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Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 4)*

—A Convenient Method for Transmission Capacity—

by Yoichi NAKAMURA** and Susumu YAMASHIRO**

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Abstract

We report an optimum operation scheduling method which can easily consider both the environmental constraint and some individual line capacities. As a typical environmental constraint, we use the NO_x pollution from every thermal generating unit.

Our method of calculation uses the LaGrangean function. For power flow estimation, any method can be applied to our method. We show in this report that we can determine the LaGrange's multipliers for each line capacity as the solution of simultaneous linear equations.

Finally, our method is tested by a model power system. By the investigation of its results, we show concretely that our method can determine the optimum operation which is economical and satisfies both the environmental constraint and the line capacities.

1. Introduction

The optimum operating schedule of an electric power system must be made considering many kinds of constraints. These constraints include many kinds of environmental constraints, the capacities of power transmission equipment, the demand supply balance, etc. These constraints should be satisfied even when the system enters a faulty state or the system load varies unexpectedly. In this report, to develop a method which may satisfy all of the above constraints, a scheduling method is reported which can satisfy the hourly total NO_x limit from every thermal unit and the line capacity limits.

The purpose of this paper is to discuss a load dispatch method which satisfies both line capacities and NO_x pollution limits economically. In this paper, the NO_x pollution limit is applied to the total value from every thermal generating unit at every hour.

Using LaGrange's multipliers method, this problem is optimized in the constrained region. The number of LaGrange's multipliers increases with the number of lines which need to be satisfied in terms of transmission capacity. These multipliers are shown to be determined as the solution of simultaneous linear equations. It is shown that a conclusion as to whether these constraints can be satisfied can be reached when the coefficients of simultaneous linear equations are estimated.

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** Department of Electrical Engineering, Kitami Institute of Technology.

Finally, the results of a model power system's simulations are shown. Investigating these results, the appropriateness of the proposed method is shown.

2. Objective Function

A power system is assumed to have M thermal units and L power transmission lines the capacities of which should be considered. Using LaGrange's multipliers method, the next objective function is constituted.

$$\phi=\sum_{m=1}^{M} f_m+\lambda\cdot(Ps-\sum_{m=1}^{M} g_m)+\mu\cdot(Y-\sum_{m=1}^{M} y_m)+\sum_{l=1}^{L}\{\nu_l\cdot(I_l-|i_l|)\} \tag{1}$$

Where, g_m is the output power of No. m thermal unit. f_m and y_m are the fuel cost and the NO_x emission of No. m thermal unit respectively, and these are estimated from eqs. (2) and (3). Ps and Y are the system load and upper limit of NO_x at every hour respectively. I_l and i_l are the transmission capacity and the power flow of No. l power line respectively. λ, μ and ν_l are LaGrange's multipliers.

$$f_m=a_m+b_m\cdot g_m+c_m\cdot g_m^2 \qquad (\underline{g_m}\leqq g_m\leqq \overline{g_m}) \tag{2}$$

$$y_m=d_m\cdot f_m \tag{3}$$

Where, a_m, b_m, c_m and d_m are the characteristic constants of No. m thermal unit. $\underline{g_m}$ and $\overline{g_m}$ are the lower and upper limits of g_m respectively. Minimizing eq. (1), the optimum operation is obtained considering the above constraints.

3. Sensitivity Matrix

The power system is assumed to have N nodes and Q power transmission lines. When i_q denotes the power flow of the q-th line which is from the j-th node to the k-th node, it becomes eq. (4) in the DC method.[1,2)]

$$i_q=(\delta_j-\delta_k)/x_q \tag{4}$$

Where, δ_j and δ_k are the voltage phase angles of the j-th node and the k-th node respectively. x_q is a reactance of the q-th line. Using a matrical equation, eq. (4) becomes eq. (5).

$$[Pb]=[B]\,[\delta] \tag{5}$$

Where, $[Pb]$ is the column vector whose element is the node power. $[B]$ is the susceptance matrix. Using sensitivity matrix $[e]$, $[i]$ becomes eq. (6).

$$[i]=[e]\,[Pb] \tag{6}$$

The size of $[e]$ is $Q\times N$, and its element e_{qn} becomes eq. (7) from eqs. (4)$\sim$(6).

$$e_{qn}=(B_{jn}^{-1}-B_{kn}^{-1})/x_q \tag{7}$$

Where, B_{jn}^{-1} and B_{kn}^{-1} are the elements of inverse matrix of $[B]$ respectively. When the voltage phase angle of the reference node is fixed at 0, the element of inverse matrix of $[B]$ which corresponds to the reference node can become 0.[3] And the element of $[e]$ which corresponds to the reference node becomes 0.

4. The Decision of the Output Power of Thermal Unit

Eq. (8) can be obtained from eq. (6).

$$\frac{di_l}{dg_m}=e_{lm} \tag{8}$$

Output power becomes eq. (9) from $\partial\phi/\partial g_m=0$ and eq. (8).

$$g_m=\frac{\lambda+\sum_{l=1}^{L}(\nu_l\cdot e_{lm})}{2\cdot c_m\cdot(1-\mu\cdot d_m)}-\frac{b_m}{2\cdot c_m} \tag{9}$$

Using one set of values of λ, μ and ν_l, the output is determined by eq. (9) between $\overline{g_m}$ and $\underline{g_m}$. λ and μ are determined by trial and error so that the system can satisfy the power supply and demand balance constraint and the emission constraint respectively.

5. The Decision of ν_l

When the n-th node power is divided into node load Pl_n and output power g_m from generators which are connected to the n-th node, i_l becomes eq. (10) from eq. (6).

$$i_l=\sum_{n=1}^{N}e_{ln}\cdot pl_n+\sum_{m=1}^{M}e_{lm}\cdot g_m \tag{10}$$

Eq. (11) is obtained when eq. (9) is substituted for eq. (10) and the result is straightened out.

$$[W]\,[\nu]=[V] \tag{11}$$

Where, elements of $[W]$ and $[V]$ are eqs. (12) and (13) respectively.

$$W_{ll'}=\sum_{m=1}^{M}\frac{e_{lm}\cdot e_{l'm}}{2\cdot c_m\cdot(1-\mu\cdot d_m)} \tag{12}$$

$$V_l=i_l-\sum_{n=1}^{N}(e_{ln}\cdot Pl_n)-\sum_{m=1}^{M}\left\{\frac{e_{lm}}{2\cdot c_m}\cdot\left(\frac{\lambda}{1-\mu\cdot d_m}-b_m\right)\right\} \tag{13}$$

Eq. (11) is the simultaneous linear equation with L unknowns which are ν_l. To satisfy every line's capacity, $[\nu]$ can be determined by solving simultaneous linear equations which are modified eq. (13) substituting I_l or $-I_l$ to i_l.

5.1 Estimating Method for $[W]$ and $[V]$ of Eq. (11)

First, λ and μ are determined so that the power supply demand balance, and emission constraint are satisfied substituting 0 to ν_l ($l=1, 2, 3, \cdots, L$). If the capacities of some lines are not satisfied by this operation, then $[\nu]$ is estimated from eq. (11). L becomes the line number whose capacity was not satisfied and it also becomes the line number whose capacity should be considered. Then, using this $[\nu]$, λ and μ are estimated so that the power supply demand balance, and emission constraints are satisfied. Each power flow is checked again. Some ν_l are treated as the constants of which lines satisfy capacities under the new $[\nu]$, but did not satisfy them under old $[\nu]$. This is explained as follows in detail. A set of lines is created and named LG (Line is Good). The elements of LG are those lines which satisfy the capacity with the newest $[\nu]$ but have formerly exceeded it with the older one. In other words, LG is the set of lines which satisfy the line capacities with ν_l ($\neq 0$). Another set LN (Line is Nogood) is created. The elements of LN are those lines which do not satisfy the line capacities with the newest $[\nu]$. They are the lines the capacities of which should be considered at the next estimating stage. Then eqs (12) and (13) are modified to eqs. (12)′ and (13)′ because ν_l of those lines which are the elements of LG are treated as constants.

$$W_{ll'} = \sum_{m=1}^{M} \frac{e_{lm} \cdot e_{l'm}}{2 \cdot c_m \cdot (1-\mu \cdot d_m)} \qquad (l, l' \in LN) \tag{12'}$$

$$\begin{aligned} V_l = I_l - \sum_{n=1}^{N} (e_{ln} \cdot Pl_n) \\ - \sum_{m=1}^{M} \left\{ \frac{e_{lm}}{2 \cdot c_m} \cdot \left(\frac{\lambda}{1-\mu \cdot d_m} - b_m \right) \right\} \\ - \sum_{l' \in LG} \sum_{m=1}^{M} \frac{e_{lm} \cdot e_{l'm}}{2 \cdot c_m \cdot (1-\mu \cdot d_m)} \cdot \nu_{l'} \end{aligned} \tag{13'}$$

In eq. (13)′, the first term of the right side is settled with $-I_l$, when the power flow of the l-th line is negative. The optimum load dispatch can be obtained which satisfies all constraints by repeated estimation for λ, μ and ν_l.

5.2 The Reduction Method for Computational Quantity and The Concluding Method Whether Constraints can be Accomplished or not

Eqs. (12)′ and (13)′ are based on eq. (9) originally. In other words, the reason for the determination of $[\nu]$ from eqs. (11), (12)′ and (13)′ is that the power output can be obtained from eq. (9). This means that $[\nu]$ from eqs. (11), (12)′ and (13)′ are obtained so that the power output satisfies eq. (9) and the line capacity. However, if the output of eq. (9) exceeds the upper or lower limit then the output is fixed at the upper or the lower limit regardless of eq. (9). In this case, the estimation must be done repeatedly.

When the capacity of the l-th line is not satisfied, if $i_l > 0$ and $e_{lm} > 0$ then the m-th generating unit must decrease its output to satisfy the l-th line's capacity. Therefore, the unit which can change its output to satisfy the line capacity is the m-th generator which is ($i_l \cdot e_{lm} > 0$ and $g_m > \underline{g_m}$) or ($i_l \cdot e_{lm} < 0$ and $g_m < \overline{g_m}$). The set of these changeable units is denoted as GG (Good Generators). Eqs. (12)′ and (13)′ are also modified to eqs. (12)″ and (13)″ respectively so that eq. (9) is used by the m-th unit which is included in GG.

$$W_{ll'}=\sum_{m\in GG}\frac{e_{lm}\cdot e_{l'm}}{2\cdot c_m\cdot(1-\mu\cdot d_m)}\qquad (l,l'\in LN)\tag{12)''$$

$$\begin{aligned}V_l=I_l&-\sum_{n=1}^{N}(e_{ln}\cdot Pl_n)-\sum_{m\notin GG}e_{lm}\cdot g_m\\&-\sum_{m\in GG}\left\{\frac{e_{lm}}{2\cdot c_m}\left(\frac{\lambda}{1-\mu\cdot d_m}-b_m\right)\right\}\\&-\sum_{l'\in LG}\sum_{m\in GG}\frac{e_{lm}\cdot e_{l'm}}{2\cdot c_m\cdot(1-\mu\cdot d_m)}\cdot\nu_{l'}\end{aligned}\tag{13)''$$

In eq. (13)'', g_m which is the third term of the right side and which is specified not to be included in GG is equal to $\underline{g}_m$ or $\overline{g}_m$. It is similar to eq. (13)' that the first term of the right side in eq (13)'' is settled $-I_l$ when the power flow of the l-th line is negative.

We have other constraints which are the power supply demand balance and the emission constraint in addition to line capacities. Then, to satisfy all constraints, it is necessary that (the number of lines which are included in LN)$+2\leqq$ (the number of units which are included in GG).

6. The Investigation for Method of Load Flow Calculation

For the purpose which [ν] can be obtained from eqs. (11), (12) and (13), it may be necessary that the load flow calculating method satisfies the following two conditions. The first condition is that the power flow becomes an explicit function of unit power (which is like eq. (10)). The second condition is that the power flow is differentiable by unit power (which is needed to get eq. (8)).

Finally, not only the DC method can be used but also any other method which satisfies the above two conditions for the load flow calculating method.

7. Calculating Results of a Model Power System

Our method was applied to a model power system which has 30 nodes and 6 thermal units, it is shown in Fig. 1. The thermal characteristic constants are

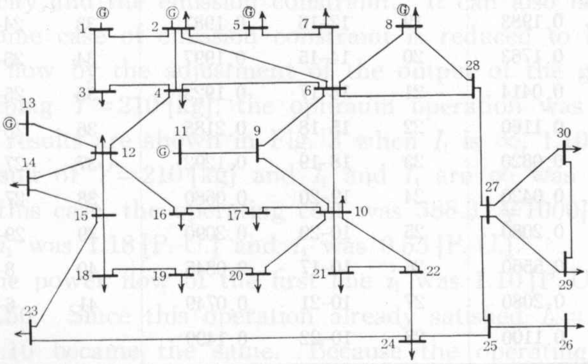

Fig. 1. Model power system.

shown in Table 1. The line constants of the model power system are shown in Table 2. Each load of nodes is shown in Table 3.

First, the capacities and environmental constraint were assumed to be infinity so that the eonommic operation could be estimated. The operating cost was 385.5 [¥1000], and the emission was 224.1 [kg] as the necessary value for the above economic operation. For the power flow of the first, fourth and 7-th lines, i_1 was 1.25 [P. U.], i_4 was 0.59 [P. U.] and i_7 was 0.56 [P. U.], respectively.

Secondly, keeping I_4 and I_7 to ∞, the capacity of the first line I_1 was settled variously. Morever, settling emission constraint Y variously, the optimum operation was estimated by the proposed method. The results are shown in Fig. 2.

Table 1. Characteristic constants of thermal units

No.	Node	$f_m=a_m+b_m\cdot g_m+c_m\cdot g_m^2$ ¥1000 a_m	b_m	$c_m\times1000$	d_m kg/¥1000	$\underline{g}_m$ MW	$\overline{g}_m$ MW
1	5	21.460	0.4170	26.10	—	15	50
2	13	6.706	1.2510	10.40	—	12	40
3	11	3.651	1.2510	10.40	0.722	10	30
4	8	2.250	1.3553	3.48	0.774	10	35
5	2	12.828	0.7298	7.30	0.750	20	80
6	1	18.640	0.8340	1.56	0.669	50	200

Note 1: The emission of No. 1, 2 units is not constrained because they are in a remote area.
Note 2: No. 6 unit is always in operation.

Table. 2. Line data

Line	Node	x	Line	Node	x	Line	Node	x
1	1- 2	0.0575	15	4-12	0.2560	29	21-22	0.0236
2	1- 3	0.1852	16	12-13	0.1400	30	15-23	0.2020
3	2- 4	0.1737	17	12-14	0.2559	31	22-24	0.1790
4	3- 4	0.0379	18	12-15	0.1304	32	23-24	0.2700
5	2- 5	0.1983	19	12-16	0.1987	33	24-25	0.3292
6	2- 6	0.1763	20	14-15	0.1997	34	25-26	0.3800
7	4- 6	0.0414	21	16-17	0.1923	35	25-27	0.2087
8	5- 7	0.1160	22	15-18	0.2185	36	28-27	0.3960
9	6- 7	0.0820	23	18-19	0.1292	37	27-29	0.4153
10	6- 8	0.0420	24	19-20	0.0680	38	27-30	0.6027
11	6- 9	0.2080	25	10-20	0.2090	39	29-30	0.4533
12	6-10	0.5560	26	10-17	0.0845	40	8-28	0.2000
13	9-11	0.2080	27	10-21	0.0749	41	6-28	0.0599
14	9-10	0.1100	28	10-22	0.1499			

Base: 100 MVA

Table 3. Load data

Node	Load [MW]	Node	Load [MW]	Node	Load [MW]
1	0	11	0	21	17.5
2	21.7	12	11.2	22	0
3	2.4	13	0	23	3.2
4	7.6	14	6.2	24	8.7
5	94.2	15	8.2	25	0
6	0	16	3.5	26	3.5
7	22.8	17	9.0	27	0
8	30.0	18	3.2	28	0
9	0	19	9.5	29	2.4
10	5.8	20	2.2	30	10.6

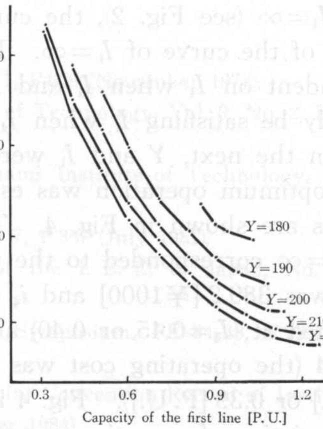

Fig. 2. Operating cost for I_1.

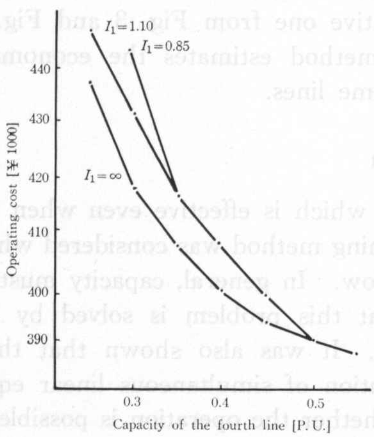

Fig. 3. Operating cost for $I_4 (Y=210)$.

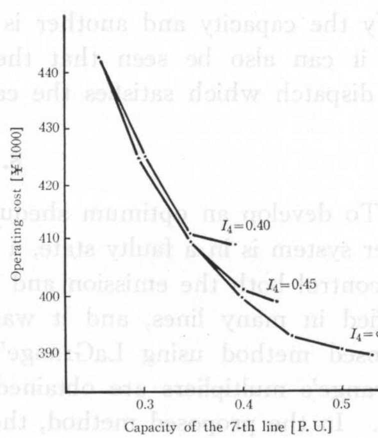

Fig. 4. Operating cost for I_7 $(Y=210, I_1=1.1)$.

From this Figure, it can be confirmed that the operating cost increases depending on the line capacity and the emission constraint. It can also be shown that the power flow in some case of emission constraint is reduced to less than a third of the economic flow by the adjustment of the output of the generating units.

Thirdly, keeping $Y=210$ [kg], the optimum operation was estimated under various I_4. The results are shown in Fig. 3 when I_1 is ∞, 1.10 and 0.85 [P. U.]. Of couse, the result of $Y=210$ [kg] and I_1 and I_4 are ∞ was similar in Fig. 2 and Fig. 3. In this case, the operating cost was 388.3 [¥1000], the power flow of the first line i_1 was 1.18 [P. U.] and i_4 was 0.55 [P. U.].

In Fig. 3, the power flow of the first line i_1 was 1.10 [P. U.] when $Y=210$, $I_1=\infty$ and $I_4=0.50$. Since this operation already satisfied $I_1=1.10$, both points of $I_1=\infty$ and 1.10 became the same. Because the operating cost was 389.2 [¥1000] and the power flow of the fourth line i_4 was 0.53 at $Y=210$, $I_1=1.10$

and $I_4=\infty$ (see Fig. 2), the curve of $I_1=1.10$ of Fig. 3 started from the middle point of the curve of $I_1=\infty$. It is known from Fig. 3 that the operating cost is dependent on I_4 when I_1 and I_4 are larger. This means that some case could already be satisfiing I_1 when I_4 was satisfied.

In the next, Y and I_1 were settled to 210 [kg] and 1.10 [P.U.] respectively. The optimum operation was estimated with various capacities of I_4 and I_7. The results are shown in Fig. 4. The optimum operation for $Y=210$, $I_1=1.10$ and I_4, $I_7=\infty$ corresponded to the points of Fig. 2 and Fig. 3 too (i. e. the operating cost was 389.2 [¥1000] and i_4 and i_7 were 0.53 [P. U.]). The resuts which were obtained at ($I_4=0.45$ or 0.40) and $I_7=\infty$ also coresponded to between Fig. 3 and Fig. 4 (the operating cost was 398.3 [¥1000] or 408.5 [¥1000] and i_7 was 0.43 [P. U.] or 0.39 [P. U.]). Fig. 4 is contrary to Fig. 3. That is domination of I_4 for the variation of operating cost when I_7 is greater, but domination of I_7 when I_7 is less. This means that some cases of less I_7 can automatically satisfy the I_4 limit.

It can be seen that one case is the exclusive combination of the lines to satisfy the capacity and another is the cooperative one from Fig. 3 and Fig. 4. And it can also be seen that the proposed method estimates the economical load dispatch which satisfies the capacity of some lines.

8. Conclusion

To develop an optimum sheduling method which is effective even when the power system is in a faulty state, a load dispatching method was considered which can control both the emission and the power flow. In general, capacity must be satisfied in many lines, and it was shown that this problem is solved by the proposed method using LaGrange's multipliers. It was also shown that these LaGrange's multipliers are obtained as the solution of simultaneous linear equations. In the proposed method, the decision whether the operation is possible or not within the line capacities can be made when the coefficients of these simultaneous linear equations are estimated. Though the DC method was used for power flow calculations in this paper, it was noted that any other calculating method can be used for the proposed load dispatching method.

The appropriateness of the proposed method was confirmed by its application to a model power system which has 30 nodes and 6 thermal generating units. The variations of the system's operating cost were shown with some kinds of constraints. From the simulations of a model power system, it is known that there are both the exclusive case and the cooperative case for the satisfaction of the capacity of the lines and these cases depend on the combination of the capacity-constrained lines. It is concretely shown that the proposed method can determine the optimum economic load dispatch when some line capacities should be considered.

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