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Probabilistic Operation of Electric Power Systems Considering Environmental Constraint (Part 1)*

—Fundamental Consideration for Instantaneous Operation—

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Abstract

We report an optimum operation method in which we regard the system load as probabilistic variable and consider both constraints of emission and probabilistic value which exceed the emission constraint. We show that when the outputs of thermal units are constrained to upper and lower limits, even if the system load is represented by the normal distribution, the stochastic character of the total thermal output differs from it and total thermal cost and emission do not become simple curves. The amount of the calculation of fault state is reduced by simplification in our method.

We apply our method to a model power system and the results are concluded as follows. Depending on whether we estimate probabilistically or determinately, different optimum groups of thermal units are obtained and the amount of emission from these groups also differ to some extent. For thermal unit' faults, our practical method can estimate appropriately the optimum operation for almost all values of emission constraints.

Nomenclature

Capital letters	:	The values for total system.
Small letters	:	The values for each unit.
a_m, b_m, c_m, d_m	:	Characteristic constants of No. m thermal unit.
$E(P)$	:	Expected value of P.
f_m	:	Fuel cost of No. m thermal unit.
F	:	System operating cost $(=\Sigma f_m)$.
$F(P)_{\mu=0}$	:	System operating cost when load equals P and economic operation is done by eq. (3).
$F(P)_{\mu\neq 0}$	:	System operating cost when load equals P and the emission constraint is considered by eq. (10).
g_m	:	Output power of No. m thermal unit.
$\bar{g}_m, \underline{g}_m$	:	Upper and lower limits of g_m.
$\bar{G}, \underline{G}$	:	These mean $\Sigma\bar{g}_m$ and $\Sigma\underline{g}_m$ respectively.
P	:	System load.

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P_{clean} : System load which amount of the emission of least emission operation is just equal $\bar{Y}$.

P_{eco} : System load which amount of the emission of economic operation is just equal $\bar{Y}$.

P_{β} : System load which the probability of greater load than it is just equal β.

Prob. $(P \leqq G)$: Probability of state of $P \leqq G$.

Prob. (unbalance) : Probability of demand supply unbalance.

Prob. (unbalance$|U_j$ state) : Probaility of demand supply unbalance when U_j is in operating state.

Prob. $(\bar{Y} < Y | U_j$ state) : Probability of $\bar{Y} < Y$ when U_j is in operating state.

Prob. (S_{jk} fault$|U_j$ schedule) : Probability that S_{jk} is faulty when U_j is scheduled.

Prob. (unbalance$|U_j$ schedule) : Probability of demand supply unbalance when U_j is scheduled.

Prob. $(\bar{Y} < Y | U_j$ schedule) : Probability of $\bar{Y} < Y$ when U_j is scheduled.

q_m : Faulty rate of No. m thermal unit.

S_{jk} : No. k group of thermal units which are in operation under U_j.

U_j : No. j group of thermal units.

y_m : NO_x emission from No. m thermal unit.

Y : Total emission for every unit.

$\bar{Y}$: Upper limit of Y.

$Y(P)_{\mu=0}$: Total emission for economic operation.

α : Constraint value for Prob. (unbalance).

β : Upper limit of Prob. $(Y > \bar{Y})$.

λ, μ : Lagrange multipliers.

Π : Symbol of products.

σ_P : Standard deviation of P.

1. Introduction

We may make the schedules for an electric power system operation under many practical conditions such as system loads, operated thermal units, environmental constraints, etc. But some conditions may vary uncertainly. For example, system loads are dependent on the weather, and may be included the forecast errors. The faults of thermal units also occur unpredictably.

In this paper, we report an electric power system scheduling method which can consider these uncertain elements. We propose an optimum operation which has the minimum expected value of operating cost under the following five conditions.

Condition 1 : System load varies probabilistically. Although our calculating method can be used for any distribution, we consider the normal distribution as a concrete example.

Condition 2 : The probability of state of demand supply unbalance is less than a constraint value.

Condition 3 : The probability which NO_x exceeds the constraint value is less than a constant. We regard the NO_x emission constraint for electric power system as a typical environmental constraint.

Condition 4 : Each generator will fault probabilistically.

Condition 5 : Through we make the observations of system load, we can always control the outputs of thermal units.

We regard the lower and upper limits of thermal output as a deterministic value. Then, although a system load depends on the normal distribution, a total thermal output does not depend on the normal distribution, and the probabilistic distribution of operating cost and NO_x emission do not become simple curves. In this paper, the expected value of the total system's operating cost is minimized considering lower and upper limits of thermal outputs.

We introduce a simple and brief calculation method for the faulty state of the thermal units. Finally, we apply our method to a model power system and discuss the appropriateness of it.

2. Decision Method Which Estimates the Optimum Group of Thermal Units without Considering the Faults of Thermal Units

We assume that a power system consists of M thermal units. We estimate the expected operating cost for 2^M groups of thermal units which is the number of every group of combinations of thermal units. The optimum group of thermal units has the minimum expected value of operating cost, and satisfies both constraints of demand supply unbalance probability and of exceeding probability for emission limit.

2.1. Operating Method not Considering NO_x Emission

Fig. 1 shows the probability density of a system load which depends on normal distribution with some expected value and standard deviation. To minimize the expected value of operating cost, load dispatch is always done economically at every load level. When f_m is estimated from eq. (1) and we regard the total f_m as the operating cost, then we can get economic load dispatch through the objective function of eq. (2) is minimized.

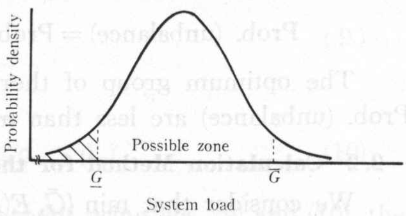

Fig. 1. Probability density of system load.

$$f_m = a_m + b_m \cdot g_m + c_m \cdot g_m^2 \qquad (\underline{g}_m \leqq g_m \leqq \bar{g}_m) \tag{1}$$

$$\phi = \sum_m f_m + \lambda \cdot (P - \sum_m g_m) \tag{2}$$

As $\partial\phi / \partial g_m = 0$,

$$g_m = \frac{\lambda - b_m}{2 \cdot c_m} \qquad (\underline{g}_m \leqq g_m \leqq \bar{g}_m) \tag{3}$$

If we neglect the upper and lower limits of g_m ($\underline{g}_m = -\infty, \bar{g}_m = \infty$), the probability density of operating cost for Fig. 1 is shown in Fig. 2 (we neglect negative outputs). In practice, the upper and lower limits are finite values and the zone of normal distribution is considered from $-\infty$ to ∞. Using $\underline{G}$ and $\bar{G}$, we consider the zone in which it is possible to operate with upper and lower output limits, as in Fig. 1.

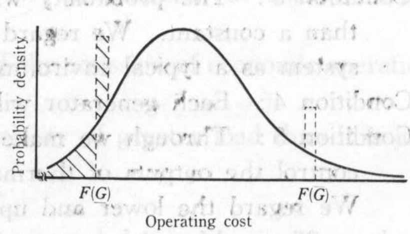

Fig. 2. Probability density of operating cost.

Because we assume that there is some supporting operation such as the operation of pumping-up power stations when the system load is less than $\underline{G}$, we can consider that outputs of thermal units are always $\underline{g}_m$ when the system load is less than $\underline{G}$. Therefore, if $P \leq \underline{G}$ then the systems' operating cost is always equal to $F(\underline{G})$ which is shown by the dotted area of $F(\underline{G})$ in Fig. 2, and its probability is equal to Prob. $(P \leq \underline{G})$. This means that the shaded areas of both sides of $F(\underline{G})$ are equal and also equal to the shaded area of Fig. 1. Since the significant value is not the probability density but the probability distribution, the dotted area is significant but the width or height of the dotted area are not. By similar consideration, we get the $F(\bar{G})$ part of Fig. 2.

The probability density of operating cost from $\underline{G}$ to $\bar{G}$ also changes through considering the upper and lower limits of each unit. The reason is that the outputs of some units attain the upper limit $\bar{g}_m$ at a lower load than $\bar{G}$ because of eq. (3).

We define the probability of demand supply unbalance such as eq. (4).

$$\text{Prob. (unbalance)} = \text{Prob. } (P > \bar{G}) + \text{Prob. } (P < \underline{G}) \tag{4}$$

The optimum group of thermal units is selected from many groups which Prob. (unbalance) are less than α.

2.2 Calculation Method for the Expected Value of Operating Cost

We consider that min $\{\bar{G}, E(P)+3\cdot\sigma_P\}$ and max $\{\underline{G}, E(P)-3\cdot\sigma_P\}$ are respectively the upper and lower load level limits for load dispatch. Dividing the load by σ_P from max $\{\underline{G}, E(P)-3\cdot\sigma_P\}$ to min $\{\bar{G}, E(P)+3\cdot\sigma_P\}$, economic load dispatch is done at every divided load level using eq. (3). Because we can generally estimate the expected operating cost from eq. (5), we can modify as in eq. (5)′ in this case.

$$E(F) = \int_{-\infty}^{\infty} F \cdot \text{Prob. } (P)\, dP \tag{5}$$

$$E(F) = \sum(\text{class mark of operating cost}) \cdot (\text{probability distribution}) \tag{5$'$}$$

2.3 Operating Method Considering NO_x Emission Constraint

NO_x emission are estimated as follow[1].

$$y_m = d_m \cdot f_m \tag{6}$$

The probability density of Y is became as Fig. 3 which is similar Fig. 2. If Prob. $(Y>\bar{Y})\leqq\beta$ at economic operation, then the optimum operation is the economic operation at every load level.

From here, we should consider the case of Prob. $(Y>\bar{Y})>\beta$ at economic operation. We should recognize Prob. $(Y>\bar{Y})=$ Prob. $(P>P_{\text{eco}})$ when economic operation is done.

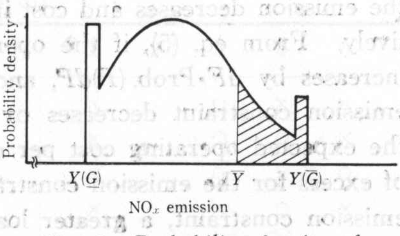

Fig. 3. Probability density of NO_x emission.

If we consider the least emission operation, the objective function is eq. (7) and its output is eq. (8).

$$\phi' = \sum y_m + \lambda\cdot(P - \sum g_m) \qquad (7)$$

$$g_m = \frac{\lambda - b_m\cdot d_m}{2\cdot c_m\cdot d_m} \qquad (\underline{g}_m \leqq g_m \leqq \bar{g}_m) \qquad (8)$$

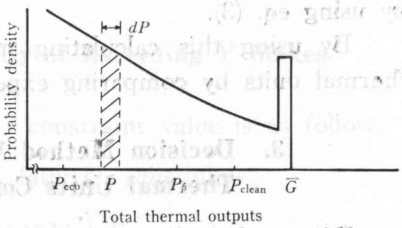

Fig. 4. Partial probability density of total thermal outputs.

We can recognize $P_{\text{eco}}\leqq P_{\text{clean}}$.

Since we must clear $\bar{Y}$ from P_{eco} to P_β at least, if $P_\beta>P_{\text{clean}}$, then we can not clear the constraint value β. From now, we should consider the case of $P_\beta\leqq P_{\text{clean}}$. Where, we can recognize $P_{\text{eco}}<P_\beta$ because of Prob. $(Y>\bar{Y})>\beta$. Fig. 4 shows the stretched probability density for Fig. 1 form P_{eco} to $\bar{G}$, whose transverse axis is replaced by total thermal outputs.

For optimum operation considering the emission constraint, we may minimize the following objective function.

$$\phi'' = \sum_m f_m + \lambda\cdot(P - \sum_m g_m) + \mu\cdot(\bar{Y} - \sum_m y_m) \qquad (9)$$

As $\partial\phi''/\partial g_m = 0$,

$$g_m = \frac{\lambda}{2\cdot c_m\cdot(1-\mu\cdot d_m)} - \frac{b_m}{2\cdot c_m} \qquad (\underline{g}_m \leqq g_m \leqq \bar{g}_m) \qquad (10)$$

If we operate considering the emission constraint using eqs. (9) and (10), then

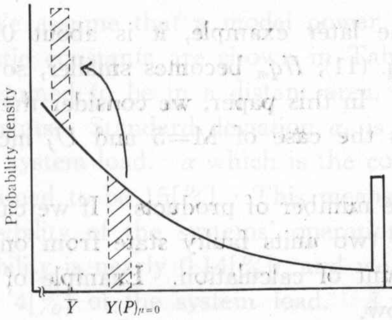

Fig. 5. Partial probability density of NO_x emission.

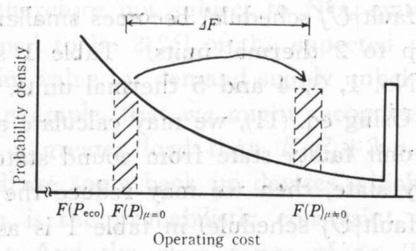

Fig. 6. Partial probability density of operating cost.

the emission decreases and cost increases likely are shown in Fig. 5 and 6 respectively. From eq. (5), if the operating cost increases by ΔF, the expected value increases by $\Delta F \cdot$Prob. $(P)dP$, and the probability distribution which exceeds the emission constraint decreases by Prob.$(P)dP$. Therefore, increasing amount of the expected operating cost per decreasing amount of the probability distribution of excess for the emission constraint becomes ΔF. Usually, when considering the emission constraint, a greater load results in a greater increase of cost. Therefore, we should operate considering the emission constraint using eq. (10) from P_{eco} to P_{β}, and for the rest of the load zone, economic operation should be done by using eq. (3).

By using this calculating method, we can decide the optimum group of thermal units by comparing expected values of operating cost for every group.

3. Decision Method Which Estimates Optimum Group of Thermal Units Considering Thermal Units' Faults

We number the groups of thermal units, and denote the j-th group as U_j. Since estimated Prob. (unbalance) or Prob. $(\bar{Y}<Y)$ in chapter 2 mean the probabilities by doing one group of operation such as U_j, we denote these as Prob. (unbalance$|U_j$ state) or Prob. $(\bar{Y}<Y|U_j$ state) respectively, henceforth.

3.1 The Calculating Method of Demand Supply Unbalance Considering Thermal Unit's Faults

When the operation of U_j is scheduled, the probability of the fault state of the whole S_{jk} is estimated from eq. (11).

$$\text{Prob.}(S_{jk}\ \text{fault}|U_j\ \text{schedule})=\{\prod_{m\in U_j\cap\overline{S_{jk}}}(1-q_m)\}\cdot\{\prod_{m'\in S_{jk}}q_{m'}\} \tag{11}$$

Where, $U_j\cap\overline{S_{jk}}$ means the group of sound thermal units, then the probability of demand supply unbalance scheduling U_j is as follow.

$$\text{Prob.}(\text{unbalance}|U_j\ \text{schedule})=\sum_k \text{Prob.}(S_{jk}\ \text{fault}|U_j\ \text{schedule})\cdot \text{Prob.}(\text{unbalance}|U_j\cap\overline{S_{jk}}\ \text{state}) \tag{12}$$

Since q_m is usually very small (in the later example, it is about 0.005~0.020), if some generators fault, then in eq. (11), Πq_m becomes smaller, so Prob. $(S_{jk}$ fault$|U_j$ schedule) becomes smaller too. In this paper, we consider the faults of up to 2 thermal units. Table 1 shows the case of M=5 and U_j including the No. 1, 3, 4 and 5 thermal units.

Using eq. (11), we may calculate a large number of products. If we calculate one unit faulty state from sound state and two units faulty state from one unit faulty state, then we may reduce the amount of calculation. Example of Prob. $(S_{j5}$ fault$|U_j$ schedule) in table 1 is as follow.

$$\text{Prob.}(S_{j5}\ \text{fault}|U_j\ \text{schedule})=\frac{q_5}{1-q_5}\text{Prob.}(S_{j2}\ \text{fault}|U_j\ \text{schedule}) \tag{13}$$

Table 1. Example of S_{jk}

m	1	2	3	4	5	m	1	2	3	4	5
U_j	○	×	○	○	○	S_{j6}	×		○	×	×
S_{j1}	×		×	×	×	S_{j7}	×		○	○	×
S_{j2}	○		×	×	×	S_{j8}	×		○	×	○
S_{j3}	○		○	×	×	S_{j9}	×		×	○	×
S_{j4}	○		×	○	×	S_{j10}	×		×	○	○
S_{j5}	○		×	×	○	S_{j11}	×		×	×	○

3.2 Operating Method in which the Probability of Exceeding $\bar{Y}$ Makes Less than β

The probability that the emission exceeds a constraint value is as follow.

$$\text{Prob.}(\bar{Y} < Y | U_j \text{ schedule}) = \sum_k \text{Prob.}(S_{jk} \text{ fault} | U_j \text{ schedule}) \cdot \text{Prob.}(\bar{Y} < Y | U_j \cap \overline{S_{jk}} \text{ state}) \quad (14)$$

If economic operation were always done, and Prob.$(\bar{Y}<Y|U_j$ schedule) was less than β, then the economic operation is the optimum. Henceforth, we consider the case of Prob.$(\bar{Y}<Y|U_j$ schedule$)>\beta$. If we consider the strict method which can achieve β, then it becomes as follows. We calculate $\Delta F \cdot$Prob.$(S_{jk}$ fault$|U_j$ schedule) and the $\bar{Y}$ considering operation is done from the load level and S_{jk} which have a minimum value of $\Delta F \cdot$Prob.$(S_{jk}$ fault$|U_j$ schedule), as described in the chapter 2.3.

For a system operator, however, he must calculate μ and the load level which should be operated considering $\bar{Y}$ at every S_{jk}, and depending on the fault and U_j, he must control the operation of the system. We may recognize that this strict method is not practical. Since we estimate β achieving operation in chapter 2.3, we can operate practically by this method independing on S_{jk}, although it is not always the true optimum.

4. Calculated Results for a Model Power System

We assume that a model power system consists of 6 thermal units. Characteristic constants are shown in Table 2. The No. 1 and No. 2 thermal units are assumed to be in a distant area, and therefore not subject to NO_x emission constraints. Standard deviation σ_P is assumed to be 2[%] of the expected value of the system load. α which is the constraint value for demand supply unbalance, is assumed to be 15[%]. This means, for example, that we might recognize the impossibility of the systems' operation for a greater load than $E(P)+3 \cdot \sigma_P$ (its probability is nearly 0.14[%]), and we could use some back up devices which are nearly 4[%] of the system load. β which is the probabilistic constraint value for exceeding $\bar{Y}$ is assumed to be 20[%]. And the class range of eq. (5)' is assumed 2[MW] what is used to consider β from P_{eco} to P_β.

Table 2. Characteristic constants of thermal units

Unit No.	$f=a+b\cdot g+c\cdot g^2$ [1,000 yen]			d	$\underline{g}$	$\bar{g}$	q
	a	b	$c\times 1{,}000$	[kg/1,000 yen]	[MW]	[MW]	[%]
1	21.460	0.4170	26.10	—	15	50	2.0
2	6.706	1.2510	10.40	—	12	40	1.0
3	3.651	1.2510	10.40	0.722	10	30	2.5
4	2.254	1.3553	3.48	0.774	10	35	2.0
5	12.828	0.7298	7.30	0.750	20	80	1.5
6	18.640	0.8340	1.56	0.669	50	200	0.5

4.1 Investigation of Economic Operation

Assuming that $\bar{Y}$ has a large enough value, we estimate economic operation for each load level. The results of operating cost and emissions are shown in Fig. 7.

The reason why cost increases simply but emission increases not simply for increasing load is as follows. The No. 1 and No. 2 thermal units are not economic (their unit cost is 1.1~1.7 times as much as the other units), the optimum group of thermal units consists of the No. 3~6 thermal unirs at 320 [MW]. The No. 2 unit comes into operation at 340 [MW], and the No. 1 unit begins at 380 [MW]. At the load level these new units are added, the outputs of the other units decrease, and the total emissions for the No. 3~6 units are decreasing.

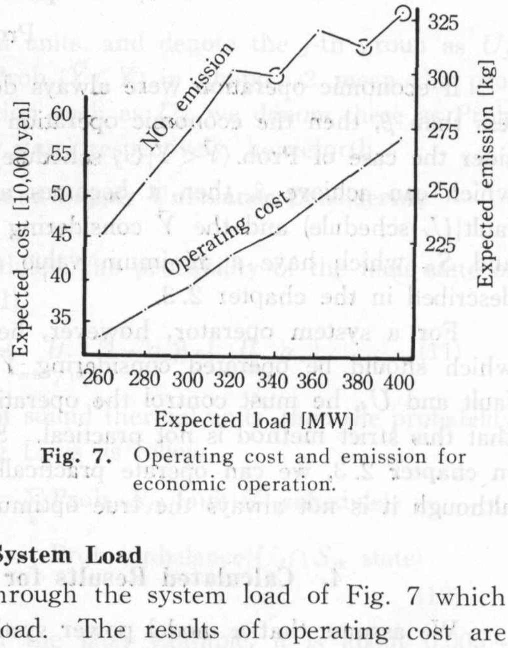

Fig. 7. Operating cost and emission for economic operation.

4.2 Comparison with Deterministic System Load

We estimate economic operation through the system load of Fig. 7 which is considered to be the deterministic load. The results of operating cost are 0.01~0.08 [¥10,000] less than the expected values of Fig. 7. The emission are 0.02~0.20 [kg] less than Fig. 7 except the ○ symbol. At 340 and 380 [MW], the emissions are 26.22 and 20.30 [kg] greater respectively. The reason for this is that the optimum group of thermal units differs whether we consider it probabilistically or determinately (for example, the No. 1 and 2 units are out of operation at 340 [MW] by deterministic estimation).

At 400 [MW], the emission is 1.84 [kg] greater than the expected value. This is because the outputs of the No. 1 and 2 units increase mainly with increasing load at this load level, and the emissions of the No. 3~6 increase only a little. This fact shows that the probabilistic density curve varies depending on the lower and upper limits of thermal units.

4.3 Investigation of Emission Constraint References

Setting 320 [MW] as the expected value of the load, we estimate the optimum operation for each emission constraint. Fig. 8 shows the expected cost (1) considering the faults of units (dotted line), and (2) neglecting them (actual line).

The reason why the operating cost curve is not simple in the case of neglecting the faults is as follows. For $\bar{Y}=320$ [kg], the optimum group of thermal units consists of the No. 3∼6 units as in the case of E(P)=320 [MW] of Fig. 7. At $\bar{Y}=300$ [kg], in order to achieve $\bar{Y}$, the No. 2 unit comes into operation and the No. 3 unit goes out of service. But this group can already achieve $\bar{Y}=280$ [kg] by economic operation (the expected value of emission is 275.4 [kg]). Therefore operations for $\bar{Y}=280$ and 300 [kg] are identical.

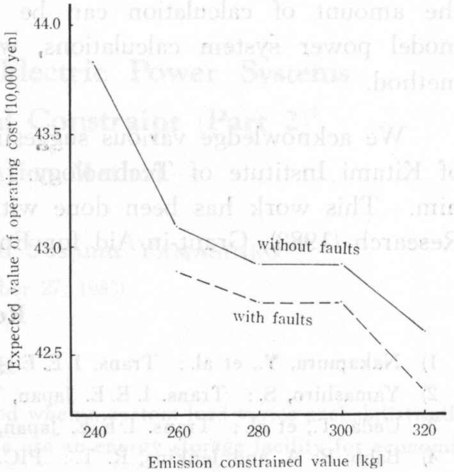

Fig. 8. Operating cost for emission constraint.

The reason that we could not get the optimum operation when $\bar{Y}$ is 240 [kg] and faults were considered is because we estimated using the practical method described in chapter 3.2, rather than using the strict method. For $\bar{Y}=240$ [kg], the best operation has 21.9 [%] for Prob. $(\bar{Y}<Y|U_j$ schedule). This does not mean that we can not clear $\bar{Y}=240$ [kg], but that we have a chance of achieving $\bar{Y}$ using the strict method.

For each emission constraint, the operating cost which does not consider faults is always greater than that considering faults. But this does not mean that faults are desirable. When thermal units fault, Prob. $(G>P)$ decreases and Prob. $(\bar{G}<P)$ increases. Therefore the expected operating cost generally decreases. We should remember that the system operating cost is a total thermal cost and the cost of a back up unit is neglected. We recognize that the estimate of optimum operation should be made considering the cost of back up operation.

For economic operation of Fig. 7, the expected cost without faults is 0.06∼0.30 [¥10,000] greater than for one with faults.

5. Conclusion

We reported an optimum operation calculating method which statisfies economically the constraints of the probability of demand supply unbalance and the probability of emission exceeding a constraint value considering probabilistic load level. We also considered upper and lower limits of thermal units' outputs, and we estimated optimum operation considering thermal unit faults which were treated probabilistically. The calculating method of the faulty state showed that the amount of calculation can be reduced by our proposed method. From model power system calculations, we could show the appropriateness of our method.

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