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Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 7)*

—A Consideration for Line Security—

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Abstract

A scheduling method is reported to satisfy the reliability limit and the environmental constraint when a line fault is regarded as probabilistic. The reliability limit is satisfied by changing the thermal output power to probabilistically achieve every line's capacity. NO_x emission from thermal stations is limited as a typical environmental constraint. These constraints are economically achieved. The LaGrange's method and the Kuhn-Tucker conditions are used to satisfy both the line capacity and the emission constraint. The LaGrange's multipliers are decided by solving the simultaneous equations. An economical method of achieving the reliability limit is described using the probability of the line fault and the increase of the operating cost.

Simulating the proposed method by a model system, the results are variously investigated. The proposed method is concretely confirmed to estimate the optimum operation which can economically satisfy the reliability limits and the emission constraints at various values. It is also shown that the proposed method can improve reliability by more than 16%.

1. Introduction

We previously reported a load dispatch method which can conveniently satisfy every line's capacity[1]. In this report, we consider a method of achieving reliability for an electric power system using our dispatch method when a probabilistic fault occurs in a transmission line.

We consider the limit of nitrogen oxide (NO_x) emission from thermal stations as a concrete example of an environmental constraint. This constraint is always achieved regardless of any line fault. That is, the operating cost is probabilistically minimized to economically achieve the limit of reliability, but on the other hand the emission limit is determinately achieved for all kinds of faulty state.

First, an economical load dispatch method is described which can satisfy both the emission constraint and the capacity of every transmission line. This method is introduced by the Kuhn-Tucker conditions. Secondly, considering every kind of faulty state, the operating costs are evaluated when operations satisfy both the emission constraint and the line capacity. To achieve economical reliability, a choice is needed as to which faulty state should be operated. Thirdly,

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a method of choice is described.

Lastly, the proposed method is applied to a model power system. From the results of many simulations, the proposed method is concretely confirmed to offer proper load dispatch and every constraint is satisfied.

2. Objective Function

The number of thermal generating units is assumed as M, L for transmission lines, N for buses. Using LaGrange's multipliers method and neglecting transmission losses, next objective function is made.

$$\phi=\sum_{m=1}^{M} f_m+\lambda\cdot\left(Ps-\sum_{m=1}^{M} g_m\right)+\mu\cdot\left(Y-\sum_{m=1}^{M} y_m\right)$$
$$+\sum_{l=1}^{L}\nu_l\cdot(I_l-i_l)+\sum_{l=1}^{L}\nu_{L+l}\cdot(i_l-I_l) \quad (1)$$

Where, f_m, g_m and y_m are the fuel cost, output power and NO_x emission of the m-th thermal unit respectively. f_m and y_m are estimated by eqs. (2) and (3) which are described later. Ps and Y are the system load and the emission limit of NO_x respectively. I_l and i_l are the capacity and the power flow of the l-th transmission line respectively. i_l is estimated by eq. (4) which is described later. λ, μ, ν_l and ν_{L+l} of eq. (1) are the LaGrange's multipliers. λ and μ are concerned with the demand supply balance and the emission constraint respectively. ν_l and ν_{L+l} are concerned with the upper limit and the lower limit of the power flow of the l-th line respectively.

$$f_m=a_m+b_m\cdot g_m+c_m\cdot g_m^2 \qquad (\underline{g_m}\leqq g_m\leqq\overline{g_m}) \quad (2)$$

$$y_m=d_m\cdot f_m \quad (3)$$

Where, a_m, b_m, c_m and d_m are the characteristic constants of the m-th thermal unit. $\underline{g_m}$ and $\overline{g_m}$ are the lower and upper limits respectively of the output power of the m-th unit.

The DC method is adopted as a power flow estimation method in this paper because eq. (1) neglects transmission losses and the estimation speed of the DC method is very fast[2]. Then, power flow $[i]$ becomes eq. (4)[3].

$$[i]=[e]\,[Pb] \quad (4)$$

Where, $[i]$ is the L-column vector whose element is i_l of eq. (1). $[e]$ is the $(L\times N)$ sensitivity matrix whose element becomes eq. (5). $[Pb]$ is the N-row vector whose element Pb_n represents the power of the n-th bus.

$$e_{ln}=(b_{jn}^{-1}-b_{kn}^{-1})/x_l \quad (5)$$

Where, b_{jn}^{-1} and b_{kn}^{-1} represent the elements of an inverse matrix of the susceptance matrix, and the direction of the l-th line is assumed from the j-th bus to the k-th bus. x_l of eq. (5) represents the reactance.

Eq. (4) is modified to eq. (6).

$$i_l = \sum_{n=1}^{N} e_{ln} \cdot Pl_n + \sum_{m=1}^{M} e_{lm} \cdot g_m \tag{6}$$

Where, Pl_n represents the load of the n-th bus.

The optimum load dispatch is obtained, when eq. (1) is minimized and $Ps = \sum_{m=1}^{M} g_m,\ Y \geqq \sum_{m=1}^{M} y_m,\ -I_l \leqq i_l \leqq I_l$.

3. Kuhn-Tucker Conditions

When the Kuhn-Tucker conditions are adopted to eq. (1), then eqs. (7)~(15) are obtained. Eq. (7) is introduced from eqs. (2), (3), (6) and $\partial\phi/\partial g_m = 0$ which is also one of the Kuhn-Tucker conditions.

$$g_m = \frac{\lambda + \sum_{l=1}^{L} e_{lm} \cdot (\nu_l - \nu_{L+l})}{2 \cdot c_m \cdot (1 - \mu \cdot d_m)} - \frac{b_m}{2 \cdot c_m} \quad (m = 1, 2, \cdots, M) \tag{7}$$

$$Ps = \sum_{m=1}^{M} g_m \tag{8}$$

$$Y \geqq \sum_{m=1}^{M} y_m \tag{9}$$

$$I_l \geqq i_l \quad (l = 1, 2, \cdots, L) \tag{10}$$

$$i_l \geqq -I_l \quad (l = 1, 2, \cdots, L) \tag{11}$$

$$\mu \cdot \left(Y - \sum_{m=1}^{M} y_m \right) = 0 \tag{12}$$

$$\nu_l \cdot (I_l - i_l) = 0 \quad (l = 1, 2, \cdots, L) \tag{13}$$

$$\nu_{L+l} \cdot (i_l + I_l) = 0 \quad (l = 1, 2, \cdots, L) \tag{14}$$

$$\mu,\ \nu_l,\ \nu_{L+l} \leqq 0 \quad (l = 1, 2, \cdots, L) \tag{15}$$

4. Estimation Method of μ

The value of $\sum_{m=1}^{M} y_m$ changes monotonically depending on μ, because the objective function is convex. Then the optimum value of μ is determined by repeated interpolation and/or extrapolation. Before starting the proposed method, μ is set at a proper imagined value. Other variables except μ are determined by a method described later. The value of μ is modified to satisfy eqs. (12) and (15). These estimations are repeated until all of eqs. (7)~(15) are satisfied. In the latter part of this report, the value of μ is regarded as a known quantity.

5. Estimation Method of ν_l and ν_{L+l}

From eqs. (2), (3) and (6), y_m and i_l can be regarded as functions of the output power of the thermal units. Among eqs. (12)~(14), the maximum number of equations of which the value of the inside of parentheses can be 0 is M-1, since eq. (8) must always be satisfied and the number of thermal units is M. For the other equations, μ, ν_l, ν_{L+l} must be 0.

First, all of ν_l and ν_{L+l} are set to 0 to get a load dispatch which does not consider any line capacity. If the result casually satisfies all of the capacity, then it is a required dispatch. Otherwise, ν_l and/or ν_{L+l} are modified to satisfy all line's capacity as follows.

New $2 \cdot L$ coefficients h_l are introduced. The value of h_l is set to 1 where l is less or equal to L, and -1 when greater than L. Then eqs. (10) and (11) become the next eqs. (16) and (17).

$$I_l \geqq h_l \cdot i_l \tag{16}$$

$$I_l \geqq h_{L+l} \cdot i_l \tag{17}$$

λ is eliminated by eqs. (7) and (8), and this result and eq. (6) are substituted to eqs. (16) and (17). An arranged form of result is eq. (18).

$$V_l \geqq \sum_{l'=1}^{2L} W_{ll'} \cdot \nu_{l'} \qquad (l=1, 2, \cdots, 2 \cdot L) \tag{18}$$

where V_l and $W_{ll'}$ are eqs. (19) and (20) respectively.

$$V_l = I_l - h_l \cdot \left[\sum_{n=1}^{N} e_{ln} \cdot Pl_n + \frac{2 \cdot Ps + \sum_{m=1}^{M} \frac{b_m}{c_m}}{\sum_{m=1}^{M} \frac{1}{c_m \cdot (1-\mu \cdot d_m)}} \cdot \sum_{m=1}^{M} \frac{e_{lm}}{2 \cdot c_m \cdot (1-\mu \cdot d_m)} - \sum_{m=1}^{M} \frac{e_{lm} \cdot b_m}{2 \cdot c_m} \right] \qquad (l=1, 2, \cdots, L) \tag{19}$$

$$W_{ll'} = h_l \cdot h_{l'} \cdot \left\{ \sum_{m=1}^{M} \frac{e_{lm} \cdot e_{l'm}}{2 \cdot c_m \cdot (1-\mu \cdot d_m)} - \frac{\left(\sum_{m=1}^{M} \frac{e_{l'm}}{c_m \cdot (1-\mu \cdot d_m)} \right) \left(\sum_{m=1}^{M} \frac{e_{lm}}{2 \cdot c_m \cdot (1-\mu \cdot d_m)} \right)}{\sum_{m=1}^{M} \frac{1}{c_m \cdot (1-\mu \cdot d_m)}} \right\}$$

$$= h_l \cdot h_{l'} \cdot \sum_{m=1}^{M} \frac{e_{lm} \cdot \sum_{m'=1}^{M} \frac{e_{l'm} - e_{l'm'}}{c_{m'} \cdot (1-\mu \cdot d_{m'})}}{2 \cdot c_m \cdot (1-\mu \cdot d_m) \cdot \sum_{m'=1}^{M} \frac{1}{c_{m'} \cdot (1-\mu \cdot d_{m'})}} \qquad (l, l' = 1, 2, \cdots, L) \tag{20}$$

When, other V_l and $W_{ll'}$ are needed whose l or l' is greater than L, then I_l, e_{ln}, e_{lm}, etc. of eqs. (19) and (20) are exchanged by I_{l-L}, $e_{l-L\,n}$, $e_{l-L\,m}$, etc. respectively. In eq. (20), although the last equation is obtained by an arrangement of the previous one, the latter is calculated faster than the former. The reason for this is that the latter has not double summation and the former has. When a model system discribed later was simulated, the former was calculated about two times as fast as the latter. The value of [ ] of eq. (19) corresponds to the l- th power flow which is estimated by all $\nu=0$.

The modification of eqs. (19) and (20) to satisfy the upper and lower output limits and to speed up the calculations has already been reported in detail[3].

Non-zero ν_l and/or ν_{L+l} are chosen among the lines whose flow exceeded the capacity. Since ν_l and ν_{L+l} are never simultaneously zero for one line because of eqs. (13) and (14), ν_l is set to 0 when the flow of the l-th line exceeds the lower limit ($i_l < -I_l$), and ν_{L+l} is set to 0 when it overflows the upper limit ($I_l < i_l$). Non-zero ν_l and/or ν_{L+l} are chosen among the remaining ν_l and/or ν_{L+l}. Because flows whose ν_l or ν_{L+l} are chosen as non-zero become I_l or $-I_l$, the inequality signs of these lines are exchanged by equality signs in eq. (18) and the resulting simultaneous equations are solved to get values of ν_l and/or ν_{L+l}. If the answer satisfies eq. (15) and every line satisfies eq. (18), then the result is acknowleged as the optimum values which are obtained by a certain μ. Otherwise, a combination of non-zero ν_l is exchanged and eq. (18) is solved repeatedly. If any combination can not satisfy eqs. (15) and (18), the constraint is impossible.

6. A Method for Economical Achievement of a Reliability Limit

In this paper, a required schedule of operation is one that should have the minimum expected operating cost and satisfy a reliability limit. That is, for all system states whose some lines are faulty and the others are healthy, a division is needed as to whether each state should be operated or not so that the expected cost is kept to a minimum and the reliability limit is achieved.

First, setting all ν to 0 and ignoring the line capacity, the load dispatch and power flows are estimated for all system states. A probability of power supply is estimated, regarding the supply as impossible even if only one line is over loaded. If the probability satisfies the reliability limit, then a required schedule is obtained. It means the power is supplied only when every flow satisfies capacity. Otherwise, the system should be operated as follows.

For the system states whose some flows are over capacity, by solving eq. (18) new operating costs are estimated which can satisfy all capacities. An order of "proper state" is estimated. "Proper state" has both a small cost increase to satisfy the capacity and a large value of the state probability. The order of it is decided by the value of (cost increse)/(probability). There are two groups of states which should be operated to supply the power. The former is consisted by the states which satisfy every capacity by $\nu=0$. The latter is consisted by other states in the order of "proper state" until the reliability limit is satisfied.

7. Results of Calculation using a Model Power System

Tables 1~3 show the model power system to which the proposed method was applied. First, the economic operation was estimated by μ, ν_l, $\nu_{L+l}=0$. From the result, the cost was $1988.75 and the emission of NO_x was 430.24 kg. A reference line capacity was set to this economic flow, then a 100% capacity specifies the value of economic flow. A faulty line was considered as only one line in the system. It was regarded as impossible to supply power if there were faults simultaneouslly in more than two lines.

Table 1. Charateristic constants of thermal units

No.	Node	$f_m=a_m+b_m\cdot g_m+c_m\cdot g_m^2$ [$]			d_m	$\underline{g_m}$	$\overline{g_m}$
		a_m	b_m	$c_m\times1000$	$\left[\frac{\text{kg}}{\$}\right]$	[MW]	[MW]
1	1	40	3.6	5.0	—	30	120
2	2	60	3.4	4.0	0.258	30	120
3	3	60	3.4	4.0	0.266	30	120
4	4	50	3.5	4.5	0.241	30	120
5	5	40	3.5	4.5	0.250	30	120

Note: No. 1 unit is not constrained for emission because it is constructed in a remote area.

Table 2. Line data

Line	Node	x	Error rate	Line	Node	x	Error rate
1	1- 9	0.50	0.030	10	5- 6	0.36	0.024
2	1-11	0.16	0.010	11	5- 9	0.16	0.010
3	2- 3	0.50	0.030	12	7- 8	0.16	0.010
4	2- 7	0.28	0.020	13	7-10	0.24	0.016
5	2-10	0.16	0.010	14	8- 9	0.36	0.024
6	3- 4	0.24	0.016	15	8-10	0.24	0.016
7	4- 6	0.28	0.020	16	8-11	0.28	0.020
8	4- 8	0.28	0.020	17	10-11	0.36	0.024
9	4- 9	0.50	0.030				

Base: 100 MVA

Table 3. Load data

Node	Load [MW]	Node	Load [MW]	Node	Load [MW]
1	0	5	0	9	121.15
2	0	6	17.31	10	43.27
3	0	7	69.23	11	43.27
4	0	8	155.77		

Table 4. Number of overflow lines when only the emission constraint is considered at $Y=410$

Fault line	Probability of state [%]	Line capacity [%] 120	140	160	180	200	220	240	260
None	71.630	3	2						
1	2.215	4	4	3	3	2	2	1	
2	0.724	5	4	4	4	4	4	4	4
3	2.215	8	4	2	2	1	1	1	1
4	1.462	7	6	3	2	1	1	1	1
5	0.724	7	5	4	4	3	3	3	2
6	1.165	10	7	7	6	5	4	2	2
7	1.462	3	2	2	1	1	1	1	1
8	1.462	10	9	8	5	5	4	4	2
9	2.215	3	3	3	2	2	1	1	1
10	1.761	3	2						
11	0.724	5	4	4	4	3	3	3	2
12	0.724	3	2						
13	1.165	3	2						
14	1.761	3	2	2	1				
15	1.165	5	4	3	2				
16	1.462	6	2	2	2	2	1	1	1
17	1.761	3	2						

The number of overflow lines is shown in Table 4 when the only emission constraint was satisfied at 400 kg. Blanks in the table mean no overload. From the table, it is known that the changed value of flow caused by a fault depends primarily upon the faulty line. The probability of power supply is shown in Fig. 1 in which the operation has not yet been artificially modified to satisfy the line capacity. The probability of supply is roughly increased depending on the line capacity in Fig. 1. However, there are a few cases in which it is not simply increased because the flows are not changed uniformly by the fault. An example is the case whose capacity changes from 230% to 270% at $Y=400$. Moreover, there are other cases whose probabilities are overturned between a lesser emission constraint and a greater one. An example of this is a case whose emission constraints are infinity and 410 and the line capacity is 300%. When

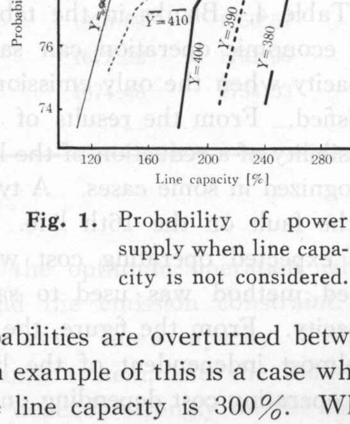

Fig. 1. Probability of power supply when line capacity is not considered.

Table 5. The operating cost when line capacity is satisfied at $Y=410$

Fault line	Line capacity [%] 120	140	160	180	200	220	240	260
None	1990.8	1990.7						
1	1991.0	1990.9	1990.8	1990.8	1990.8	1990.7	1990.7	
2	—	—	—	—	—	—	—	—
3	—	1992.7	1991.6	1991.4	1991.2	1991.1	1991.0	1991.0
4	—	—	—	—	—	—	—	1999.9
5	—	—	—	1997.9	1995.3	1993.2	1991.8	1990.9
6	—	—	—	—	1997.8	1995.9	1994.6	1993.4
7	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—
9	—	1995.9	1995.4	1994.9	1994.6	1994.4	1994.2	1994.0
10	1990.9	1990.7						
11	—	—	—	—	—	2012.0	2008.6	2006.2
12	1990.8	1990.7						
13	1991.0	1990.7						
14	1991.0	1990.8	1990.7	1990.7				
15	—	1992.8	1991.6	1990.9				
16	1991.0	1990.8	1990.7	1990.8	1990.8	1990.7	1990.7	1990.7
17	1990.8	1990.7						

—: impossible

the line capacity is less the probability is changed in a wider zone, and on the contrary at greater capacity the change of the probability is narrower.

Table 5 shows the new operating costs which were obtained to satisfy the line capacity by the proposed method corresponding to Table 4. Blanks in the table mean that the economic operation can satisfy the line capacity when the only emission constraint is satisfied. From the results of the table, the possibility of a reduction of the line capacity is recognized in some cases. A typical example is the fault on the 16th line. Fig. 2 shows the expected operating cost when the proposed method was used to satisfy the line capacity. From the figure, the expected cost is almost independent of the line capacity. The reason is the small change of the operating cost depending on the line capacity, and this can also be confirmed

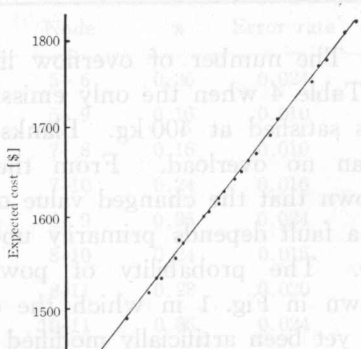

Fig. 2. Operating cost when emission constraint is satisfied at 410 kg.

by Table 5. The small change in Table 5 is also explained by the small values of the characteristic constants c_m of the thermal units compared with a_m and b_m. This means that the fuel cost characteristics become quadratic curves which are like linear lines. Since the incremental fuel cost does not widely change, the difference of operating costs is small between the economic operation and other optimum operations. Then it is not always the case that the expected cost is independ of the line capacity as in Fig. 2. If the characteristic constant c_m is greater, then the expected cost is thought less for a greater line capacity. Because the cost is not changed by the line capacity in this case, the division as to whether each system state should artificially be operated or not which was described in chapter 6 becomes insignificant. In other words, a random division may sometimes be permissible, but as has been described, not always.

In this report, the operating cost was considered as the total of the fuel cost of the thermal units. This means that cost occurs only when power is supplied and no cost occurs when there is no service. In Fig. 2, the reason for the cost decreasing with less probability of power supply is the decrease of the number of operated states. Fig. 2 does not show that less probability of supply is desirable because of the lower cost.

Finally, the expected costs are shown in Table 6 where operation satisfied the reliability limits and emission constraints using the proposed method at line capacity of 160%. In the table, — means an impossible constraint. It is similar to previous cases that the cost increase is small when the emission constraint is satisfied and the reason is less c_m. The table shows concretely that the proposed method can decide the optimum operation which can satisfy the reliability limit and the emission constraint. A case is confirmed from the table in which the reliability can be improved by more than 16%.

Table 6. The expected cost when the reliability limit is satisfied

Emission constraint	Reliability limit 72%	76%	80%	84%	88%
∞	1431.91	1511.46	1591.00	1670.56	1750.24
420	1432.17	1511.74	1591.31	1670.89	1750.57
410	1433.32	1512.94	1592.57	1672.22	1751.96
400	1435.24	1514.98	1594.72	1674.48	1754.33
390	1438.05	1517.94	1597.85	—	—

8. Conclusion

A scheduling method was reported to plan the optimum operation which could economically satisfy the reliability limit and the emission constraint. A load dispatch method was also reported to satisfy every line's capacity.

Many simulations were attempted by the model system, and the following results were obtained. The power flows' change depends strongly on the faulty line. The proposed method can widely control the power flows. When the line capacities and the emission constraint are satisfied, the change in the value of the operating cost depends on the characteristics of the thermal units. The proposed method was concretely shown to economically satisfy both the reliability limit and the emission constraint.

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