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Determination of the Loads Restoration Sequence Taking into Account the Cost of Interruption of Electrical Service

(Consideration of the Deviation of the Cost of Interruptions of Electrical Service)*

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Abstract

In the restoration state of a power system, the time taken for a dropped generator to parallel in, and the cost of interruption of electrical service vary according to the operating conditions just before the fault concerned, the amount of the load expected to be consumed during the fault and accidental factors.

In this paper, the authors propose a method of determining the loads restoration sequence which takes into account the cost of interruption of electrical service, in which the above factors are considered.

The authors develop a new calculating method which takes into account the distributions of these values. Comparing this method to the one previously presented which used fixed expected values[1)], the authors confirm that the former method is fully capable of determining the loads restoration sequence.

1. Introduction

The cost of interruption of electrical service (we use "cost of interruption" hereafter) depends upon not only the kind of load but also the magnitude and the duration of the interruption, etc.[1,6)]

Accordingly, it does not minimize the cost of interruption to restore the loads by using fixed priority levels of loads defined long before the fault occurs at the individual electric utility. Therefore, the authors have proposed a method of determining the loads restoration sequence which takes into account the cost of interruption, in which fixed values of the cost of interruption and expected time taken for a generator which has dropped out to parallel in, are used[1,2)]. But in the restoration state of a power system, the time taken for a generator which has dropped out to parallel in and the cost of interruptions vary according to operating conditions just before the fault occurs, the amount of load to be consumed during the fault and accidental factors. Therefore, considering the

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distribution of deviations of these values, there arises a storng suspicion that results which take into account the distributions of these values will differ from those using fixed expected values.

In this paper, the authors at first develop a new calculating method which takes into account the distributions of these values[3,5]. Secondly, the authors give examples using actual size model systems, and in conclusion, compare this method to the one using fixed expected values, and so, confirm that the method using fixed expected values is adequate to determine the loads restoration sequence.

2. Formulation considering the distribution of deviation of the cost of interruption and the time taken for a dropped generator to parallel in

2.1 Assumptions of the formulation

The following assumptions are made for the formulation.

(1) The magnitude of load is some integer times a fixed value Δ. (we can divide the magnitude of the load by Δ, and the result is referred to as a "divided load".)

(2) Loads are never reinterrupted.

(3) Minimizing the cost of interruption is the only objective, minimizing the generating cost or the loss of transmission is not considered.

(4) Magnitude of the loads are constant throughout the period under consideration.

(5) Paralleling the generator is the first priority if the generators are dropped as a result of the fault, and restoration of the interrupted loads is the next priority. It is assumed that the generator buses have been charged before this calculation starts.

(6) The total output of generators increases to meet the interrupted loads with in a finite period, and the operations for paralleling the interrupted loads finish before the time when the total output of generators increases to meet Δ.

(7) Abnormal voltage and over-loads never occur during restorative operations for paralleling the designated load.

It is assumed that output of a generator g increases as described by the following equations.

$$G_g(t_g)=d_g(t_g-t_{g1})+e_g \tag{1}$$

Where,

$$\left.\begin{array}{lll} \text{at zero output } (t_g<t_{g1}) & : & d_g=0,\ e_g=0 \\ \text{at increasing output } (t_{g1}\leqq t_g<t_{g2}) & : & d_g>0\ \ e_g\geqq 0 \\ \text{at constant output } (t_{g2}\leqq t_g) & : & d_g=0\ \ e_g=p_g(t_g) \end{array}\right\} \tag{2}$$

$G_g(t_g)$: output of generator g at t_g.

t_g : relative time that the generator g is tripped out is considered 0

$$t_g = u + t_{gers} \tag{3}$$

t_{gers} : elapsed time to start this calculation after generator g is dropped out.

u : relative time assuming that the time this evaluating calculation starts is 0.

t_{g1} : time required for generator g to start to pick up load after its dropping out.

t_{g2} : time required for output of generator g reach the rated outout after it starts to pick up load.

$$t_{g2} = \left\{p_{g0} - (e'_g - d_g \cdot t_g)\right\} / d_g \tag{4}$$

e_g : minimum output of generator g, or output before the fault if it did not drop.

p_g : rated output of generator g.

G_g at t_g is determined if d_g, e'_g and t_g are decided. e_g is decided from the status of generator g and d_g is the value predetermined by design. As t_{g1} varies according to the generator status before the fault and the plant operating situation after the fault and other factors, in this paper, t_{g1} is treated as a stochastic variable. Although distribution of t_{g1} is different for individual generators, it is estimated from the past operating pattern, and its probability density function is represented by $w'(t_{g1})$.

Although the cost of interruption for the ith divided load is approximated as a quadratic function[1], it is not always exactly as described by the quadratic function and varies according to the status of the load just before the fault or the expected power consumption during the outage and other factors. After that, the cost of interruption covers a wide distribution as shown in Fig. 1. The distribution of cost of interruption varies according to individual divided loads. The cost of interruption $F_i(t_i)$ is also treated as a stochastic variable and its probability density function is represented as $r'\{F_i(t_i)\}$. If the divided load contains some different kind of load, $r'\{F_i(t_i)\}$ is defined as the weighted summation of the individual probability density function weighted by the magnitude of the loads[1]. In this case, the socially important loads must be correctly evaluated through weight factors.

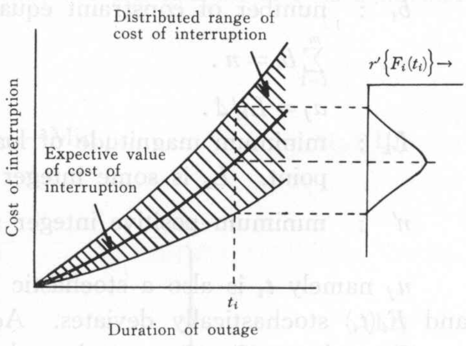


Fig. 1. The cost of interruptions of electrical service.

2.2 Cost minimizing problem

The cost of interruption is represented as F_{ij}, if the ith divided load is restored at the point of time that the total output of generators increases just sufficient to meet $j\Delta$. Considering the constraint of the generators' output increasing pattern, the problem of minimizing the total cost of interruption for all devided loads is formulated as a minimum cost flow problem, as follows.

Objective function :

$$F=\sum_{i=1}^{m}\sum_{j=1}^{n'}F_{ij}\cdot x_{ij}\longrightarrow \text{Min} \quad (5)$$

Constraints :

$$\left.\begin{aligned}&\sum_{i=1}^{m}x_{ij}=a_j \qquad (\text{For } j=1,2,\cdots,n)\\&\sum_{j=1}^{n'}x_{ij}=b_i \qquad (\text{for } i=1,2,\cdots,m)\\&x_{ij}\geqq 0 \qquad (\text{For } i=1,2,\cdots,m,\ j=1,2,\cdots,n')\end{aligned}\right\} \quad (6)$$

Where,

$$x_{ij}=\begin{cases}1: \text{at } i\text{th divided load is restored at } u_j\\0: \text{otherwise}\end{cases}$$

$$F_{ij}=F_i(t_i) \quad (7)$$

$$t_i=u_j+t_{iers} \quad (8)$$

u_j : the relative time that the total output of generators takes to increase to meet $j\Delta$, assuming the time when this calculation starts is 0.

t_{iers} : the time to start this calculation after the ith divided load is dropped.

n : number of divided loads. $m=n-n_d$.

n_d : number of abbreviated constraint equations formed by binding several equations described divided loads with the same cost of interruption.

b_i : number of constraint equations binding to ith equation, and

$\sum_{i=1}^{m}b_i=n$.

$a_j=L_k/\Delta$.

L_k : minimum magnitude of load which must be restored at the ith time point. L_k is some integer times Δ.

n' : minimum positive integer satisfing $\sum_{i=1}^{n'}a_j=n$

u_j namely t_i is also a stochastic variable, because t_{g1} is a stochastic variable and $F_i(t_i)$ stochastically deviates. Accordingly, F_{ij} is a stochastic variable by eq. (7), and eq. (5), (6) must be solved as a stochastic programming problem. Generally the stochastic programming problem is solved under the constraints as eq. (6)~(8) by introducing a preference function $H(f)$ as[8]

$$H\{f(F:x_{ij})\}\longrightarrow \text{Min or Max}$$

$f(F:x_{ij})$ is a distribution function of F in eq. (5) which is calculated at a determined x_{ij} assuming that the simultanious distribution function of F_{ij} is given.

Usually, a preference function is selected for the purpose of one of the following three optimizations[9].

(a) Minimizing the variance of the cost of interruption, with the constraint of achieving the predefined expected cost of interruption. (this is referred to as an "expected cost type problem".)

(b) Defining the expected utility function of the cost of interruption and minimizing it. (This is referred to as a "utility function type proble,".)

(c) Setting a probability so that the cost of interruption is under a certain level, and minimizing this level. (This is referred to as a "cost level type problem".)

2.3 Formulation of the problem

As the expected cost to be achieved itself is a value given by solving this problem, it is difficult to apply the expected cost type problem to such a case as this. So, the remaining two methods shall be considered.

(1) Cost level type problem

The cost level type problem is defined as minimizing F_e under the constraint of

$$p(F_e \geqq \sum_i \sum_j F_{ij} \cdot x_{ij}) \geqq \eta \qquad (9)$$

Where, $p(r)$ means the probability that r is true. This problem is solved by minimizing F_e illustrated in Fig. 2, for every combination of F_{ij}, and is known as $\eta\%$ bound. Namely the purpose of this problem is to minimize F_e given that the probability that F of eq. (5) is under F_e is $\eta\%$. Accordingly, this problem is formulated as a nonlinear programming problem as follows, assuming that the distribution of F is a normal distribution. (See section 3.1.)

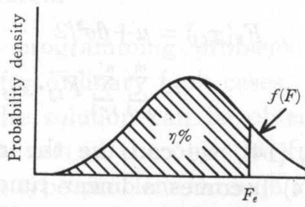


Fig. 2. $\eta\%$ bound.

Objective function :

$$F_e = \sum_{i=1}^{m} \sum_{j=1}^{n'} \overline{F_{ij}} x_{ij} + \alpha \Big(\sum_{i=1}^{m} \sum_{j=1}^{n'} \sigma_j \cdot x_{ij} \Big)^{1/2} \longrightarrow \text{Min} \qquad (10)$$

Constraints :

$$\left.\begin{array}{ll} \sum_{i=1}^{m} x_{ij} = a_j & (\text{for } j=1, 2, \cdots, n') \\ \sum_{j=1}^{n'} x_{ij} = b_i & (\text{for } i=1, 2, \cdots, m) \\ x_{ij} = 1 \text{ or } 0 & (\text{for } i=1, 2, \cdots, \text{m}, \ j=1, 2, \cdots, n') \end{array}\right\} \qquad (11)$$

Where, $\overline{F_{ij}}$ means the expected value of F_{ij}, σ_{ij} means the variance of F_{ij}.

The problem of minimizing eq. (10) under the constraints of eq. (11) must be solved fundamentally by means of a combinational enumeration method. So, as m or n' increases, large memory capacity and much computing time become necessary proportional to those increased values of m and n', even if we use the branch and bound method or the lexicographical enumeration method[14].

(2) Utility function type problem

If F and θ mean the total cost of interruption and the positive constant respectively the utility function is defined as follows.

$$H(F_e)=\varepsilon^{\theta F}-1 \tag{12}$$

As F becomes greater, the value of the utility function monotonically increases. Since the distribution of F can be considered as a normal distribution (See section 3. 2.), for the purpose of minimizing the expected value of eq. (12) under constraint of eq. (11), the objective function is as follows[9].

$$\begin{aligned} E\{H(F)\} &= \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty}(\varepsilon^{\theta F}-1)\cdot\varepsilon^{-\frac{(F-\mu)}{\sigma^2}}\cdot dF \\ &= \varepsilon^{\theta\cdot(\mu+\theta\sigma^2/2)}-1 \longrightarrow \text{Min} \end{aligned} \tag{13}$$

Where, μ means the expected value of F, and σ^2 means the variance of F.

After all, eq. (13) can be rewritten as eq. (14) in the appropriate range of θ.

$$\begin{aligned} F_k(x_{ij}) &= \mu+\theta\sigma^2/2 \\ &= \sum_{i=1}^{m}\sum_{j=1}^{n'}\overline{F_{ij}}\cdot x_{ij}+\frac{\theta}{2}\sum_{i=1}^{m}\sum_{j=1}^{n'}\sigma_{ij}^2\cdot x_{ij} \longrightarrow \text{Min} \end{aligned} \tag{14}$$

In eq. (14), we can use the relation of $x_{ij}=x_{ij}^2$, as x_{ij} takes only 0 or 1, and eq. (14) becomes a linear function of x_{ij}.

An appropriate value of θ must be selected, because the value of eq. (13) widely varies according to μ and σ^2. This can be done by relating θ to α in the cost level type problem as follows.

(3) Relation of θ and α

Let us assume that the value of the objective function of eq. (10) or eq. (14) is at a minimum at the x_{ij}'s combinational pattern A_2, for a determined θ and α, and also assume such a pattern A_1 exists that the value of the objective function becomes less than that of pattern A_2 as θ and α increases. Let us also assume θ_0 and α_0 are the exact values of θ and α respectively when the values of the objective functions of the two patterns are the same. Then, the objective function of pattern A_1 becomes less than that of pattern A_2 by a small increase of θ or α from θ_0 or α_0. For the pattern A_1 and A_2, F_{A1}, S_{A1}, F_{A2}, S_{A2} are defined as follows respectively,

$$\left.\begin{aligned} F_{A1} &= \sum_{i=1}^{m}\sum_{j=1}^{n'}F_{ij}\cdot x_{ij}^{A1} \\ S_{A1} &= \sum_{i=1}^{m}\sum_{j=1}^{n'}\sigma_{ij}^2\cdot x_{ij}^{A1} \\ F_{A2} &= \sum_{i=1}^{m}\sum_{j=1}^{n'}F_{ij}\cdot x_{ij}^{A2} \\ S_{A2} &= \sum_{i=1}^{m}\sum_{j=1}^{n'}\sigma_{ij}^2\cdot x_{ij}^{A2} \end{aligned}\right\} \tag{15}$$

In the cost level type problem, for $\alpha\geqq\alpha_0$, eq. (17) holds.

$$(F_{A1}+\alpha\sqrt{S_{A1}})-(F_{A2}+\alpha\sqrt{S_{A2}})\leqq 0 \tag{16}$$

$$\alpha\geqq(F_{A1}-F_{A2})/(\sqrt{S_{A2}}-\sqrt{S_{A1}}) \tag{17}$$

In the utility function type problem, similarly eq. (18) holds for $\theta \geqq \theta_0$.

$$\theta/2 \geqq (F_{A1}-F_{A2})/(S_{A2}-S_{A1}) \tag{18}$$

The sign of equality holds in eq. (17) and eq. (18) just at $\alpha=\alpha_0$, $\theta=\theta_0$, and it is also a boundary where the value of the objective function of pattern A1 becomes smaller than that of pattern A2. So, it can be considered that θ_0 is exactly equivalent to α_0. Taking a ratio of $\theta_0/2$ and α_0,

$$(\theta_0/2)/\alpha_0 = 1/(\sqrt{S_{A1}}+\sqrt{S_{A2}}) \tag{19}$$

$$\theta_0 = 2\alpha_0/(\sqrt{S_{A1}}+\sqrt{S_{A2}}) \tag{20}$$

Accordingly, θ_0 can be defined as equivalent to α_0, if S_{A1} and S_{A2} are known.

3. Solution of the problem

The cost level type problem is a nonlinear programming problem and it takes an enormous calculating time to solve even for ordinary fault cases. However, the meaning of α is easily understood, so the solution can be obtained in reasonable computing time and the value of coefficients of the objective function are easily determined by solving the utility function type problem through relating θ to α.

3.1 Calculation of $\overline{F_{ij}}$ and σ_{ij}^2

$\overline{F_{ij}}$ and σ_{ij}^2 in eq. (14), the objective function of the utility function type problem, must be calculated. Let us assume $s(u_j)$ and $s'(u_j)$ are the distribution function and the probability density function of u_j respectively. u_j is the time required for the total output of generators to increase to meet $j\Delta$.

On the other hand, we can calculate t_j corresponding to u_j from eq. (8), we can also calculate as follows the expected value $\overline{F_{ij}}$ and the variance σ_{ij}^2 of the cost of interruption of the ith divided load at the point of time when the total output of the generators increases exactly to meet $j\Delta$.

$$\begin{aligned}\overline{F_{ij}} &= \int_{-\infty}^{\infty} s'(u_j)\cdot\int_{-\infty}^{\infty} F_i(t_i)\cdot r'\{F_i(t_i)\}\cdot dF_i(t_i)\cdot du_j \\ &= \int_{-\infty}^{\infty} s'(u_j)\cdot\overline{F_i(t_i)}\cdot du_j \\ &\doteqdot \sum_{u_j=0}^{n_m} s'(u_j)\cdot\overline{F_i(t_i)}\cdot\Delta u_j\end{aligned} \tag{21}$$

$$\begin{aligned}\sigma_{ij}^2 &= \left\{\overline{F_i(t_i)}\right\}^2-\left\{\overline{F_{ij}}\right\}^2 \\ &= \int_{-\infty}^{\infty} s'(u_j)\cdot\int_{-\infty}^{\infty}\left\{F_i(t_i)\right\}^2\cdot r'\left\{F_i(t_i)\right\}\cdot dF_i(t_i)\cdot du_j-\left\{\overline{F_{ij}}\right\}^2 \\ &= \sum_{u_j=0}^{u_m} s'(u_j)\cdot\Big[\sum_{F_i(t_i)=F_{i\min}}^{F_{i\max}(t_i)}\left\{F_i(t_i)\right\}^2\cdot r'\left\{F_i(t_i)\right\}\cdot\Delta F_i(t_i)\Big]\cdot\Delta u_j-\left\{\overline{F_{ij}}\right\}^2\end{aligned} \tag{22}$$

Where, u_m, $F_{i\min}(t_i)$ and $F_{i\max}(t_i)$ indicate the boundaries of the practical integral domain of the probability density function.

$s'(u_j)$ is not always a normal distribution even if the distribution of t_{g1} is a normal distribution, because of the upper and lower limits of generator output. So, distribution of F_{ij} is not always a normal distribution either. However, we can approximate the distribution of eq. (5) as a normal distribution regardless of the distribution pattern of t_{g1} and $F_i(t_i)$ etc. This is because, we can apply the central limit theorem[10,11] to the calculation, as F is a linear summation of F_{ij}, and u_j is appropriately concentrated around the expected value $\overline{u_j}$[10,11].

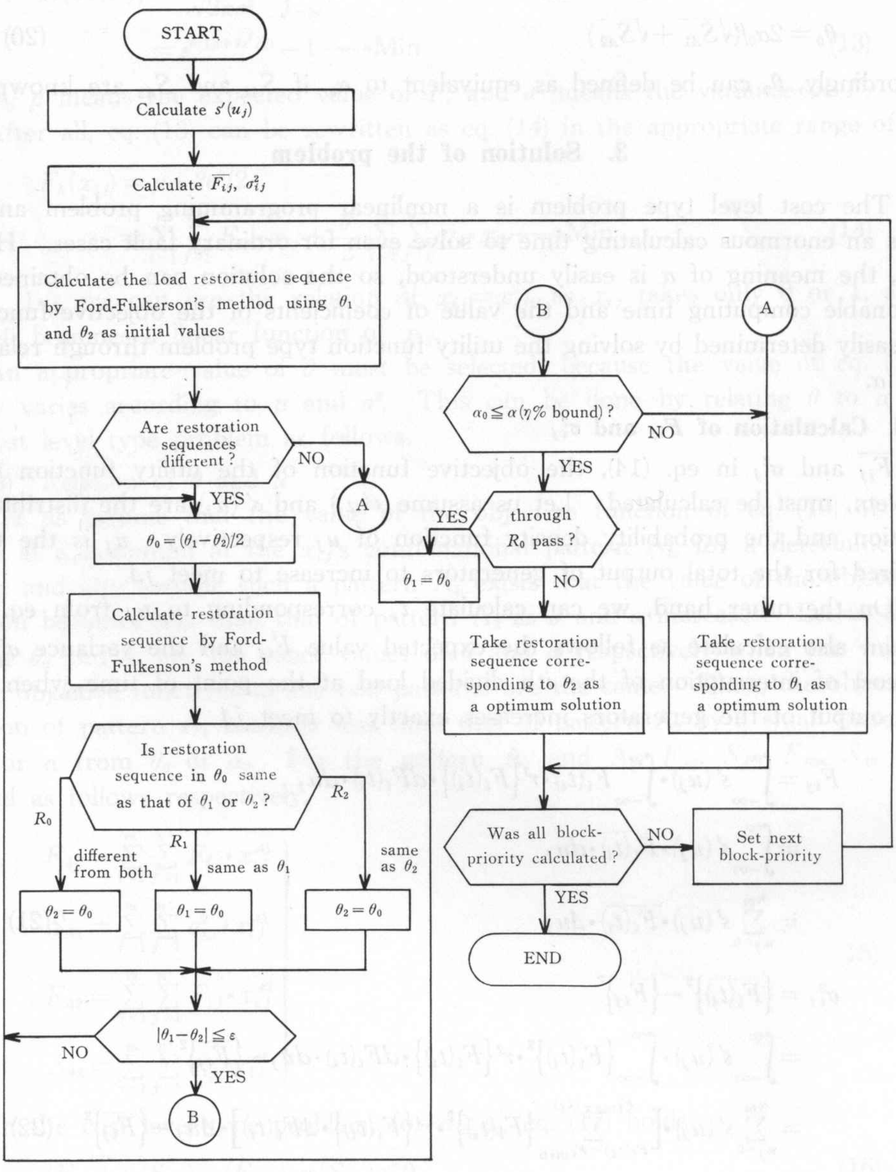


Fig. 3. Flowchart.

3.2 Algorithm

The problem of minimizing eq. (14) under the constraint of eq, (11) becomes a linear programming problem (minimum cost flow problem), and it is easily solved by Ford-Fulkerson's method[1,7]. The flowchart for this algorithm is shown in Fig. 3. In Fig. 3, at first $s'(u_j)$ is calculated by the method shown in Appendix, in the next place, $\overline{F_{ij}}$ and σ_{ij}^2 are calculated by eq. (21) and eq. (22). Then, eq. (14) can be minimized under the constraints of eq. (11) by Ford-Fulkerson's method, if θ can be determined. Although the value of θ is not yet known here, we can solve the problem as follows.

Let θ_1 mean the value of θ corresponding to $\eta=50\%$ (equivalent to the value using expected values), and θ_2 also mean the value of θ corresponding to $\eta=100\%$. θ_1 and θ_2 are estimated by $\sum_j \mathrm{Min}\{\sigma_{ij}^2\}$ and eq. (20), and we can develop the following calculation by using θ_1, θ_2 as intial values. That is, if restoration sequences calculated by Ford-Fulkerson's method are the same for θ_1 and θ_2, we can take the restoration sequence calculated by using θ_1 or θ_2 as a optimum solution because the optimum loads restoration sequence does not change over the range of $\eta=50$–$100(\%)$. If the loads restoration sequence are different for θ_1 and θ_2, θ_0, the point of θ where the optimum loads restoration sequence changes, must be calculated by the bisection method[13] and α_0 will be calculated corresponding to θ_0 by eq. (20). If the calculated α_0 is smaller than the predefined α corresponding to $\eta\%$ (predefined standard level), the optimum solution is that corresponding to θ_2, and similarly when α_0 is greater than the predefined α, the optimum solution is that of θ_1. Moreover, we must calculate through R_0 in Fig. 3, if other loads restoration sequences are obtained in the process of the bisection method's calculation.

Besides, if we take into account the block priority[1], we can only do the over all calculation after the calculation of $\overline{F_{ij}}$ and σ_{ij}^2, in accordance with the priority sequence as described in reference [1].

4. Examples and considerations

4.1 Model system

The method mentioned above is applied to a 30 buses 9 generator model as described in Fig. 4. The model system has 1 water turbine generator and 8 thermal generators. The startup and output-increasing characteristics of generators are shown in Table 1. The coefficients of quadratic functions for the expected values of the costs of interruptions and their variances are shown in Table 2. Although $w'(t_{g1})$ and $r'\{F_i(t_i)\}$ are of course arbitrary functions, in this example they are assumed to be normal distribution functions in order to simplify the problem.

4.2 Consideration of the results derived from the examples

Simulations of the model system have been done for the 4 cases shown in Table 3, assuming that there are no minimum output constraints for the gen-

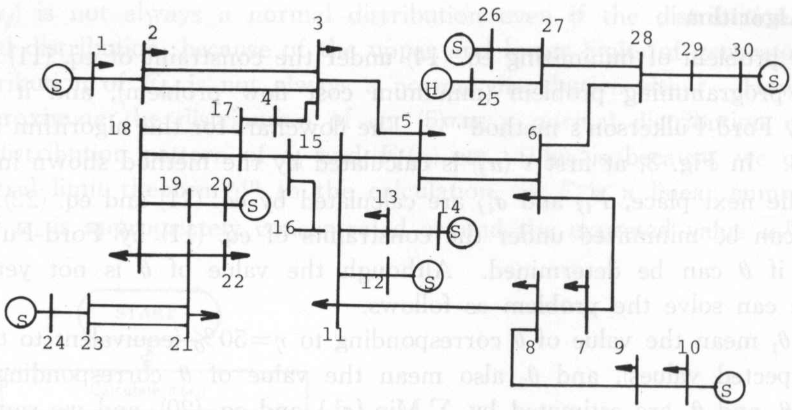


Fig. 4. Power system model.

Table 1. Characteristics of generators

Generator Bus No.	P_{g0}	d_g	t_{g1}	σ_{tg1}	Generator Bus No.	P_{g0}	d_g	t_{g1}	σ_{tg1}
1	0.396	0.01	—	—	24	1.340	0.02	—	—
10	0.480	0.05			25	0.072	0.02	5.0	3.0
12	0.617	0.01	20.0	5.0	26	0.400	0.04	15.0	3.0
14	0.955	0.05	10.0	3.0	30	0.265	0.05	10.0	3.0
20	1.570	0.01	—	—					

Table 2. Characteristics of loads

Load Bus No.	Magni-tude of load	a^1	b^1	c^1	a^2	b^2	c^2	Priority level	$3\sigma_{Ft}$
5	1.2	175.0	1,500.0	0.0	125.0	7,500.0	−180,000.0	1	$0.3\times F_i(t_i)$
6	0.2	50.0	10,000.0	0.0	50.0	10,000.0	0.0	2	$0.2\times F_i(t_i)$
7	1.1	75.0	0.0	0.0	45.0	3,600.0	−108,000.0	6	$0.5\times F_i(t_i)$
8	0.5	50.0	17,500.0	0.0	75.0	14,500.0	90,000.0	5	$0.2\times F_i(t_i)$
11	0.2	100.0	0.0	0.0	45.0	6,500.0	−192,000.0	3	$0.4\times F_i(t_i)$
13	0.1	25.0	5,000.0	0.0	61.25	650.0	130,500.0	4	$0.3\times F_i(t_i)$

a^h, b^h, c^h: $h=1$; $0\leqq t<60$ (min)
$h=2$, $60\leqq t<1440$ (min)
σ_{F_t}: Standard deviation of the cost of interruptions of ith devided load at t_i.

Table 3. Fault cases

Case No.	Total load dropped	Dropped load Bus No.	Generator output P.U. (dropped generator, if output=0)								
			#1	#10	#12	#14	#20	#24	#25	#26	#30
1	2.8	5, 6, 7, 11, 13	0.1	0.6	0.0	0.0	1.255	1.345	0.0	0.0	0.0
2	2.6	5, 6, 7, 13	0.285	0.6	0.0	0.0	1.270	1.345	0.0	0.0	0.0
3	1.5	5, 11, 13	0.396	0.552	0.0	0.0	1.570	1.345	0.072	0.4	0.265
4	0.3	11, 13	0.396	0.454	0.343	0.955	1.570	1.345	0.072	0.4	0.265

Table 4. The results of computation

Case No.	$\alpha=0.0$ Value of the objective function $F_1(F_0)$ ×1,000Yen	Load restoration sequence	$\alpha=0.1645$ Value of the objective function F_2 ×1,000Yen	Load restoration sequence	$(F_2'-F_2)/F_2$
1	12,149 (11,923)	6→5→13→11→7	13,077	6→5→11→7→13	4.48×10^{-4}
2	11,717 (11,511)	6→5→13→7	12,659	6→5→7→13	5.28×10^{-4}
3	13,230 (12,943)	5→11→3	13,230	5→11→13	0.0
4	761 (724)	13→11	761	13→11	0.0

erators. The conclusions are shown in Table 4. In Table 4, F_0 means the total cost of interruption calculated by using the expected values of the costs of interruptions of divided loads, and the condition $F_1 \neq F_0$ arises from distortions in the distribution of the generators' output, and from errors inherent in integral calculations in eq. (21). F_2' is the value of the objective function when $\alpha=1.645$ using the restoration sequence calculated with $\alpha=0.0$.

The following results are shown in Table 4.

If we take $\eta=95\%$ ($\alpha=1.645$), as the pre-defined standard level, the optimum restoration sequence of cases 1 and 2 are different from those using expected values, but in cases 3 and 4, they are the same. In cases 1 to 3, $F_{A1}<F_{A2}$, $S_{A1}>S_{A2}$ hold. But in case 4, $F_{A1}<F_{A2}$, $S_{A1}<S_{A2}$, and the restoration sequence will never change for any value of α. In case 3, the difference in standard deviations between S_{A1} and S_{A2} is so small in comparison with that of the expected values F_{A1} and F_{A2}, that a large α is necessary to change the optimum restoration sequence ($\alpha=28.6$, $\eta=100$). So, in $\eta=95(\%)$, the loads restoration sequence is the same as that of using expected values.

In this example, it is considered that the difference of the values of the objective functions (the total cost of interruption at $\eta\%$) is very small even if a difference in the restoration sequences exists between this method and that of using expected values. This difference depends on the expected value F and the variance S, and we will investigate this difference in the rest of this section[5].

We will consider only the cost level type problem hereafter, because the utility function type problem can be related to it.

Let us assume two patterns A1 and A2 exist, and a value of eq. (10) namely F_e for the pattern A1 is different from A2 at the same α.

Where,

$$F_{A1}>F_{A2}, \quad \sqrt{S_{A1}}<\sqrt{S_{A2}} \tag{23}$$

and F_{A1}, F_{A2}, S_{A1}, S_{A2} are defined in eq. (15).

Let it also be assumed

$$\left.\begin{aligned} \sqrt{S_{A1}} &= q_1 F_{A1} \\ \sqrt{S_{A2}} &= q_2 F_{A2} \\ F_{A1} &= \beta F_{A2} \end{aligned}\right\} \tag{24}$$

Then the difference (d) in the value of the objective function for patterns A1 and A2 is as follows.

$$d=(1+\alpha q_1)F_{A1}-(1+\alpha q_2)F_{A2}<0 \tag{25}$$

From eq. (23)~(25),

$$1<\beta<(1+\alpha q_2)/(1+\alpha q_1)=\xi \tag{26}$$

Dividing d by the value of the objective function of pattern A1,

$$D=\left|\frac{d}{F_{A1}+\sqrt{S_{A1}}}\right|=\left|1-\frac{(1+\alpha q_2)}{\beta(1+\alpha q_1)}\right|=\left|1-\frac{\xi}{\beta}\right| \tag{27}$$

The relationship of ξ, β, D is illustrated in Fig. 5 on refering to eq. (25). The relationship of ξ and β is

$$\xi=\frac{1-\alpha q_2}{1-\alpha q_1}=\frac{\gamma}{1+\gamma}+\frac{c\gamma}{1+\gamma}\beta \tag{28}$$

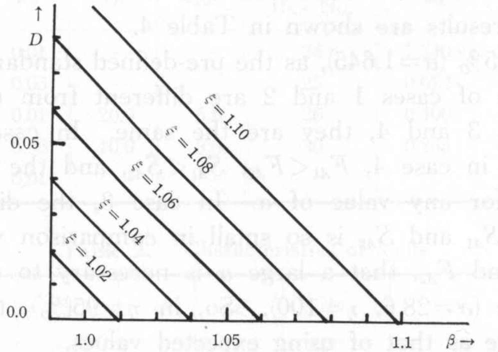
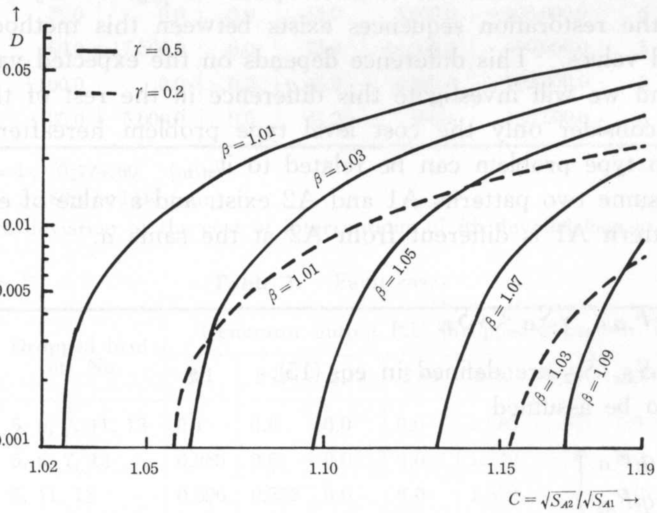


Fig. 5. Relation D and β.



$C=\sqrt{S_{A2}}/\sqrt{S_{A1}}\rightarrow$

Fig. 6. Relation between D and $c=\sqrt{S_{A1}}/\sqrt{S_{A2}}$.

Where,

$$c=\sqrt{S_{A2}}/\sqrt{S_{A1}} \text{ and } \gamma=\alpha q_1.$$

From eq. (27) and eq. (28), the relationshop of c and D at the parameters β and γ, is illustrated in Fig. 6 for $\gamma=0.3$ and $\gamma=0.5$.

Fig. 5 and Fig. 6 show that the difference in the values of the objective functions is $1\sim5$ percent at most, according to the relation of ξ and β or the relations among c, γ and β, even if the loads restoration sequence calculated taking into account the distribution of the cost of interruption and the time taken for the generators which have dropped out to parallel in, differs from that calculated by use of these expected values.

It can be considered that only a slight difference exists between the two calculating methods, because errors of quite a few percent are included in the distribution of the expected values of the cost of interruptions. Moreover, the necessary calculating time of the method taking into account the deviation is up to several hundred times longer than that using expected values.

Accodingly, we can conclude that the method using expected values is sufficient to calculate the optimum loads restoration sequence.

5. Conclusions

In this paper, the authors developed a method of determining a loads restoration sequence taking into account distribution of the cost of interruptions of electrical service and the time taken for generators which have dropped out to parallel in. The conclusion from this method is compared with that from the method using expected values.

The examples and the considerations show that for the α corresponding to a η percent bound, the optimum restoration sequence is different from that using expected values, but the difference in the values of the objective functions (total costs of interruption) for the same α is less than a few % for most of the cases. Accordingly, it is confirmed that the method using expected values is sufficient to decide the loads restoration sequence, taking into account computing time and uncertainty of the cost of interruption of electrical service, irrespective of the distribution pattern.

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References

1) K. Nara, S. Yamashiro and T. Koike: "Determination of the Load Restoration Sequence Taking into Account the Cost of Interruptions Electrical Service". Trans. of IEEJ, Vol. 101-B, No. 2 (1981).
2) K. Nara and T. Koike: "Power Systems Automatic Restorative Operation with Over-load Removal Based on the Loop Closing —Formulation Considering the CB Operation Time—". Record of 1980-National Conv. of IEEJ, No. 836, Tokyo.
3) K. Nara, S. Yamashiro and T. Koike: "Determination of the Load Restoration Sequence Taking into Account the Cost of Interruptions Electrical Service —Considering the Deviation of the Cost of Interruptions Electrical Service—". Proceeding of the Information Processing Symposium of IEEJ, IP-80-29 (1980), Osaka.
4) K. Nara, S. Yamashiro and T. Koike: Determination of the Load Restoration Sequence Taking into Account the Cost of Interruptions Electrical Service —On th Case of Considering the Extended Fault and Factor of Indefiniteness—". Record of 1980-Joint Four Inst. Elec. Engrs., Japan, No. 59, Hokkaido.
5) K. Nara, S. Yamashiro and T. Koike: "Determination of the Load Restoration Sequence Taking into Account the Cost of Interruptions Electrical Service —Relation of the Two Problem Including Probabilities and Improvement of Optimum Value of Objective Function—". Record of 1981-National Conv. of IEEJ, No. 881, Tokyo.
6) For an example C. A. DeSalvo, C. H. Hoffman and R. G. Hooke: "The Application of Planning Criteria to the Determination of Generator Service Data by Operating Gaming". AIEE Trans. PAS, Feb. (1960).
7) L. R. Ford and D. R. Fulukerson: "Flows in Networks". PRINCETON UNIVERSITY PRESS, (1962).
8) For an example, Miyawaki, Mine and Fujisawa: "Operations Research —Science for Investment, Planning and Management—". Kyoritsu Shuppan, Japan, (1957).
9) T. Nishida: "OR Handbook". Asakura Shoten, Japan, (1972).
10) W. Feller: "A Introduction to Probability Theory and Its Applications". John Willy and Sons, Inc. (1957).
11) A. Popoulis: "Probability, Random Variables, and Stochastic Processes". McGraw-Hill, (1965).
12) Б. В. Гнеденко: "Курс теории вероятностеи" Издание Пятое, (1969).
13) For an example, J. M. McCormik and M. G. Salivadori: "Numerical Methods in FORTRAN". Prentice-Hall, (1964).
14) For an example, I. Nabeshima: "Scheduling Theorem". Morikita Shuppan, Japan, (1974).

Appendix

Calculation of $s'(u_j)$

G_g, output of generator g at the arbitrary time u_j, is given as follows from eq. (1).

$$G_g = d_g(u_f + t_{gers} - t_{g1}) + e_g \tag{App. 1}$$

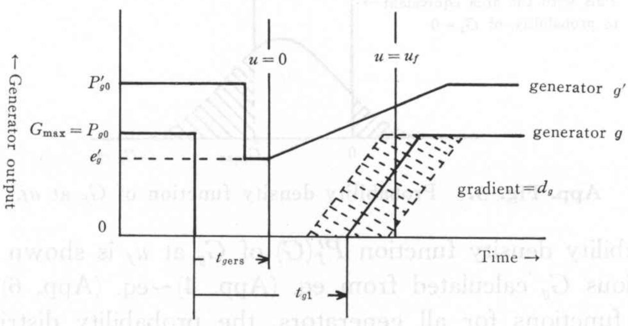
Then, from App. Fig. 1, for the range of $0 \leqq G_g \leqq G_{g\max}$, the following equation holds.

$$(u_f + t_{gers}) + e_g/d_g \geqq t_{g1} \geqq (u_f + t_{gers}) + e_g/d_g - G_{g\max}/d_g \tag{App. 2}$$

Where,

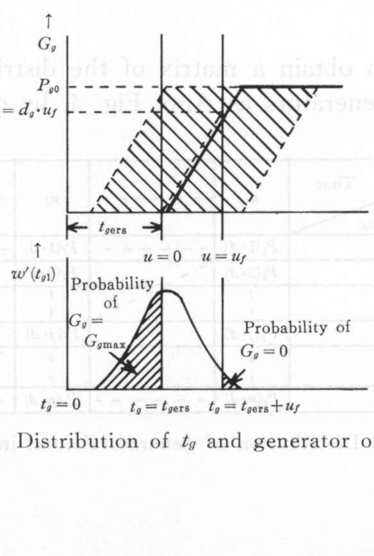
$$\left.\begin{array}{ll} G_g = 0 & (t_{g1} > u_f + t_{gers} + e_g/d_g) \\ G_g = G_{g\max} & (t_{g1} < u_f + t_{gers} + e_g/d_g - G_{g\max}/d_g) \end{array}\right\} \tag{App. 3}$$

$G_{g\max}$: rating or maximum output of generator g, can be calculated from eq. (8).



App. Fig. 1. Output of generators.

t_{g1} is distributed in the range of $t_{gers} \leqq t_{g1} \leqq \infty$, and so, $P'_g(G_g)$, the probability density function of G_g at u_f is given as follows for the range of $0 < G_g < G_{g\max}$ from App. Fig. 2.



App. Fig. 2. Distribution of t_g and generator output at u_f.

$$P'_g(G_g) = w'(u_f + t_{gers} + e_g/d_g - G_g/d_g) \tag{App. 4}$$

As $u>0$, the probability of $G_g=0$ or $G_g=G_{g\max}$ can be considered as the shaded area of App. Fig. 2, then,

$$P_g(G_g=0) = 1 - w(u_f + t_{gers} + e_g/d_g) \tag{App. 5}$$

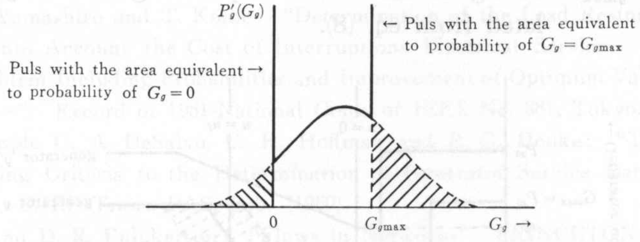
$$P_g(G_g=G_{g\max}) = w(u_f + t_{gers} + e_g/d_g - G_{g\max}/d_g) \tag{App. 6}$$

Where, the following exist.

$$0 < u_f + t_{gers} + e_g/d_g - G_{g\max}/d_g \tag{App. 7}$$

$G_{g\max}$ must be calculated as follows.

$$G_{g\max} = \begin{cases} u'_f d_g + e_g & : \text{ at } u_f + t_{gers} + e_g/d_g - (\text{rated output})/d_g < 0 \\ (\text{rated output}) & : \text{ at } u_f + t_{gers} + e_g/d_g - (\text{rated output})/d_g \geqq 0 \end{cases} \tag{App. 8}$$



App. Fig. 3. Probability density function of G_g at u_f.

The probability density function $P'_f(G)$ of G_g at u_f is shown as App. Fig. 3. $P'_f(G_g)$ for various G_g calculated from eq. (App. 4) ~ eq. (App. 6), then, by convoluting these functions for all generators, the probability distribution function $P(G)$ for the total incremental output of generators G can be calculated as follows, because functions $P_f(G)$ are independent of each other[12].

$$P_f(G \leqq j\Delta) = \iint \cdots \int \cdots \int_{\Sigma G_g \leqq j\Delta} P'_f(G_1) \cdot P'_f(G_2) \cdots P'_f(G_k) \cdot dG_1 \cdot dG_2 \cdots dG_k \tag{App. 9}$$

Accordingly we can obtain a matrix of the distribution pattern of the total incremental output of generators as App. Fig. 4, by calculating eq. (App. 1) ~ eq.

Time / Total incremental generator output	u_1	- - - - - -	u_f	- - - - - -	$u_{\max}$
$1\cdot\Delta$	$P_1(1\cdot\Delta)$	- - - - - -	$P_f(1\cdot\Delta)$	- - - - - -	$P_{\max}(1\cdot\Delta)$
$2\cdot\Delta$	$P_1(2\cdot\Delta)$		$P_f(2\cdot\Delta)$		¦
¦	¦		¦		¦
$j\cdot\Delta$	$P_1(j\cdot\Delta)$		$P_f(j\cdot\Delta)$		¦
¦	¦		¦		¦
$n\cdot\Delta$	$P_1(n\cdot\Delta)$	- - - - - -	$P_f(n\cdot\Delta)$	- - - - - -	$P_{\max}(n\cdot\Delta)$

App. Fig. 4. Distribution of generators' total incremen tal output.

(App. 9) as the values of u_f and j are increased from $u_f=0$ and $j=1$ respectively.

$P_f(j\Delta)$ monotonically decreases as j increases from 1 to n at the column u_f in App. Fig. 4, because total generator output monotonically increases as time passes. On the other hand, at the line $j\Delta$, it is guaranteed that $P_f(j\Delta)$ monotonically increases as f (subscript of u_f) increases from 1 to maximum (u_1 to $u_{\max}$). It can be considered that the line $j\Delta$ in App. Fig. 4 decribes the distribution of u_f, the time required for the total output of generators to increase to meet $j\Delta$.

Consequently, the probability density function $s'(u_j)$ can be calculated for u_f, the discrete value of u_j, as follows.

$$s'(u_j=u_f)=P_f(j\Delta)-P_{f-1}(j\Delta) \qquad \text{(App. 10)}$$

Where, u_j is the time required for the total output of generators to increase to meet $j\Delta$, explained above.