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Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 6)*

—An Improvement of The Convenient Method—

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Abstract

A method for economic load dispatch is expected to satisfy many constraints. The constraints, in this paper, are the supply demand balance, the emission constraint and the line capacities. A convenient method and a strict method have already been reported in the previous memoir. The purpose of this report is to present some improvements on the previously reported convenient method, in terms of no estimating error, application to many transmission capacity constraints and rapid estimation. These improvements make use of the Kuhn-Tucker conditions. It is shown that the number of non-zero LaGrange multipliers must be not more than the number of thermal generating units when the constraints can be satisfied. In the proposed method, a proper combination of non-zero multipliers is sought. The selection of the multipliers for non-zero is done by the partial differential coefficients. The proposed method and the strict method are applied to a model system. From many simulation results, it is shown that the proposed method can rapidly estimate the optimum load dispatch without error even if many line capacities are constrained.

1. Introduction

To estimate the economic load dispatch which can satisfy the NO_x emission constraint and the transmission capacities, we previously reported two methods[1]. They are the convenient method and the strict method. The convenient method is very fast in comparison with the latter method. However not only there is a little estimating error but also the transmission capacities can not be considered whose number is greater than that of generating units minus 2.

In this report, we improve the convenient method so that it can consider many transmission capacities without estimating-error and yet retain fast estimation. The improvement is achieved by making use of one of the Kuhn-Tucker conditions. This condition shows that when every constraint can be satisfied the number of non-zero LaGrange's multipliers which are concerned with the transmission power flow should be less than the number of generating units minus 2. When a proper combination is chosen for non-zero LaGrange's multipliers, the optimum dispatch is obtained for our problem. A method of choosing the proper combination is also described.

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The proposed method is applied to a model power system. The strict method is also applied to the model system, and the estimating error is investigated. Setting many transmission capacities, the proposed method is checked for whether the optimum results are obtained or not. Calculating time is also compared. The appropriateness of the proposed method is considered.

2. Objective Function

The number of thermal generating units is assumed as M, L for transmission lines. The next objective function is adopted using LaGrange's multipliers method.

$$\phi=\sum_{m=1}^{M} f_m+\lambda\cdot(Ps-\sum_{m=1}^{M} g_m)+\mu\cdot(Y-\sum_{m=1}^{M} y_m) + \sum_{l=1}^{L}\{\nu_l\cdot(I_l-i_l)\}+\sum_{l=1}^{L}\{\nu_{L+l}\cdot(i_l+I_l)\} \quad (1)$$

Where, f_m, g_m and y_m are the fuel cost, output power and NO_x emission of the m-th thermal unit respectively. f_m and y_m are estimated by eqs. (2) and (3) which are described later. λ, μ, ν_l and ν_{L+l} of eq. (1) are the LaGrange's multi pliers. λ and μ are concerned with the demand supply balance and the emission constraint respectively. ν_l and ν_{L+l} are concerned with the upper limit and the lower limit of the power flow of the l-th line respectively. Ps and Y of eq. (1) are the system load and the emission limit respectively, and these are the instantaneous constraints. I_l and i_l are the capacity and the power flow of the l-th line respectively.

$$f_m=a_m+b_m\cdot g_m+c_m\cdot g_m^2 \qquad (\underline{g_m}\leqq g_m\leqq\overline{g_m}) \quad (2)$$

$$y_m=d_m\cdot f_m \quad (3)$$

Where, a_m, b_m, c_m and d_m are the characteristic constants of the m-th thermal unit. $\underline{g_m}$ and $\overline{g_m}$ are the lower and upper limits respectively of the output power of the m-th unit.

3. The Optimum Conditions for The Power Lines

When the Kuhn-Tucker conditions are applied to the constraint of the line capacity of eq. (1), then eqs. (4) and (5) are introduced.

$$\nu_l\cdot(I_l-i_l)=0 \quad (4)$$

$$\nu_{L+l}\cdot(i_l+I_l)=0 \quad (5)$$

When the number of nodes of the system is denoted as N and the element of the sensitivity matrix[1)] is shown as e_{ln}, the power flow becomes eq. (6)[2)].

$$i_l=\sum_{n=1}^{N} e_{ln}\cdot Pb_n+\sum_{m=1}^{M} e_{lm}\cdot g_m \quad (6)$$

Where, Pb_n is the load of the n-th node. When eq. (6) is substituted to eqs. (4) and (5), the variables excepting ν_l and ν_{L+l} become only g_m, and their number is M. Because the number of lines is L, the total number of equations (4) and (5) becomes $2 \cdot L$. That is, the number of terms within parentheses of eqs. (4) and (5) which can be made into zero is equal or less than M. To satisfy eqs. (4) and (5), ν_l and ν_{L+l} must be zero for the equations which the value of contents of parenthese is not zero. In other words, the number of non-zero ν_l and ν_{L+l} is M at the most.

For our problem, two other constraints must be satisfied. They are the supply demand balance and the emission constraint. Because these constraints depend on g_m, the number of lines of which the power flow can be equal to I_l or $-I_l$ becomes M-2. And then, the number of non-zero ν_l and ν_{L+l} becomes M-2 at the most.

4. The Optimum Condition for The Thermal Units

Eqs. (2), (3) and (6) are substituted to eq. (1). To minimize the objective function, $\partial\phi/\partial g_m$ is set to zero and then the thermal output becomes eq. (7).

$$g_m = \frac{\lambda + \sum_{l=1}^{L}\left\{e_{lm} \cdot (\nu_l - \nu_{L+l})\right\}}{2 \cdot c_m \cdot (1-\mu \cdot d_m)} - \frac{b_m}{2 \cdot c_m} \tag{7}$$

5. The Optimum Conditions for ν_l and ν_{L+l}

From eqs. (4) and (5), since if ν_l and ν_{L+l} should be simultaneously zero for the same line then the power flow of it must become simultaneously I_l and I_l, they are never simultaneously zero. For the l-th line, a new coefficient h_l is introduced. When ν_l is selected non-zero h_l is substituted 1, and when ν_{L+l} is made non-zero h_l is substituted -1. At this time, the power flow must become $h_l \cdot I_l$. This value of the power flow and eq. (7) are substituted to eq. (6), and the result is straightened out. The next simultaneous equation is introduced.

$$[W][\nu] = [V] \tag{8}$$

Where, the elements of $[W]$ and $[V]$ become as follows.

$$W_{ll'} = \sum_{m=1}^{M} \frac{h_{l'} \cdot e_{lm} \cdot e_{l'm}}{2 \cdot c_m \cdot (1-\mu \cdot d_m)} \tag{9}$$

$$V_l = h_l \cdot I_l \cdot \sum_{n=1}^{N}(e_{ln} \cdot Pb_n) - \sum_{m=1}^{M}\left\{\frac{e_{lm}}{2 \cdot c_m} \cdot \left(\frac{\lambda}{1-\mu \cdot d_m} - b_m\right)\right\} \tag{10}$$

A method how ν_l and ν_{L+l} should be selected as non-zero is described later (in the next section). Eq. (8) is solved for selected ν_l and ν_{L+l}. Because eq. (7) was introduced without considering the upper and the lower limits of the output power, modifications of the elements estimation of eqs. (9) and (10) may be needed. The method of the modifications was described in detail in the previous paper[1]. The result for the modifications is written as in eqs. (9)′ and (10)′.

$$W_{ll'} = \sum_{m \in GG} \frac{h_{l'} \cdot e_{lm} \cdot e_{l'm}}{2 \cdot c_m \cdot (1-\mu \cdot d_m)} \tag{9$'$}$$

$$V_l = h_l \cdot I_l - \sum_{n=1}^{N} (e_{ln} \cdot Pb_n) - \sum_{m \notin GG} (e_{lm} \cdot g_m) - \sum_{m \in GG} \left\{ \frac{e_{lm}}{2 \cdot c_m} \circ \left(\frac{\lambda}{1 - \mu \cdot d_m} - b_m \right) \right\} \tag{10'}$$

Where, GG is a set of thermal units which satisfy $(i_l \cdot e_{lm} > 0$ and $g_m > \overline{g_m})$ or $(i_l \cdot e_{lm} < 0$ and $g_m < \underline{g_m})$. The value of g_m in eq. (10)' is $\underline{g_m}$ or $\overline{g_m}$. And when a thermal unit comes to belong to GG, it is necessary to repeat the solution for eq. (8).

6. A Method for The Selection of Non-zero ν_l and ν_{L+l}

To get an initial result which neglects all line capacity, all ν_l and ν_{L+l} are set at zero. If some power flow of the calculated result exceeds the line capacity, then the selection for non-zero ν_l or ν_{L+l} is necessary. Of course, the other ν_l and ν_{L+l}, the power flows of which satisfy their capacities, should be kept at zero. And if the l-th line exceeds the upper limit, then ν_{L+l} should be fixed at zero. If it exceeds the lower limit, then ν_l should be done at zero. Non-zero ν_l and ν_{L+l} are selected from among the rest of ν_l and ν_{L+l}. The selected ν_l and ν_{L+l} should be effective for every overflow lines. Substituting eq. (7) in eq. (6), a partial derivative becomes as follow.

$$\frac{\partial i_l}{\partial \nu_{l'}} = \frac{\partial i_l}{\partial \nu_{L+l'}} = W_{ll'} \tag{11}$$

Because of the Kuhn-Tucker conditions for eq. (1), ν_l and ν_{L+l} are never positive. Therefore, when the power flow of the l-th line exceeds the upper limit, then the favorable $\nu_{l'}$ or $\nu_{L+l'}$ must be has the greater value of $\partial i_l / \partial \nu_{l'}$ or $\partial i_l / \partial \nu_{L+l'}$. On the contrary, exceeding the lower limit, the lesser of $\partial i_l / \partial \nu_{l'}$ or $\partial i_l / \partial \nu_{L+l'}$ must be favorable. Therefore, the non-zero $\nu_{l'}$ or $\nu_{L+l'}$ should be selected which has the largest $h_l \cdot W_{ll'}$ possible. When every line capacity is satisfied and all ν_l and ν_{L+l} are not positive, then the proposed load dispatch is obtained. Otherwise other combination of non-zero ν_l and ν_{L+l} must be checked repeatedly.

7. Calculating Results of a Model Power System

A model power system which is shown in Fig. 1 was used to simulate the proposed method. The characteristic constants of the thermal units are shown in Table 1. The line constants of the model power system are shown in Table 2. The load of each node is shown in Table 3.

Fig. 2 shows the results of the case in which the first and the fourth line capacities are constrained at various limits and NO_x emission at 210 [kg]. In Fig. 3, the constraint of the 7th line capacity is added to the constraints of Fig. 2. The detailed investigations for each result have been described in previous papers[1]. Furthermore, when the fifth line capacity is added, the results become as in Fig. 4. The number of line capacities for Fig. 4 is four. The improvement in the convenient method for many capacities can be confirmed. The quadratic

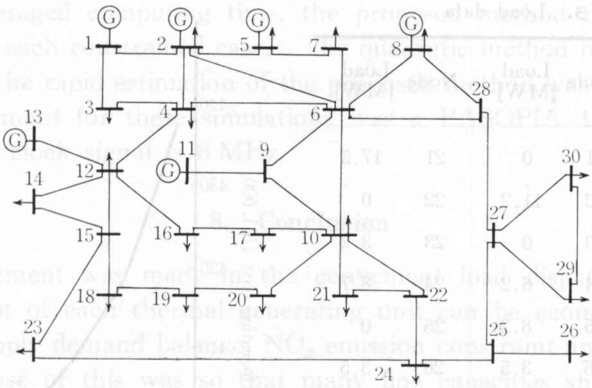

Fig. 1. Model power system.

Table 1. Characteristic constants of thermal units.

No.	Node	$f_m = a_m + b_m \cdot g_m + c_m \cdot g_m^2$ ¥1000			d_m	$\underline{g_m}$	$\overline{g_m}$
		a_m	b_m	$c_m \times 1000$	kg/¥1000	MW	MW
1	5	21.460	0.4170	26.10	—	15	50
2	13	6.706	1.2510	10.40	—	12	40
3	11	3.651	1.2510	10.40	0.722	10	30
4	8	2.254	1.3553	3.48	0.774	10	35
5	2	12.828	0.7298	7.30	0.750	20	80
6	1	18.640	0.8340	1.56	0.669	50	200

Note: The emissions of No. 1, 2 units are not constrained because they are in a remote area.

Table 2. Line data.

Line	Node	x	Line	Node	x	Line	Node	x
1	1– 2	0.0575	15	4–12	0.2560	29	21–22	0.0236
2	1– 3	0.1852	16	12–13	0.1400	30	15–23	0.2020
3	2– 4	0.1737	17	12–14	0.2559	31	22–24	0.1790
4	3– 4	0.0379	18	12–15	0.1304	32	23–24	0.2700
5	2– 5	0.1983	19	12–16	0.1987	33	24–25	0.3292
6	2– 6	0.1763	20	14–15	0.1997	34	25–26	0.3800
7	4– 6	0.0414	21	16–17	0.1923	35	25–27	0.2087
8	5– 7	0.1160	22	15–18	0.2185	36	28–27	0.3960
9	6– 7	0.0820	23	18–19	0.1292	37	27–29	0.4153
10	6– 8	0.0420	24	19–20	0.0680	38	27–30	0.6027
11	6– 9	0.2080	25	10–20	0.2090	39	29–30	0.4533
12	6–10	0.5560	26	10–17	0.0845	40	8–28	0.2000
13	9–11	0.2080	27	10–21	0.0749	41	6–28	0.0599
14	9–10	0.1100	28	10–22	0.1499			

Base: 100 MVA

Table 3. Load data

Node	Load [MW]	Node	Load [MW]	Node	Load [MW]
1	0	11	0	21	17.5
2	21.7	12	11.2	22	0
3	2.4	13	0	23	3.2
4	7.6	14	6.2	24	8.7
5	94.2	15	8.2	25	0
6	0	16	3.5	26	3.5
7	22.8	17	9.0	27	0
8	30.0	18	3.2	28	0
9	0	19	9.5	29	2.4
10	5.8	20	2.2	30	10.6

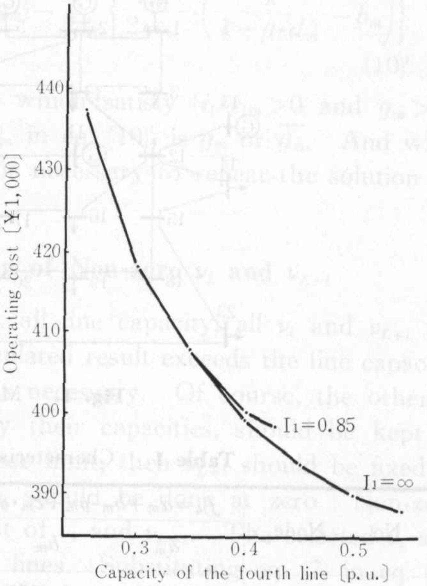

Fig. 2. Operating cost for I_4 ($Y=210$).

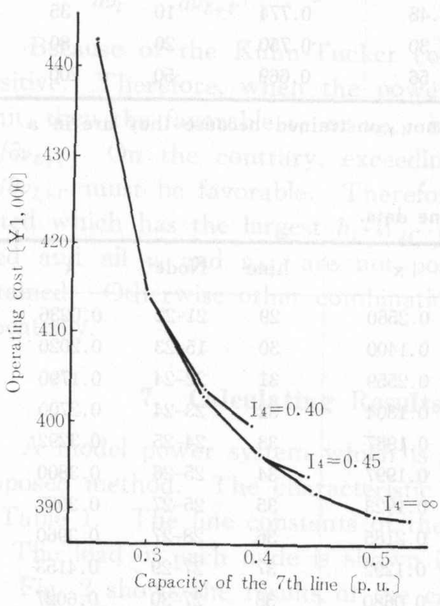

Fig. 3. Operating cost for I_7 ($Y=210$, $I_1=1.1$).

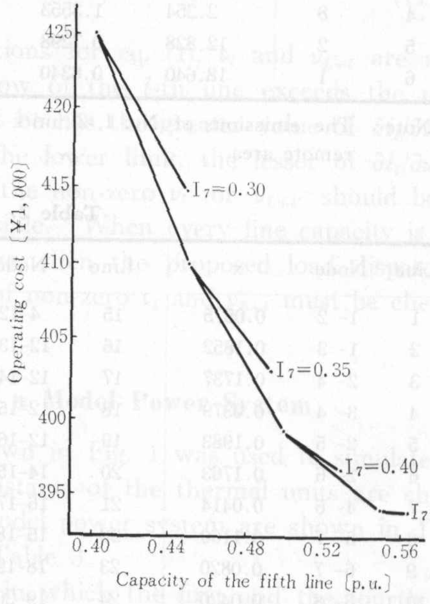

Fig. 4. Operating cost for I_5 ($Y=210$, $I_1=1.1$, $I_4=0.45$).

programming method[1] was also applied to the model system. Its results for each constrained case agreed completely with those of the proposed method. Thus, it can be confirmed that our proposed method should have no estimating error.

For the averaged computing time, the proposed method needed 3 minutes 30.7 seconds for each constrained case. The quadratic method needed 21 minutes 38.0 seconds. The rapid estimation of the proposed method was confirmed. The computing equipment for these simulations was a PASOPIA 16 whose CPU is 8088+8087, the clock signal is 6 MHz.

8. Conclusion

An improvement was made in the convenient load dispatch method with which the output of each thermal generating unit can be economically specified to satisfy the supply demand balance, NO_x emission constraint and the line capacities. The purpose of this was so that many line capacities should be satisfied, estimating error should not appear and yet the results should be estimated rapidly.

The convenient method was improved by using the Kuhn-Tucker conditions. It was introduced from one of the Kuhn-Tucker conditions that the number of non-zero LaGrange's multipliers is not more than the number of the thermal units when all the constraints are accomplished. The proper combination of non-zero multipliers was required. Because there are two multipliers which are concerned with the supply demand balance and the emission constraint, the number of non-zero multipliers which are concerned with the line capacities is at most two less than the number of the thermal units. The choice method was also shown for the combination of non-zero multipliers. These were chosen so that their partial differential coefficients are as great as possible.

A model power system was used for the proposed method and for the strict method of the quadratic programming method. By using many line capacity constraints, the appropriateness of the proposed method was confirmed. Constraining many line capacities, it was confirmed that the proposed method can estimate the optimum load dispatch even if many line capacities are given. For estimating precision, both results completely agree with each other and no estimating error in the proposed method was also confirmed. For the speed of calculation, it was shown that the calculating time of the proposed method is 6.2 times faster than that of the quadratic method in the case of a 30 node model system.

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