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Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 5)*

—A Strict Method For Transmission Capacity—

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Abstract

An optimum operation scheduling method is reported using quadratic programming. In this report, the optimum operation is considered as that which economically satisfies all line capacities and environmental constraint. This method is developed in order to extend another method whose schedule satisfies many of the constraints of a faulty state in an electric power system. The limit of NO_x pollution is considered as a typical environmental constraint.

Kuhn-Tucker conditions are shown for this problem. Except for the NO_x pollution constraint, the Kuhn-Tucker conditions are solved using quadratic programming. The kinds of artificial variables introduced, which are necessary for an initial feasible solution, are kept to a minimum so that the calculation can be done compactly. The cost function of the quadratic programming is constituted by the quadratic equations which are selected from Kuhn-Tucker conditions. The cost function is modified to a linear equation.

The results of the model system simulations are compared with the quadratic programming method and LaGrange's multipliers method. Investigating them, the following is shown: LaGrange's method is very fast for calculating time, but it is weak if there are many and severe constraint. The quadratic method can estimate the optimum operation strictly even when the constraints are many and severe.

1. Introduction

An optimum operation scheduling method was reported in a previous paper[1] which used LaGrange's multipliers method. The previous method can apply only when there are less constraints than generating units.

In this report, a new scheduling method is shown which uses quadratic programming. It can schedule for many constraints, even if they exceed the number of generating units.

Since the NO_x emission constraint is a quadratic one, it is excepted from the optimization of quadratic programming. That is, the LaGrange's multiplier corresponding to emission is assumed previously and suitably. The quadratic programming is solved, then emission is checked. The above multiplier is modified by trial and error, and the quadratic programming is solved repeatedly.

Investigating the sign of the constant term of constraint equations, artificial

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variables are introduced which are needed for an initial feasible solution. The cost function of quadratic programming becomes essentially a quadratic equation. Using a derived function of itself, this cost function is modified linearly. The coefficient of modified function is estimated repeatedly and effectively.

Finally, the proposed method is applied to a model power system which is the same as that used for a previous paper. By comparing the results of both methods, the appropriateness of the proposed method is shown.

2. Formulation of The Problem

It is assumed that the power system has M thermal generating units, L power lines and N nodes. NO_x pollution from the thermal units is regarded as one of the typical environmental constraints. When this problem is formulated, it will become as follows.

Minmize

$$\sum_{m=1}^{M} f_m \tag{1}$$

Subject to

$$Ps = \sum_{m=1}^{M} g_m \tag{2}$$

$$\underline{g_m} \leqq g_m \leqq \overline{g_m} \tag{3}$$

$$-I_l \leqq i_l \leqq I_l \tag{4}$$

$$\sum_{m=1}^{M} y_m \leqq Y \tag{5}$$

Where, f_m, g_m, $\underline{g_m}$, $\overline{g_m}$ and y_m are fuel cost, output power, lower limit of g_m, upper limit of g_m and NO_x emission for the m-th thermal unit, respectively. f_m and y_m are estimated from following eqs. (6) and (7). Ps is the system load, and it is the total of each node load. I_l and i_l are the line capacity and power flow of the l-th power line. Y is the upper limit of NO_x emission.

$$f_m = a_m + b_m \cdot g_m + c_m \cdot g_m^2 \tag{6}$$

$$y_m = d_m \cdot f_m \tag{7}$$

Where, a_m, b_m, c_m and d_m are the characteristic constants of the m-th thermal unit.

3. LaGrangean Function and Power Flow Estimation

Since the constraints of the lower limit $\underline{g_m}$ are intended to be treated especcially, the LaGrangean function becomes eq. (8) from eqs. (1) $\sim$ (5) excepting $\underline{g_m}$.

$$\begin{aligned}\phi = &\sum_{m=1}^{M} f_m + \lambda \cdot \left(Ps - \sum_{m=1}^{M} g_m\right) + \sum_{m=1}^{M} \left\{\nu_m \cdot (g_m - \overline{g_m})\right\} - \sum_{l=1}^{L} \left\{\nu_{M+l} \cdot (I_l + i_l)\right\} \\ &+ \sum_{l=1}^{L} \left\{\nu_{M+L+l} \cdot (i_l - I_l)\right\} + \mu \cdot \left(\sum_{m=1}^{M} y_m - Y\right)\end{aligned} \tag{8}$$

Where, λ, ν_l and μ are LaGrange's multipliers respectively. On the other side, the power flow becomes as follows when the DC method is used.

$$i_l=\sum_{n=1}^{N}(e_{ln}\cdot Pl_n)+\sum_{m=1}^{M}(e_{lm}\cdot g_m) \tag{9}$$

Where, e_{ln} is the element of the sensitivity matrix, and is estimated from eq. (10). Pl_n is the node load of the n-th node.

$$e_{ln}=(B_{jn}^{-1}-B_{kn}^{-1})/x_l \tag{10}$$

Where, B_{jn}^{-1} and B_{kn}^{-1} are the elements of the inverse matrix of the susceptance matrix. x_l is a reactance of the l-th power line which is connected from the j-th node to the k-th node.

4. The kuhn-Tucker Conditions of Our Problem

The kuhn-Tucker conditions of eq. (8) become as follows.

$$\frac{\partial\phi}{\partial g_m}=(1+\mu\cdot d_m)\cdot\frac{df_m}{dg_m}-\lambda+\nu_m-\sum_{l=1}^{L}(\nu_{M+l}\cdot e_{lm})+\sum_{l=1}^{L}(\nu_{M+L+l}\cdot e_{lm})\geqq 0 \tag{11}$$

$$g_m\cdot\frac{\partial\phi}{\partial g_m}=0 \tag{12}$$

$$Ps-\sum_{m=1}^{M}g_m=0 \tag{13}$$

$$g_m\leqq\overline{g_m} \tag{14}$$

$$\nu_m\cdot(g_m-\overline{g_m})=0 \tag{15}$$

$$-I_l\leqq i_l \tag{16}$$

$$i_l\leqq I_l \tag{17}$$

$$\sum_{l=1}^{L}\{\nu_{M+l}\cdot(I_l+i_l)\}=0 \tag{18}$$

$$\sum_{l=1}^{L}\{\nu_{M+L+l}\cdot(i_l-I_l)\}=0 \tag{19}$$

$$\sum_{m=1}^{M}y_m\leqq Y \tag{20}$$

$$\mu\cdot(\sum_{m=1}^{M}y_m-Y)=0 \tag{21}$$

5. Expression for Quadratic Programming

To use quadratic programming, this section is considered excepting eqs. (20) and (21). This means that the quadratic problem is solved with one value of μ. And the optimum value of μ which satisfies eqs. (11)$\sim$(21) is estimated by repeated trial and error.

Another proper method[2] is known to treat the upper limit of the variable

$\overline{g_m}$, but this constraint is handled simply and similarly to other constraints, because unit number M is generally sufficiently less than line number L. Even if this special method is used for the upper limit $\overline{g_m}$ of this problem, the number of equations of quadratic programming can be decreased a little.

To consider the lower limit $\underline{g_m}$, a new variable gx_m is introduced which is estimated to satisfy eq. (22).

$$gx_m = g_m - \underline{g_m} \tag{22}$$

5. 1 Expression for Constraint Equations

Using eq. (22), the following can be formulated from eqs. (11), (13), (14), (16) and (17) respectively.

$$2\cdot c_m\cdot(1+\mu\cdot d_m)\cdot gx_m - \lambda + \nu_m - \sum_{l=1}^{L}(\nu_{M+l}\cdot e_{lm}) + \sum_{l=1}^{L}(\nu_{M+L+l}\cdot e_{lm}) - Sl_m$$
$$= -(b_m + 2\cdot c_m\cdot \underline{g_m})\cdot(1+\mu\cdot d_m) \tag{23}$$

$$\sum_{m=1}^{M} gx_m + Ar_m = Ps - \sum_{m=1}^{M}\underline{g_m} \tag{24}$$

$$gx_m + Sl_{M+m} = \overline{g_m} - \underline{g_m} \tag{25}$$

$$\sum_{m=1}^{M}(e_{lm}\cdot gx_m) - Sl_{2M+l} + Ar_{M+l} = -I_l - \sum_{n=1}^{N}(e_{ln}\cdot Pl_n) - \sum_{m=1}^{M}(e_{lm}\cdot \underline{g_m}) \tag{26}$$

$$\sum_{m=1}^{M}(e_{lm}\cdot gx_m) + Sl_{2M+L+l} - Ar_{M+L+l} = I_l - \sum_{n=1}^{N}(e_{ln}\cdot Pl_n) - \sum_{m=1}^{M}(e_{lm}\cdot \underline{g_m}) \tag{27}$$

Where, Sl_m is a slack variable for the inequality constraint and Ar_m is an artificial variable for an initial feasible solution.

gx_m, λ, ν_m, ν_{M+l} and ν_{M+L+l} should be selected as the non-base of initial feasible solution. The reason is the negative right side of eq. (23), because $\mu \geqq 0$[3] and $(b_m + 2\cdot c_m\cdot \underline{g_m})$ is equal to the incremental fuel cost at $\underline{g_m}$ and it is positive. The right sides of eqs. (24) and (25) are always positive. In eq. (26), when the right side is positive, Sl_{2M+l} should be selected as non-base and Ar_{M+l} as base, or vice versa. Eq. (27) should be treated similarly to select the base and non-base. The rest of the variables become base, and they are Sl_m, Ar_m and Sl_{M+m}.

5. 2 Expression for Cost Function

The cost function is constituted by artificial variables and the rest of the Kuhn-Tucker conditions, which are eqs. (12), (15), (18) and (19). Then, the next cost function F can be obtained, and is transformed as follows:

$$F = \sum_{m=1}^{M} gx_m\cdot Sl_m + \sum_{m=1}^{M}\nu_m\cdot Sl_{M+m} + \sum_{l=1}^{L}\nu_{M+l}\cdot Sl_{2M+l} + \sum_{l=1}^{L}\nu_{M+L+l}\cdot Sl_{2M+L+l}$$
$$+ \sum_{m=1}^{M+2L} Ar_m = \frac{1}{2}\left\{\sum_{m=1}^{M} gx_m\cdot\frac{\partial F}{\partial gx_m} + \sum_{m=1}^{M} Sl_m\cdot\frac{\partial F}{\partial Sl_m} + \sum_{m=1}^{M}\nu_m\cdot\frac{\partial F}{\partial \nu_m}\right.$$
$$+ \sum_{m=1}^{M} Sl_{M+m}\cdot\frac{\partial F}{\partial Sl_{M+m}} + \sum_{l=1}^{L}\nu_{M+l}\cdot\frac{\partial F}{\partial \nu_{M+l}} + \sum_{l=1}^{L} Sl_{2M+l}\cdot\frac{\partial F}{\partial Sl_{2M+L}}$$
$$\left. + \sum_{l=1}^{L}\nu_{M+L+l}\cdot\frac{\partial F}{\partial \nu_{M+L+l}} + \sum_{l=1}^{L} Sl_{2M+L+l}\cdot\frac{\partial F}{\partial Sl_{2M+L+l}}\right\} \tag{28}$$

Where, the values of $\partial F/\partial gx_m$, $\partial F/\partial Sl_m$, $\partial F/\partial \nu_m$, $\partial F/\partial Sl_{M+m}$, $\partial F/\partial \nu_{M+l}$, $\partial F/\partial Sl_{2M+l}$, $\partial F/\partial \nu_{M+L+l}$ and $\partial F/\partial Sl_{2M+L+l}$ of eq. (28) are estimated by the substitution of the former stage values of the corresponding variables in repeated linear programming calculations.[4] That is, the optimum operation can be obtained by minimizing eq. (28) in the region of eqs. (23)~(27).

6. The Calculation Results of a Model Power System

The proposed method was applied to the same model power system as used in the previous paper, which used LaGrange's multipliers method.

Firstly, the optimum operation was estimated with the various emission constraints and the first line capacity, as in the previous paper. The results are shown in Fig. 1. This corresponds completely with the results of the previous method. Then it can be confirmed that both methods are proper.

Secondly, keeping the emission constraint as 210 [kg], the optimum operation was estimated with the various capacities for the first and fourth lines, also as in the previous paper. The results are shown in Fig. 2. The cases of $I_1=\infty$ and 1.10 are little different, so both curves correspond. To compare the results of both methods, the results of the previous method are shown by a dotted line. The case of $I_1=\infty$ corresponds completely in both methods, but when the constraints of line capacity become severer then the estimating error of the previous method becomes large. The maximum error for the operating cost is about 5.6 [%] which is at $I_1=0.85$ and $I_4=0.3$ [P. U.]. From Fig. 2, it is known that when the constraints are lax the previous method estimates the true optimum operation, and that the quadratic programming method can estimate the optimum

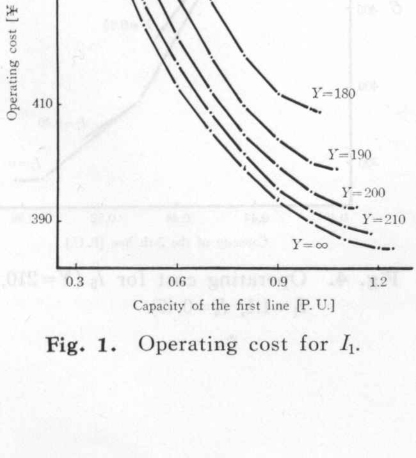

Fig. 1. Operating cost for I_1.

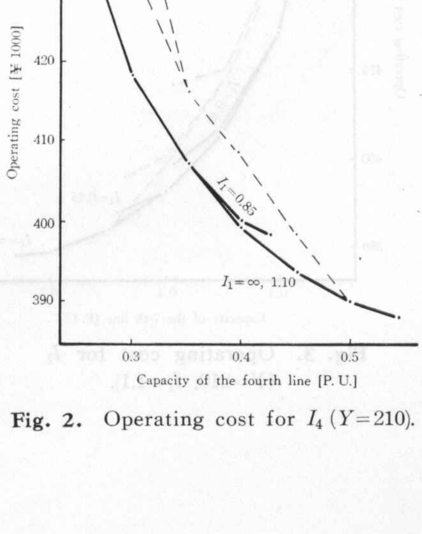

Fig. 2. Operating cost for I_4 ($Y=210$).

operation even if the constraints become severe and many.

Thirdly, keeping at $Y=210$ and $I_1=1.10$, the optimum operation was estimated with various capacities for the fourth and 7-th lines which is also as in the previous case using LaGrange's multipliers method. Together with the previous results, these results are shown in Fig. 3. Also in these results, the previous method has an estimating error when the constraints are severe, but the case of $I_7=0.25$ is thought to be an exception. The maximum error for the operating cost is 2.2 [%] at $I_4=0.40$ and $I_7=\infty$ in this case. Since this maximum error is thought to be small, it can be seen that the previous method does not always result in a large error, even if the constraints are severe and/or many.

The computing time is considered for both methods. These optimum operations were estimated by PASOPIA 16 whose CPU is 8088 and clock signal is 6 MHz. The average computing time for every case of Fig. 2 and Fig. 3 in the previous method was 56.6 seconds, and this quadratic method was 26 minutes 33.8 seconds. That is, the previous LaGrange's is very fast although it is weak when there are severe and many constraints. The quadratic programming method is always strict. When the number of constraints is greater than the number of generating units, then the quadratic programming method can be used but LaGrange's method can not.

Finally, the optimum operation was estimated with 4 lines capacity. The results are shown in Fig. 4. It could be shown concretely that the quadratic programming method can estimate the optimum operation even if many lines have capacity.

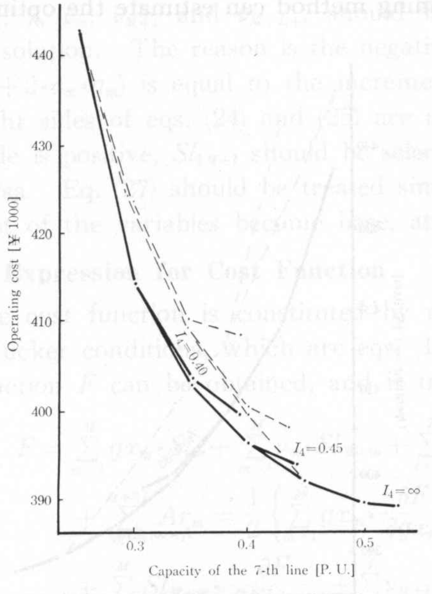

Fig. 3. Operating cost for I_7 ($Y=210$, $I_1=1.1$).

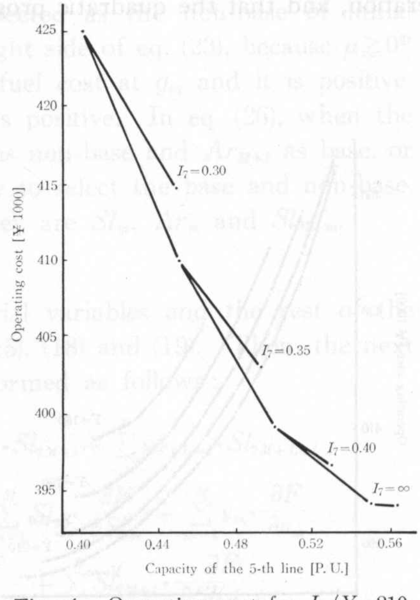

Fig. 4. Operating cost for I_5 ($Y=210$, $I_1=1.1$, $I_4=0.45$).

7. Conclusion

To develop a schedulling method which can be used in a fault state, an economic load dispatch method which satisfies many line capacities and the NO_x pollution limit was reported using the quadratic programming method. Although the previously reported method using LaGrange's multipliers can apply only to a lesser number of constraints than the number of generating units, this proposed method was shown to be able to apply to any number of constraints.

The cost function of quadratic programming was constituted by quadratic equations in the Kuhn-Tucker conditions of this problem. It was shown to be possible to modify this cost function to a linear function by using a derived function of itself.

The simulations of the proposed method were done using a model power system. The results were compared with the previous results. From investigations of this comparison, the following was shown: The method using LaGrange's multipliers can estimate in 1/30 of the calculating time of the quadratic method, but there is an estimating error for severe and many constraints. The proposed method using quadratic programming can estimate the optimum operation strictly even when the constraints are severe and many. The optimum operations with a greater number of constraints than generating units were estimated concretely by the proposed method.

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