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Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 3)*

—A Method Considering Transmission Capacity—

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Abstract

We have previously reported an optimum operation decision method which would be economical and satisfy environmental constraint. In this report, we consider the constraints of transmission capacity. Using the same method used for environmental constraint, we can also satisfy the transmission capacity constraints. But if we use this method, we must be prepared for the increased kinds of the numbers of Lagrange's multipliers. The decision of these multipliers generally becomes increasingly difficult as the numbers increase.

In this proposed method, only two Lagrange's multipliers are used. That is to say, as a first step we consider only two constraints; the power supply and demand balance constraint and the environmental constraint. The result obtained from the above step is modified to satisfy the transmission capacity constraints and also to minimize the increase in operating costs.

In the first half of this report, we consider the deterministic system load. In the latter half, we consider the case in which a system load varies probabilistically. Finally we show the results of model system simulations.

1. Introduction

When we consider the schedule of electric power system operation, we must pay attention to many problems. For example, consumers tolerate little service interruption, and environmental pollution is a serious problem too. In this report, we consider transmission capacities for each line, in addition to the above two problems. In other words, we report the scheduling method for electric power systems which can give us economic operation considering demand supply balance, air pollution constraint and transmission capacities. The operation discussed here is instantaneous. We also consider the hourly NO_x emission constraint as a concrete example of an environmental constraint.

Although we consider the deterministic operation in the first half of this report, we also consider, in the latter half, the probabilistic one in which the electric load varies probabilistically. When we estimate probabilistic optimum operation, we do not concretely calculate each probabilistic load level, but calculate the increase in the expected cost when the load varies probabilistically. We

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regard the unit commitment with minimum expected cost as the optimum operation.

Finally, we apply our method to a model power system. We show the results which are obtained under some environmental constraints and/or transmission capacity constraints.

2. Objective Function

We assume that a power system has M thermal units and L power transmission lines. When we extend the objective function to include the constraints of transmission capacity simply, it may become eq. (1).

$$\phi=\sum_{m=1}^{M} f_m+\lambda\cdot(P-\sum_{m=1}^{M} g_m)+\mu\cdot(Y-\sum_{m=1}^{M} y_m)+\sum_{l=1}^{L}\nu_l\cdot(I_l-|i_l|) \tag{1}$$

Where, g_m is the output power of No. m thermal unit. f_m and y_m are the fuel cost and the NO_x emission of No. m thermal unit respectively, and these are estimated from eqs. (2) and (3). P and Y are system load and upper limit of NO_x every hour respectively. I_l and i_l are the transmission capacity and the power flow of No. l power line respectively. λ, μ and ν_l are Lagrange's multipliers.

$$f_m=\begin{cases} a_m+b_m\cdot g_m+c_m\cdot g_m^2 & (\underline{g}_m\leqq g_m\leqq \bar{g}_m) \\ fs_m & (g_m=0)\end{cases} \tag{2}$$

$$y_m=d_m\cdot f_m \tag{3}$$

Where, a_m, b_m, c_m and d_m are the characteristic constants of No. m thermal unit. $\underline{g}_m$ and $\bar{g}_m$ are lower and upper limits of g_m respectively. And fs_m is the start-up cost of No. m thermal unit.[3)]

Minimizing eq. (1), we can reach optimum operation considering the above constraints. But eq. (1) has many Lagrange's multipliers, the kinds of these are $(2+L)$. That usually makes the problem difficult to deal with, because we must decide so many multipliers efficiently. Therefore, we use another objective function of eq. (1)$'$.

$$\phi'=\sum_{m=1}^{M} f_m+\lambda\cdot(P-\sum_{m=1}^{M} g_m)+\mu\cdot(Y-\sum_{m=1}^{M} y_m) \tag{1$'$}$$

In eq. (1)$'$, we ignore the constraint of transmission capacities. After estimating the load dispatch considering only the demand supply balance and the emission constraint by minimizing eq. (1)$'$, we modify the result to satisfy the transmission capacity constraints as follows.

3. Decision Method for Which Unit's Power Should be Changed to Satisfy the Capacity Constraints

When the result of minimizing eq. (1)$'$ satisfies all the transmission capacity constraints, the required operation is attained. In the following, we report a method to satisfy the transmission capacity constraints when the result of eq. (1)$'$ can not satisfy them.

3.1 Estimation of The Cost Change by A Power Increase of No. m Unit

To decide which unit's power should be increased to satisfy the capacity constraints, let's imagine No. m thermal unit's power to be increased by ΔG. The increase of power of another unit m' becomes eq. (4).[4]

$$\Delta g_{m'} = \frac{\Delta G}{c_{m'} \cdot (1-\mu \cdot d_{m'}) \cdot \sum_{m'' \neq m} \frac{1}{c_{m''} \cdot (1-\mu \cdot d_{m''})}} \qquad (m' \neq m) \qquad (4)$$

And the increase of fuel cost becomes eq. (5) from eq. (2).

$$\Delta f_m = \Delta g_m \cdot b_m + \Delta g_m \cdot c_m \cdot (2 \cdot g_m + \Delta g_m) \qquad (m=1, 2, 3, \cdots, M) \qquad (5)$$

Where, Δg_m of eq. (5) implies ΔG or $\Delta g_{m'}$ of eq. (4). The emission increases by $d_m \cdot \Delta f_m$ for every unit because of eq. (3), then the increase of the total system becomes $\sum_{m=1}^{M} (d_m \cdot \Delta f_m)$. When the system decreases the emission by $\sum_{m=1}^{M} (d_m \cdot \Delta f_m)$ to satisfy the emission constraint, the increase in the system's operating cost becomes $-\mu \cdot \sum_{m=1}^{M} (d_m \cdot \Delta f_m)$.[4] Therefore, ΔF of eq. (6) is the increase of the operating cost for the total system when No. m thermal unit's power is increased by ΔG and the emission constraint is satisfied.

$$\Delta F = \sum_{m=1}^{M} \left\{ (1-\mu \cdot d_m) \cdot \Delta f_m \right\} \qquad (6)$$

3.2 Estimation of the Power Flow Change of Each Line

Because of its small computational quantity,[5] we adopt the DC method for load flow calculations. In the DC method, we estimate the node power $[P]$ by eq. (7).

$$[P] = [B]\,[\delta] \qquad (7)$$

Where, $[P]$ and $[\delta]$ are the power and the voltage phase of each node, and these are column vectors. When the numbers of nodes are N, these sizes are $N-1$. $[B]$ is the susceptance matrix and its size is $(N-1) \times (N-1)$.[6] Power flow is estimated from eq. (8).[7]

$$[i] = [W]\,[\delta] \qquad (8)$$

Where, $[i]$ is the column vector which means the power flow of each line and its size is L. $[W]$ is the transformation matrix and its size is $(L) \times (N-1)$. The elements of $[W]$ are susceptance or 0. When the sensitivity matrix is defined by eq. (9), the power flow becomes eq. (10) from eqs. (7), (8) and (9).

$$[e] = [W]\,[B]^{-1} \qquad (9)$$

$$[i] = [e]\,[P] \qquad (10)$$

Where, $[B]^{-1}$ means the inverse matrix of $[B]$. From eq. (10), the increase of power flow of line l becomes eq. (11) when the thermal units increase output as described above.

$$\Delta i_l = \sum_{m=1}^{M} (e_{lm} \cdot \Delta g_m) \tag{11}$$

3.3 To decide which Unit's Output Should be Changed

By ΔI_l, we denote the value which must be increased to satisfy the transmission capacity of No. l line.

To consider a simple case, we begin with the assumption that only one line (which is No. l) has the capacity constraint. Because we must increase the output of No. m thermal unit by $\Delta I_l/\Delta i_l$ times with ΔG, the increasing cost of the total system may become eq. (12) approximately.

$$\Delta H(m) = \Delta F \cdot \Delta i_l / \Delta i_l \tag{12}$$

There are two principal reasons why eq. (12) is approximate. The first reason is that other units except No. m are dispatched by eq. (1)′ independently of the line capacity. Another reason is that we regard the cost change ΔF as constant until capacity is satisfied. However, we can decide which unit's output should be changed by ΔG, because we estimate the optimum unit to adjust the output repeatedly until capacity is satisfied. Of course the optimum unit has the minimum $\Delta H(m)$, and we repeat the estimation from eq. (4) to eq. (12) while capacity is not satisfied. By the sign of $\Delta I_l/\Delta i_l$, we can know whether the output of No. m unit should be increased or decreased, i. e. if $\Delta I_l/\Delta i_l > 0$, then increase; otherwise decrease. Moreover, because $-\Delta F$ means the cost increase when No. m unit's output is decreased, $\Delta H(m)$ means the cost increase when No. m unit's output is changed to suit the capacity.

When we must consider some constraints of capacity, more calculation may be needed, because, one line capacity may expect the No. m unit to increase output, but the other line may expect it to decrease. In our method, the lines of the system are divided into two groups. One group has lines which expect to increase and another has lines which expect to decrease. We sum up the increase of cost of eq. (12) with each group. When we denote these groups as $S1$ and $S2$, this summation becomes eqs. (13) and (14).

$$\Delta H1 = \sum_{l \in S1} (\Delta F \cdot \Delta I_l / \Delta i_l) \tag{13}$$

$$\Delta H2 = \sum_{l \in S2} (\Delta F \cdot \Delta I_l / \Delta i_l) \tag{14}$$

Where, if $S1$ expects to increase the output, then the system operating cost would change in the direction shown by the sign of $\Delta H1$ when the output of No. m is increased until every line of $S1$ can satisfy the capacity constraints. However, the power flow of some lines in $S2$ will become further removed from capacity constraints.

We would prefer rather less increase in cost for $\Delta H1$ and $\Delta H2$. It will be disturbed by a greater one, because the direction of output change for $\Delta H1$ and $\Delta H2$ are opposite. In our method, we give the penalty cost of $\sum |\Delta F \cdot \Delta I_l|$ to this disturbance. For example, when $\Delta H1$ is less than $\Delta H2$, the output change is assessed by $\Delta H1 + \sum_{l \in S2} |\Delta F \cdot \Delta I_l|$. Although the above assessment does not show the cost increase when all capacity is satisfied, we consider that it is convenient to decide the optimum unit of which the output should be changed. Similarly, as in the case when only one capacity is considered, we change the output of the unit which has the minimum assessment, repeatedly.

4. Consideration for Probabilistic Variation of System Load

When system load varies probabilistically, we assume that the system is operated at each load level, as we described in chapter 3. We consider the normal distribution as a typical one for the system load variation.

4.1 The Method for Estimating The Expected Value of Fuel Cost

When the system load varies probabilistically, then the output of each unit varies probabilistically too. The standard deviation of thermal output and the expected value of fuel cost become eqs. (15) and (16) respectively.

$$\sigma_m = \frac{\sigma_p}{c_m \cdot \sum_{m'} \frac{1}{c_{m'}}} \tag{15}$$

$$E(f_m) = a_m + b_m \cdot E(g_m) + c_m \cdot \left[\left\{ E(g_m) \right\}^2 + \{\sigma_m\}^2 \right] \tag{16}$$

Where, σ_m and σ_p are standard deviations of output of No. m unit and the system load respectively. $E(\ )$ means the expected value. Because thermal units are constrained by both upper and lower output limits, then in some cases we may need to consider these limits. In this paper, we consider σ_m so that $E(g_m) \pm 3 \cdot \sigma_m$ can be involved in the possible zone. That is, when $E(g_m) \pm 3 \cdot \sigma_m$ exceeds the upper or lower limit by eq. (15), σ_m should be modified by eq. (17).[8)]

$$\sigma_m = \frac{\bar{g}_m - E(g_m)}{3} \quad \text{or} \quad \frac{E(g_m) - \underline{g}_m}{3} \tag{17}$$

4.2 The Estimation of the Cost Increase when A System is Constrained Probabilistically

Because variation of system load is usually small as compared with its expected value, the cost increase when the probability constraint is considered can be estimated by its gradient. For example, when we must operate the system so that the probability of exceeding line capacity is less than a particular constrained value (it is denoted by α), then the cost increase is estimated as follows. We must satisfy the line capacity in some power flow levels which is from I_l to I_α.[9)] Where, the probability of exceeding I_α is α (i. e. Prob $(i_l > I_\alpha) = \alpha$). We can estimate the gradient of cost from eq. (18) when we control the power flow.

$$\frac{\partial (\sum_m f_m)}{\partial i_l} = \sum_{m'} \frac{\partial (\sum_m f_m)}{\partial g_{m'}} \cdot \frac{\partial g_{m'}}{\partial i_l} \tag{18}$$

Where, we can get $\partial g_{m'} / \partial i_l$ from $[e]^{-1}$. Then, we can modify the expected value of the system operating cost by using the value of which line flow must be controlled, its probability distribution and eq. (18).

5. Calculation Results for a Model Power System

We adopted a model power system which has 30 nodes and 6 thermal units. The system is shown in Fig. 1, and the line constants are shown in Table 1. Each load of nodes is shown in Table 2 and the thermal characteristic constants shown in Table 3. The standard deviation of each load was assumed to be 8%.

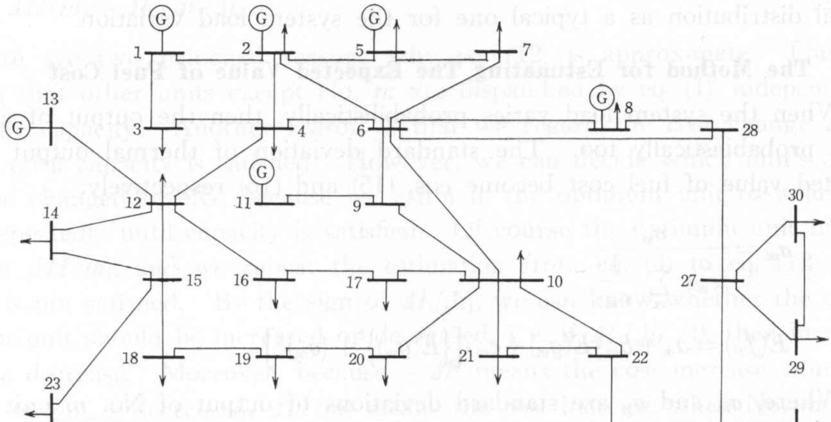

Fig. 1. Model power system.

Table 1. Line data

Line	Node	X	Line	Node	X	Line	Node	X
1	1—2	0.0575	15	4—12	0.2560	29	21—22	0.0236
2	1—3	0.1852	16	12—13	0.1400	30	15—23	0.2020
3	2—4	0.1737	17	12—14	0.2559	31	22—24	0.1790
4	3—4	0.0379	18	12—15	0.1304	32	23—24	0.2700
5	2—5	0.1983	19	12—16	0.1987	33	24—25	0.3292
6	2—6	0.1763	20	14—15	0.1997	34	25—26	0.3800
7	4—6	0.0414	21	16—17	0.1923	35	25—27	0.2087
8	5—7	0.1160	22	15—18	0.2185	36	28—27	0.3960
9	6—7	0.0820	23	18—19	0.1292	37	27—29	0.4153
10	6—8	0.0420	24	19—20	0.0680	38	27—30	0.6027
11	6—9	0.2080	25	10—20	0.2090	39	29—30	0.4533
12	6—10	0.5560	26	10—17	0.0845	40	8—28	0.2000
13	9—11	0.2080	27	10—21	0.0749	41	6—28	0.0599
14	9—10	0.1100	28	10—22	0.1499			

Base : 100 MVA

Table 2. Load data

Node	Load [MW]	Node	Load [MW]	Node	Load [MW]
1	0	11	0	21	17.5
2	21.7	12	11.2	22	0
3	2.4	13	0	23	3.2
4	7.6	14	6.2	24	8.7
5	94.2	15	8.2	25	0
6	0	16	3.5	26	3.5
7	22.8	17	9.0	27	0
8	30.0	18	3.2	28	0
9	0	19	9.5	29	2.4
10	5.8	20	2.2	30	10.6

Table 3. Characteristic constants of thermal units

No.	Node	$f_m = a_m + b_m \cdot g_m + C_m \cdot g_m^2$ 1000 yen			d_m	$\underline{g_m}$	$\overline{g_m}$	fs_m
		a_m	b_m	$c_m \times 1000$	kg/1000 yen	MW	MW	1000 yen
1	5	21.460	0.4170	26.10	—	15	50	3.8
2	13	6.706	1.2510	10.40	—	12	40	2.6
3	11	3.651	1.2510	10.40	0.722	10	30	2.2
4	8	2.254	1.3553	3.48	0.774	10	35	2.4
5	2	12.828	0.7298	7.30	0.750	20	80	3.2
6	1	18.640	0.8340	1.56	0.669	50	200	—

Note 1 : No. 1, 2 units are not constrained for emission because these are constructed in distant area.
Note 2 : No. 6 unit is always in operation.

Firstly, we assumed the capacity and environmental constraint to be infinity so that economic operation could be estimated. We then considered a simple case in which only No. 10 line has a capacity constraint. The result of the above economic operation was that the expected cost of system operation was 366,700 yen, the expected emission was 256.0 kg and the expected line flow of No. 10 was 0.638 p. u.

Secondly, we regarded the only environmental constraint as infinity. The following results were obtained by $\Delta G = 30$ MW and $\alpha = 0.5$. The expected costs are shown as $Y = \infty$ curve in Fig. 2. From this curve, we can certify the increase of operating cost depending on line capacity.

Thirdly, we constrained the emission at 90%, 80% and 70% of the economic emission, which is 256.0 kg. When line capacity was regarded as infinity, then the results are the points which are shown at the right edges of each curve in Fig. 2. Constraining the line flow at each value, we could obtain each curve of Fig. 2. When $Y = \infty$ and $I = 0.538$ p. u. then the expected cost was 377,090 yen,

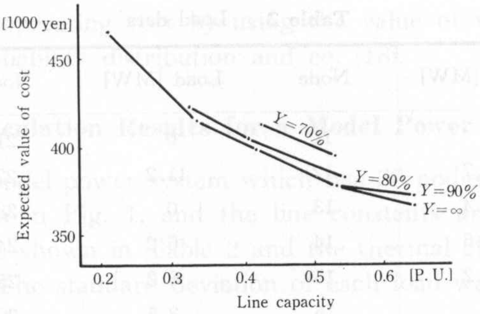

Fig. 2. Operating cost for emission and capacity constrains

and when $Y=230.4$ kg (which is 90% of 256 kg) and $I=0.541$ p. u., the expected cost was 377,060 yen. These two points were so close that the latter point coincides with another in Fig. 2. When $Y=\infty$ and $I=0.438$ p. u., then the expected value of emission was 228.2 kg. Because this emission could satisfy the constraint of $Y=90\%$, then the other parts of these curves became the same. From the curves of Fig. 2, we can certify that the possible zone of line flow shows a reducing trend depending on the emission constraint.

6. Conclusion

We reported a system scheduling method which could satisfy the line capacities and the emission constraint economically. We show that the objective function has many Lagrange's multipliers when it is made simply. In our objective function, we ignore the capacities to reduce the numbers of the multipliers. We also reported the method which modifies the result to satisfy the line capacities economically.

We also reported the method of probabilistic calculation. When we consider the constraints probabilistically, we do not calculate each variation in detail but calculate the cost increase by its gradient. In this paper, we described the method of the gradient calculation.

Finally, we used our method for a model power system and reported the results. Through the model calculation, we can verify that our method can schedule the operation which can satisfy the constraints, and also it is appropriate. From the calculation results, we can show how the operating cost depends on the line capacity and the emission constraint.

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