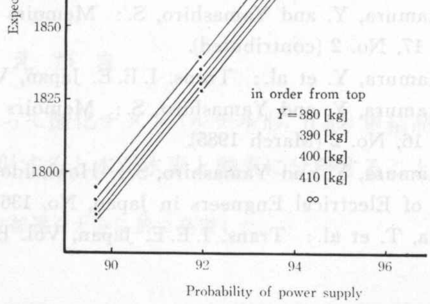

Fig. 3. Operating cost for each emission constraint (line capacity is 220%).

been explained in detail in the previous report, is that it is introduced by the small c_m of the characteristics of the thermal units. The expected cost of the strict method and the approximate one was nearly equal so that the difference could not show in the figure.

Fig. 3 shows the expected cost for each emission constraint when the reliability limit is economically achieved at line capacity of 220%. The reason why the difference of cost between emissions is small at each reliability limit is also explained by small c_m. Fig. 3 indicates that the proposed method can estimate the optimum schedule to economically satisfy both the reliability limit and the emission constraint.

8. Conclusion

The scheduling method to give the optimum operation which satisfies economically both the reliability limit and the emission constraint was reported. Two kinds of method were described to estimate the change of the output power of the healthy units after a fault. They were the strict method and the approximate method. From the concrete simulations of the model system, the speedy calculations of the approximate method were shown. Its speed was about 15 times faster compared with the strict method when the change of thermal output was estimated with a 5 thermal unit system. There was little estimating error in the approximate results.

The reliability limit was satisfied probabilistically achieving every line capacity. The line capacity was achieved by LaGrange's method. Using a model power system, the proposed method was concretely confirmed to estimate the optimum operation which can economically satisfy the reliability limit and the emission constraint. From various investigations into the results of the simulations, the appropriateness of the proposed method was shown.

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