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On Flexural Wave Velocity Propagating in Fiber Reinforced Rectangular Bar

Toshiyuki OHSHIMA*, Shuichi MIKAMI*, Masaru INOUE**
and Sumio G. NOMAHI***

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Abstract

The phase and group velocities of flexural wave propagating in rectangular bar which have been reinforced by fibers oriented in rectangular array over the cross section, is solved by means of "Finite Integration Transforms".

The dispersion charâcteristics of wave velocity with wave number which are effected by the orientation of the fibers as well as by the shape of the cross section, are numerically illustrated in the figures.

I. Introduction

Fraser[1], Nigro[2], Tanaka et al.[3] investigated stress wave propagation in homogeneous isotropic rectangular bar and reported on their dispersion properties in relation to cross sectional size.

Achenbach et al.[4] and Hegemier et al.[5] studied stress wave propagation in infinite fiber reinforced composites whose fiber interval may cause the dispersion of the wave.

When a stress wave propagates in a rectangular bar reinforced by longitudinal fibers, the wave should disperse on account of both effects.

The aim of this paper is to find how these effects appear in a harmonic flexural wave travelling in a fiber reinforced rectangular bar.

The customary approach consists in replacing the composite by a homogeneous medium whose material constants are determined in terms of the material constants and the geometry of each constituent of the composite[6].

The authors herein attempt to handle the problem with the aid of dynamic finite prism method and finite integration transforms[7] to keep the discreteness of the matter.

It is, however, assumed that fiber reinforcement are equi-distantly spaced and carry concentrated mass and stiffness, and that the fiber react as "Timoshenko beam" in the transverse direction[8].

* Member of Japan Society of Civil Engineering, Department of Development Engineering, Kitami Institute of Technology, Kitami-city.

** Member of Japan Society of Civil Engineering, Hokkaido Branch of Daiho Construction Co., Ltd., Sapporo.

*** Member of Japan Society of Civil Engineering, Department of Civil Engineering, Hokkaido University, Sapporo.

The dispersion characteristics of wave velocity with wave number are numerically obtained, making use of iteration method for eigenvalue matrix of phase velocity.

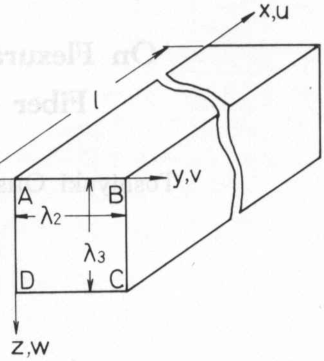

Fig. 1. Finite prism element.

II. Theory

As discribed above, the fibers are in rectangular array over the cross section, each array represented by a prism element[7].

Let the prism element hold linear displacement distribution over the cross section.

Multiplying the dynamic equations of equilibrium of forces with the weight functions and integrating them over all volume with consideration of the boundary conditions of the crosswise surfaces, we have as follows ;

$$\left.\begin{aligned}
&\int_V L_1\left(\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}-\rho\frac{\partial^2 u}{\partial t^2}\right)dV=0\,,\\
&\int_V L_2\left(\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}-\rho\frac{\partial^2 v}{\partial t^2}\right)dV=0\,,\\
&\int_V L_3\left(\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\sigma_z}{\partial z}-\rho\frac{\partial^2 w}{\partial t^2}\right)dV=0\,.
\end{aligned}\right\}\qquad(1)$$

where $L_1=\cos\frac{m\pi}{l}\,x\cdot f_1(y)\cdot g_1(z)$,

$L_2=\sin\frac{m\pi}{l}\,x\cdot f_2(y)\cdot g_2(z)$,

$L_3=\sin\frac{m\pi}{l}\,x\cdot f_3(y)\cdot g_3(z)$,

V = total volume,

l = length of bar,

ρ = density of material

We obtain a matrix expression relating the nodal forces to the nodal displacement with respect to the prism element.

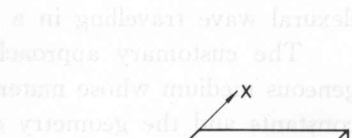

Fig. 2. Fiber-reinforced composite bar.

Again taking the equilibrium of forces at the node, involving the three components of mass force, the axial stiffness force of the fibers and its transversal Timoshenko beam stiffness force, we finally get the expression, as follows ;

$$\left([\mathbf{K}]-\frac{\partial^2}{\partial t^2}[\mathbf{M}]\right)\{\mathbf{u}\}=0 \qquad (2)$$

where [K] is a symmetrical matrix of size 5×5

and the elements of the matrix are as follows;

$$k_{11}=\frac{2\mu+\lambda}{36}\bar{\Delta}_y^2\bar{\Delta}_z^2D_x^2+\frac{\mu}{6\lambda_2^2}\Delta_y^2\bar{\Delta}_z^2+\frac{\mu}{6\lambda_3^2}\Delta_z^2\bar{\Delta}_y^2+E_f\eta D_x^2,$$

$$k_{12}=\frac{\mu+\lambda}{12\lambda_2}\Delta_y\bar{\Delta}_z^2D_x,\qquad k_{13}=\frac{\mu+\lambda}{12\lambda_3}\Delta_z\bar{\Delta}_y^2D_x,$$

$$k_{22}=\frac{\mu}{36}\bar{\Delta}_y^2\bar{\Delta}_z^2D_x^2+\frac{2\mu+\lambda}{6\lambda_2^2}\Delta_y^2\bar{\Delta}_z^2+\frac{\mu}{6\lambda_3^2}\Delta_z^2\bar{\Delta}_y^2+\mu_fK_f\eta D_x^2,$$

$$k_{23}=\frac{\mu+\lambda}{4\lambda_2\lambda_3}\Delta_y\Delta_z,\qquad k_{24}=k_{25}=\mu_fK_f\eta D_x,$$

$$k_{33}=\frac{\mu}{36}\bar{\Delta}_y^2\Delta_z^2D_x^2+\frac{2\mu+\lambda}{6\lambda_3^2}\Delta_z^2\Delta_y^2+\frac{\mu}{6\lambda_2^2}\Delta_y^2\bar{\Delta}_z^2+\mu_fK_f\eta D_x^2,$$

$$k_{44}=k_{55}=\mu_fK_f-D_x^2E_fr_f^2,$$

and other elements are zero.

$$\{\mathbf{u}\}^T=\{u,v,w,-\theta^y,-\theta^z\},\qquad D=d/dx,$$

$$\bar{\Delta}^2=\Delta^2+6,\qquad \Delta^2f(y)=f(y+1)-2f(y)+f(y-1),$$

$$\Delta f(y)=f(y+1)-f(y-1),\qquad \eta=A_f/A_p,\qquad A_p=\lambda_2\lambda_3,$$

μ, λ denote Lamè's elastic constants and ρ density of the matrix matter. [M] is a diagonal matrix in which

$$m_{11}=m_{22}=m_{33}=\frac{\rho}{36}\bar{\Delta}_y^2\bar{\Delta}_z^2+\rho_f\eta,\qquad m_{44}=m_{55}=-\rho_fr_s^2$$

and E_f, u_f, r_f, A_f, κ_f and ρ_f are elastic modulus, elastic shear modulus, radius of gyration, sectional area, shear coefficient and density of fiber, respectively.

In the case of a plane wave with respect to the y and z coordinates, Eq. (2) turns out to the following forms

$$[C_{11}+E_f\eta]\,Du=\rho^*u'' \qquad (3)$$

$$[C_{44}+\mu_fK_f\eta]\,D_x^2v-\mu_fK_f\eta D_x\theta^y=\rho^*v'' \qquad (4)$$

$$[C_{44}+\mu_fK_f\eta]\,D_x^2w-\mu_fK_f\eta D_x\theta^z=\rho^*w'' \qquad (5)$$

where $C_{11}=2\mu+\lambda$, $C_{44}=\mu+\eta\mu_f$, $\rho^*=\rho+\eta\rho_f$
and $'=d/dt$,
which coincide with the equations of Achenbach et al.[4)]

The harmonic wave propagating in the x direction may be written by the following expressions

$$\left.\begin{aligned} u&=U\cos\frac{2\pi}{l}(x-ct), & v&=V\sin\frac{2\pi}{l}(x-ct),\\ w&=W\sin\frac{2\pi}{l}(x-ct), & \theta^y&=\Theta^y\cos\frac{2\pi}{l}(x-ct),\\ \theta^z&=\Theta^z\cos\frac{2\pi}{l}(x-ct) & & \end{aligned}\right\} \qquad (6)$$

where c, l, θ^y and θ^z are phase velocity, wave length and slopes produced by the rotation of the fiber with respect to the z and y axes, respectively.

Substitution of Eq. (6) in Eq. (2) yields finite difference equations, which are easily treated by means of "the finite inegration transforms".[7]

In the process of analysis considering the symmetry and asymmetry of the flexural modes and applying the following boundary conditions;

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0\ ; \quad \text{at the top and bottom surfaces, and}$$

$$\sigma_y = \tau_{yz} = \tau_{xy} = 0\ ; \quad \text{at the right and left side surfaces,}$$

we finally find the eigen value matrix from which we can calculate the phase velocity of the flexural wave by iteration method.

III. Numerical results and the remarks

(1) Discussion of Accuracy

In order to make sure the accuracy with the number of prism elements, the wave velocity rate c/c_s in the homogeneous isotropic square bar is taken into account; where c_s is the shear wave velocity of the matter.

The results calculated by Fraser[1], Nigro[2] and Tanaka et al.[3], by the Timoshenko beam equation and by the presenting method are tabulated in Table 1 by changing the value of γb^*, where $\gamma = 2\pi/l$ and $b^* = b/2$.

Table 1. Comparison of accuracy

γb^*	Fraser	Nigro	Timoshenko	Tanaka Thind	author 48 div	24 div	8 div
0.1	0.0925	0.0925	0.1376		0.0921	0.0921	0.0900
0.3	0.2643	0.2642	0.2236		0.2640	0.2639	0.2640
0.5	0.4058	0.4058	0.4048		0.4056	0.4055	0.4056
1.0	0.6316	0.6316	0.6279	0.6316	0.6313	0.6313	0.6325
2.0	0.8081	0.8081	0.7400	0.8083	0.8080	0.8082	0.8122
3.0	0.8664	0.8666	0.8008	0.8668	0.8664	0.8671	0.8747
4.0	0.8893	0.8902	0.8365	0.8901	0.8897	0.8909	0.9032
5.0	0.8990	0.9008	0.8589	0.9000	0.8994	0.9016	0.9194
6.0	0.9020	0.9063	0.8738	0.9043	0.9035	0.9067	0.9303
7.0	0.9030	0.9098	0.8840		0.9053	0.9098	0.9387
8.0	0.9030	0.9128	0.8914		0.9063	0.9121	0.9456
10.0		0.9181	0.9042		0.9077	0.9164	0.9567
14.0		0.9285	0.9117		0.9108	0.9253	0.9715

The numerical result obtained by the presenting method tends to Nigro's result as γb^* increases.

Three different number of prism elements (8, 24, 48) give almost the same result for γb^* less than 5, but they yield values a little different from one another for γb^* more than 5.

For instance, the result using eight division gives 4.7% more than Fraser's and 3.6% more than Nigro's for $\gamma b^*=8$, and it also tends to the surface wave velocity of the matrix in the case of infinite γb^*.

(2) Effect of the density ratio between fiber and matrix ρ_f/ρ

Fig. 3 and 4 show the phase velocity curves of flexural wave in a square

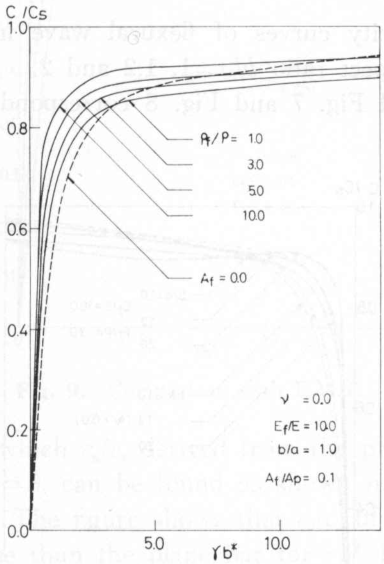

Fig. 3. Effect of density ratio $\rho_f/\rho\,(\nu=0)$.

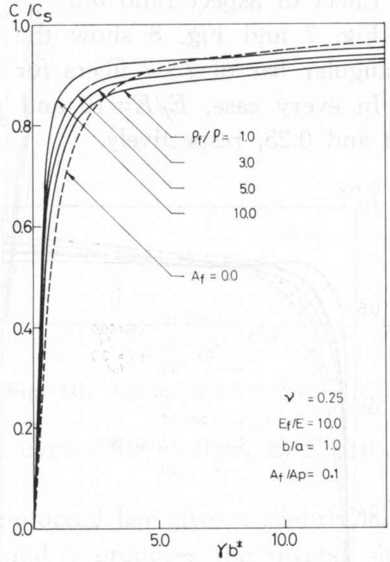

Fig. 4. Effect of density ratio $\rho_f/\rho\,(\nu=0.25)$.

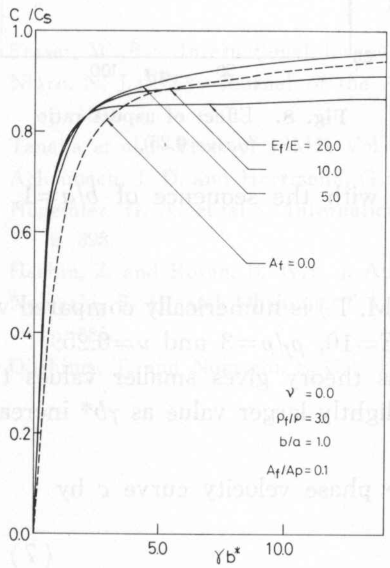

Fig. 5. Effect of elastic modulus ratio $E_f/E\,(\nu=0)$.

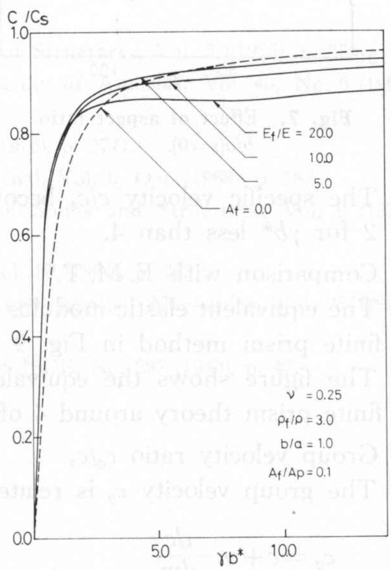

Fig. 6. Effect of elastic modulus ratio $E_f/E\,(\nu=0.25)$.

bar of 7×7 fibers where $\rho_f/\rho=1$, 3, 5 and 10 including the dotted line for $A_f=0$. The elastic modulus ratio between fiber and matrix E_f/E is 10 and Poisson's ratio of the matrices are 0 and 0.25 for Fig. 3 and 4, respectively.

(3) Effect of the elastic modulus ratio between fiber and matrix E_f/E

In taking E_f/E as 20, 10 and 5 in a square bar of 7×7 fibers, the phase velocity curve are shown in Fig. 5 ($\nu=0$) and Fig. 6 ($\nu=0.25$).

(4) Effect of aspect ratio b/a

Fig. 7 and Fig. 8 show the phase velocity curves of flexural wave in a rectangular bar of 7×7 fibers for different aspect ratio $b/a=1$, 1.2 and 2.

In every case, $E_f/E=10$ and $\rho_f/\rho=3$, and Fig. 7 and Fig. 8 correspond to $\nu=0$ and 0.25, respectively.

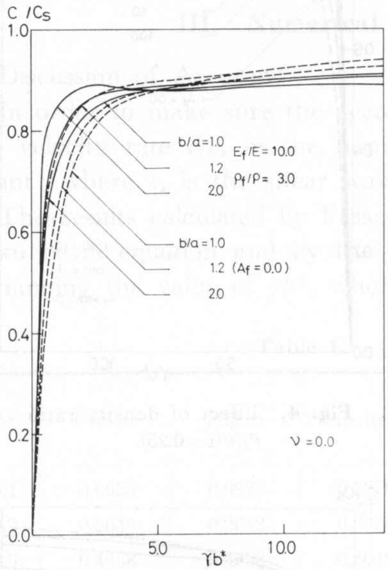

Fig. 7. Effect of aspect ratio $b/a(\nu=0)$.

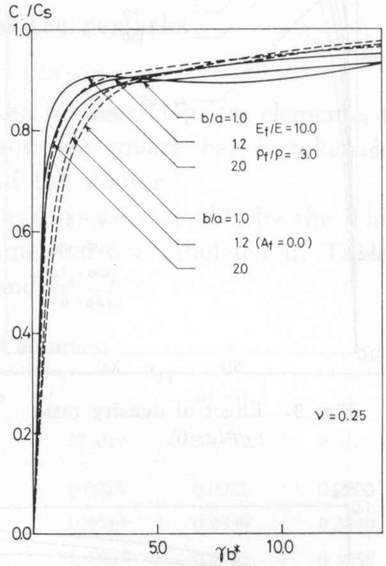

Fig. 8. Effect of aspect ratio $b/a(\nu=0.25)$.

The specific velocity c/c_s becomes larger with the sequence of $b/a=1$, 1.2 and 2 for γb^* less than 4.

(5) Comparison with E. M. T.[6)]

The equivalent elastic modulus thoery (E. M. T.) is numerically compared with the finite prism method in Fig. 9 where $E_f/E=10$, $\rho_f/\rho=3$ and $\nu=0.25$.

The figure shows the equivalent modulus theory gives smaller values than the finite prism theory around 4 of γb^* and slightly larger value as γb^* increases.

(6) Group velocity ratio c_g/c_s

The group velocity c_g is related with the phase velocity curve c by

$$c_g=c+m\frac{dc}{dm} \tag{7}$$

where m is wave number,

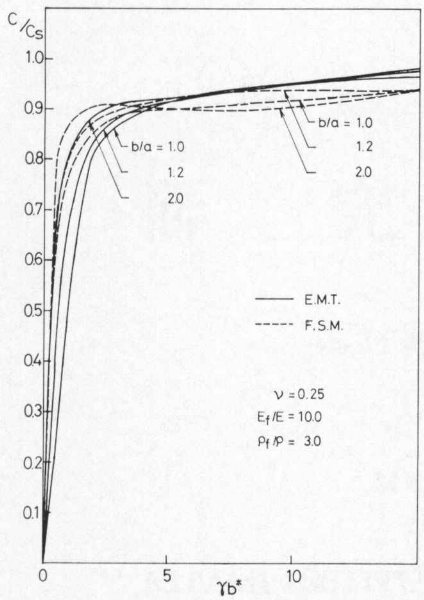

Fig. 9. Comparison with E.M.T.

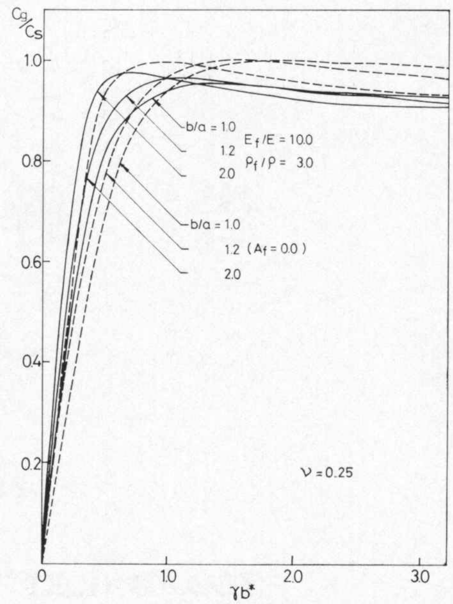

Fig. 10. Group velocity ratio c_g/c_s.

by which c_g/c_s derived from the phase velocity curves for $\nu=0.25$, $E_f/E=10$ and $\rho_f/\rho=3$, can be found as shown in Fig. 10.

The figure shows that c_g/c_s of the fiber reinforced bar gives a slightly larger value than the plane bar for γb^* less than 1 and it produces the reverse situation for γb^* more than 1.

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