Temperature Dependence of Thermal Conductivity of Frozen Soil*

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Abstract

Using a hand-made thermal probe, measurements were made by a non-stationary absolute method as to the dependence on temperature of thermal conductivity of frozen soil by changing temperature in a range from 20 to −170°C and keeping as invariable as possible such other factors as moisture content, particle size distribution, mineral composition, dry density, etc.

Soil samples were prepared from volcanic ashes, which were sieved so that they comprised particles of 0.105 to 0.250 mm in diameter. The moisture content of ecah sample was arranged from 0 to 60 percent by adding a corresponding amount of water to oven-dried soil, so every sample had the same dry density.

It was revealed that the following three formulas are in a good agreement with the results of the experiment.

(i) Thermal conductivity of frozen soil varies with temperature in the following relation: $K=A\cdot T^B$ (A and B are constants; T is the absolute temperature.)

(ii) Thermal conductivity and moisture content have the following relation: $K=C\cdot e^{D\cdot w}$ (C and D are constants; w is the moisture content.)

(iii) The value of thermal conductivity K_G calculated from the geometric mean by the following formula agrees well with the value of observed thermal conuctivity K: $K_G=K_s^{\phi_s}\cdot K_a^{\phi_a}\cdot K_w^{\phi_w}\cdot K_i^{\phi_i}$, where K_s, K_a, K_w, K_i express thermal conductivity for soil particle, air, water and ice respectively; ϕ_s, ϕ_a, ϕ_w, ϕ_i express volumetric ratio of each component in the same order.

1 Introduction

It is known that the thermal conductivity of soil is affected by density, mineral composition, moisture content, texture of soil, particle size distribution, etc. Many reports have been published on relations between thermal conductivity of soil and these factors. For instance, Kersten (1949)[1] studied the effects of moisture content and dry density on the thermal conductivity of soil. Smith (1938[2], 1939[3], 1942[4]), Rooyen and Winterkorn (1959)[5] showed for dry and moist soil that the thermal conductivity is influenced by organic matter contained and quality of texture. Horai and Simmons (1969)[6] measured thermal conductivities of powderd rocks, from which they estimated thermal conductivities of 119 rock-forming minerals. Woodside and Messmer (1961)[7] came up with a three-element resister model, which consists of soil particles and a saturated fluid. All of the foregoing reports, however, dealt only with unfrozen soil. Relating to frozen

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soil, Kersten (1963)[8] and Higashi (1953)[9] presented each empirical formula as to relations between thermal conductivity and both of moisture content (or ice content) and dry density. Penner (1970)[10] and Penner, Johnston and Goodrich (1975)[11] studied the effect of soil-water-ice interactions on thermal conductivity, especially the influence of unfrozen moisture, whereby they measured the thermal conductivity of frozen soil for a temperature range from 5 to -26°C. Katayama et al. (1969)[12] reported the application of Woodside's model to water-saturated sand in a frozen condition. In all of these investigations, however, little attention has been paid to the influences of temperature, as was pointed out by Rooyen and Winterkorn[5].

This paper presents the results of measurements of thermal conductivities of frozen and unfrozen soil samples in temperatures ranging from 20 to -170°C with other factors kept as invariable as possible, that is, moisture content, particle size distribution, mineral composition, dry density, etc. Measurements were carried out by a transient heat-flow method using a hand-made thermal probe. A discussion is also made in this paper about the results.

2 Material

Soil samples were prepared from volcanic ashes, which were taken from the field of the Frozen Soil Laboratory in Kitami Institute of Technology. Sieves were used to select particles 0.105 to 0.250 mm in diameter. Their specific gravity measured by a pycnometer was 2.42. The maximum dry density was 1.07 g/cm^3 and the optimum moisture content was 37 percent for this density, which were obtained by a compaction test carried out according to Specification A-1210 of Japanese Industrial Standard using a mold 10 cm in diameter and a rammer 2.5 kg in weight. As to the mineral composition of the soil samples, volcanic glass represented 70 percent by weight, and scoria 15 percent, the rest being quartz, feldspar, etc. Soil was dried in an oven for 24 hours at 110°C. A soil with a given moisture content was prepared by adding a corresponding amount of water to the oven-dried soil and mixing them well. It was stuffed compactly into a polyvinyl chloride pipe 80 mm in inside diameter and 300 mm in length so that all the samples had a definite volume. Since the same oven-dried soil was used to prepare samples with different moisture contents, they had the same dry density. To make a frozen sample ready for measurement of thermal conductivity each sample was frozen to a concrete-like solid rapidly below -40°C so that ice segregation did not take place in the texture. Then measurements followed.

3 Simple Theory of Measuring Method

The method adopted in this experiment to measure thermal conductivity is the non-stationary absolute measurement method generally called the "probe method". Only a brief outline is presented here, since the theory of this method has been described in detail by Van der Held and Van Drunen (1949)[13], de Vries

(1952)[14], de Vries and Peck (1958)[15], Blackwell (1954)[16], Saito and Okagaki (1956)[17], and Rooyen and Winterkorn (1959)[5].

We suppose that a continuous linear heat source exists in an infinite homogeneous and isothermal medium. If supplying of the continuous heat $Q\,(W/m)$ starts at time $t=0$, when the medium is at the initial temperature θ_0 (°C), the temperature θ (°C) at the time t (sec.) and the distance r (m) from the heat source is given by the following formula, which was derived by Carslaw and Jaeger (1946)[18]:

$$\theta(r,t)=\theta_0+\frac{Q}{4\pi K}\int_{\frac{r^2}{4\alpha t}}^{\infty}e^{-u}\frac{du}{u}=\theta_0-\frac{Q}{4\pi K}E_i\left(-\frac{r^2}{4\alpha t}\right), \qquad (1)$$

where K is the thermal conductivity of the medium, α is the thermal diffusivity, u is the integration variable, and $-E_i(-x)$ is the exponential integral. Formula (1) leads to the following by asymptotic expansion.

$$\theta(r,t)=\theta_0+\frac{Q}{4\pi K}\left\{-\gamma-\ln\frac{r^2}{4\alpha t}+\frac{r^2}{4\alpha t}-\frac{1}{2\cdot 2!}\cdot\left(\frac{r^2}{4\alpha t}\right)^2+\frac{1}{3\cdot 3!}\cdot\left(\frac{r^2}{4\alpha t}\right)^3-\cdots\right\}, \qquad (2)$$

where $\gamma\,(=0.57721\cdots)$ is Euler's constant.

If the temperature is measured at a point near the line-shaped heat source and the time t is sufficiently large, formula (2) leads approximately to formula (3):

$$\theta(r,t)=\theta_0+\frac{Q}{4\pi K}\left(\ln\frac{4\alpha t}{r^2}-\gamma\right). \qquad (3)$$

Formula (4) is obtained if $\Delta\theta$, namely, the temperature difference between t_1 and t_2, is measured.

$$K=\frac{Q}{4\pi\Delta\theta}\left(\ln\frac{t_2}{t_1}\right). \qquad (4)$$

The thermal conductivity of the medium can be determined by the use of formula (4), provided that the times t_1 and t_2 are so large that the term of $r^2/4\,\alpha t-\cdots$ in formula (2) can be neglected.

In the experiment the distance between the heat source and the thermocouple junction is about 0.15 cm, namely, $r=0.0015$ m, and α remains between $0.002\sim0.007$ cm^2/s in an ordinary case, so the value of $r^2/4\alpha t$ becomes nearly zero when t, the lapse of time after heating, is 30 seconds or more. Hence, the passage of a short period of time allows the measurement of the thermal conductivity K.

4 Apparatus and Operation

4.1 Probe

The detailed illustration of a hand-made probe is given in Fig. 1. The thermocouple wires are of copper and constantan, 0.3 mm in diameter. The junction is inserted into a middle of a glass tube about 3 mm in outside diameter and 240 mm in length. A nichrome wire 0.127 mm in diameter and 800 mm in length is twined around the glass tube and fixed by an adhesive. It is covered with a thin layer of paper which serves as an electrical insulation from an enclosing copper tube. The glass tube has an outside diameter of 3 mm an inside diameter of 0.1 mm, and a length of 240 mm. It is inserted into the copper tube. Liquid paraffin fills up the crevice between the glass tube and the copper tube. This probe method has several merits; for example, rapid measurements are possible and a temperature gradient in a sample is so small that the influence of water migration in the sample is negligible.

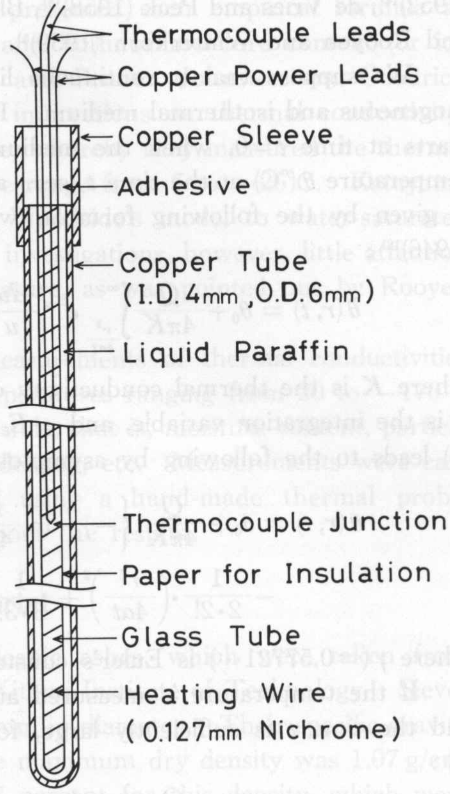

Fig. 1. Detail of a thermal conductivity probe.

4.2 Method

The schematic diagram of the apparatus is shown in Fig. 2. A sample has a shape of a cylindrical column 80 mm in diameter and 280 mm length. The probe is inserted into the axial portion of this sample from the top. This assemblage of probe and sample is placed in a cryostat which works from 10 to −190°C by liquid N_2. Operation is started after the sample becomes isothermal, which is ensured by leaving the sample for thirty to sixty minutes after confirming that the temperature at the surface of the sample measured by another thermocouple becomes the same as the temperature inside the probe. The current to the probe heater is switched on, using a D. C. regulated power supply; from this instant on the temperature rise is recorded for about 15 minutes. The recorded data is used to calculate the thermal conductivity of the sample at the initial equilibrium temperature, as shown by the description of a measurement later. The

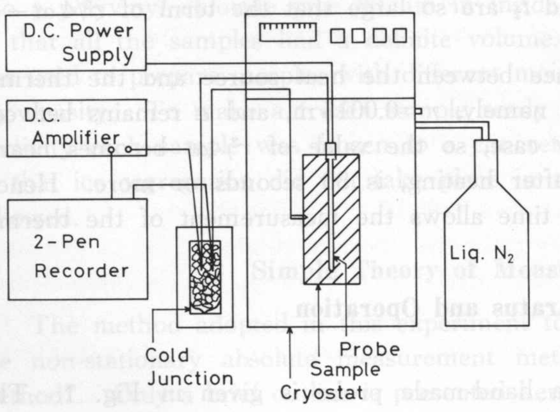

Fig. 2. Schematic diagram of apparatus.

power supply was controlled in these experiments to see to it that the temperature rise of the probe remained within 5°C. Furthermore, the thermal conductivity of the same sample is measured at different temperatures by changing the temperatures of the cryostat accordingly. Liquid N_2 was supplied continuously for a week to avoid the change of the freezing condition of the sample as the result of melting. Thus 17 data were obtained per sample on the average, whereby their moisture content, dry density, etc., remained the same. Consumption of liquid N_2 averaged about 300 liters per sample.

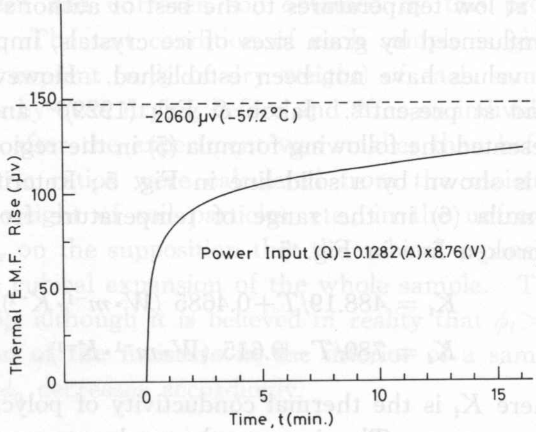

Fig. 3. Tpyical heating curve of the probe.

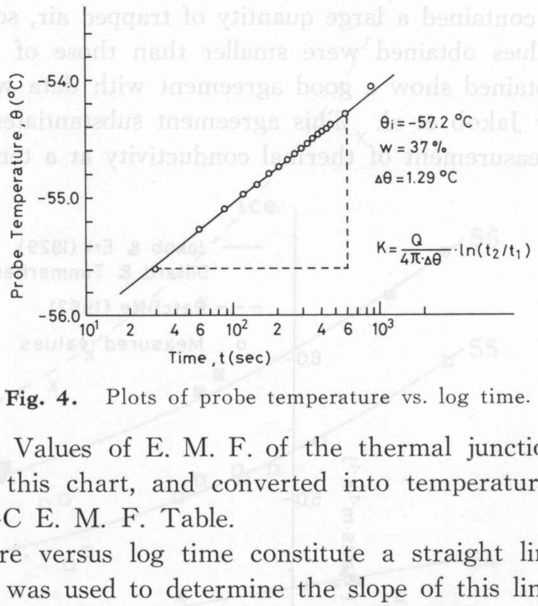

Fig. 4. Plots of probe temperature vs. log time.

4.3 Measured Examples

A typical record of a heating curve of a probe is shown in Fig. 3. The recorder had a chart speed of 1 cm/min.; sensitivity of E. M. F. was 12.5 μV/cm, as amplified by a D. C. amplifier. Values of E. M. F. of the thermal junction were read every 30 seconds from this chart, and converted into temperatures by Japanese Industrial Standard C-C E. M. F. Table.

The plots of probe temperature versus log time constitute a straight line in Fig. 4. An electronic computer was used to determine the slope of this line, $\ln(t_2/t_1)/\Delta\theta$, using the least squares method. The temperature difference $\Delta\theta$ of the probe was 1.29°C in this sample, as $\theta_{600sec.}$ was -54.30°C and $\theta_{30sec.}$ was -55.59°C. The rate of heat generation per unit length of the probe is: $Q = 0.1282\,(A) \times 8.76\,(V)/0.22\,(m) = 5.105\,(\text{Watts}\cdot m^{-1})$. Thus, the thermal conductivity K is given by $K = 5.105/(4\pi \times 1.29) \times \ln(20) = 0.943$ $(\text{Watts}\cdot m^{-1}\cdot K^{-1})$. The initial thermal equilibrium temperature of this sample was -57.2°C.

5 Results of Experiments

Plots in circles in Fig. 5 represent thermal conductivities of polycrystalline ice measured with an aim to test the function of the probe. Only a limited number of reports are available on the thermal conductivity of polycrystalline ice at low temperatures to the best of author's knowledge. Since the conductivity is influenced by grain sizes of ice crystals, impurities and trapped air, its acceptable values have not been established. However, two empirical formulas are on hand at present[19]. Jakob & Erk (1929)[20] and Dillard & Timmerhause (1966)[21] presented the following formula (5) in the region of temperature from 0 to -165°C as is shown by a solid line in Fig. 5 ; Ratcriffe (1962)[22] submitted the following formula (6) in the range of temperature from 0 to -150°C, as is shown by a broken line in Fig. 5 :

$$K_i = 488.19/T + 0.4685\ (W\cdot m^{-1}\cdot K^{-1}), \tag{5}$$

$$K_i = 780/T - 0.615\ (W\cdot m^{-1}\cdot K^{-1}), \tag{6}$$

where K_i is the thermal conductivity of polycrystalline ice, and T is the absolute temperature. The ice sample used was prepared by filling distilled water in a polyvynil chloride tube and freezing it in a domestic freezer at -18°C. Hence, it contained a large quantity of trapped air, so it was expected that the measured values obtained were smaller than those of bubbleless ice. However, the data obtained show a good agreement with data which have been reported, especially by Jakob et al. This agreement substantiates the capability of the probe in the measurement of thermal conductivity at a temperature range from 0 to -170°C.

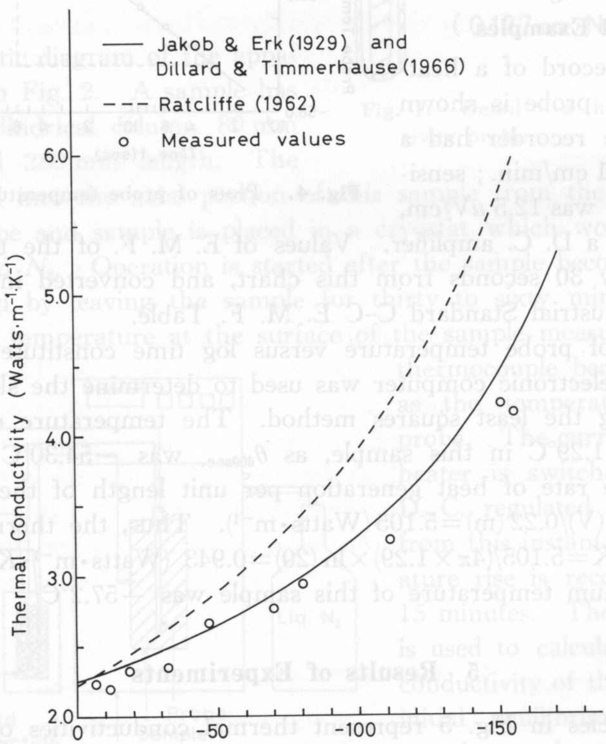

Fig. 5. Dependence of thermal conductivity on temperature for ice.

Thermal conductivities of frozen and unfrozen soil obtained by this probe are shown in Fig. 6 and Table 1. The test condition of each sample is given in Table 2, in which the moisture content (w% of dry weight) of each sample is the average of two measurements by oven drying before and after an individual experiment. The moisture content after the experiment was smaller than before by about 5 percent. The volumetric ratios were calculated from the moisture content, dry density, true specific weight of soil particles, etc., in the unfrozen state. It was presumed that $\phi_i = \phi_w$ on the supposition that the cubical expansion of water by freezing is equal to the cubical expansion of the whole sample. This paper uses the approximation $\phi_i = \phi_w$, although it is believed in reality that $\phi_i > \phi_w$ because of the volumetric expansion of the moisture in the interior of a sample as the result of freezing and that ϕ_a decreases accordingly.

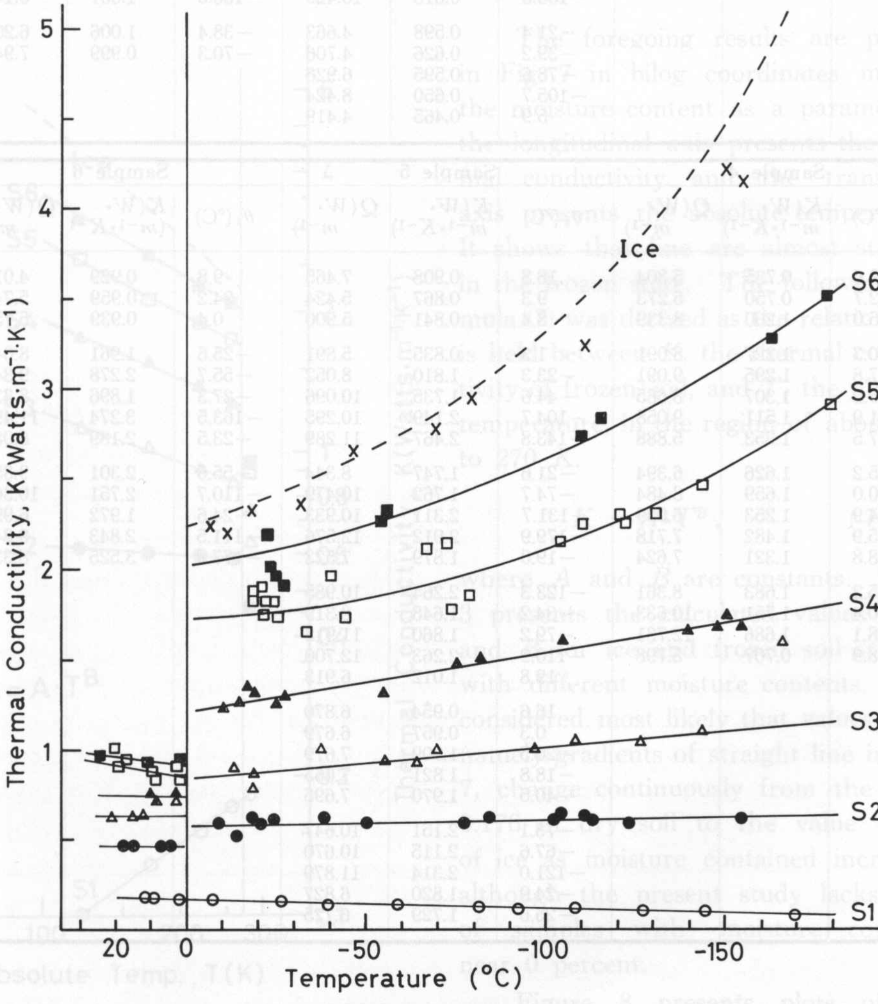

Fig. 6. Dependence of thermal conductivity on temperature for soil.

Table 1. Results of measurement of thermal conductivity

Sample 1			Sample 2			Sample 3		
θ_i(°C)	$K(W\cdot m^{-1}\cdot K^{-1})$	$Q(W\cdot m^{-1})$	θ_i(°C)	$K(W\cdot m^{-1}\cdot K^{-1})$	$Q(W\cdot m^{-1})$	θ_i(°C)	$K(W\cdot m^{-1}\cdot K^{-1})$	$Q(W\cdot m^{-1})$
9.5	0.178	1.490	16.1	0.466	7.343	20.6	0.615	5.928
−8.1	0.166	1.479	15.1	0.468	4.385	12.7	0.623	5.186
9.2	0.177	1.609	4.5	0.470	4.432	15.0	0.618	5.154
−27.8	0.155	1.613	−15.2	0.518	6.336	2.7	0.713	5.182
−81.8	0.132	1.578	−10.0	0.585	4.250	−19.0	0.765	5.205
−109.0	0.128	1.579	−50.9	0.595	5.475	−19.1	0.869	5.139
−170.0	0.090	1.607	−103.5	0.620	6.682	−64.3	0.930	5.153
−59.7	0.146	1.560	−19.1	0.622	4.948	−97.6	1.016	5.174
−92.6	0.126	1.564	−111.9	0.634	6.999	−144.4	1.102	5.203
−144.9	0.105	1.585	−25.1	0.606	5.919	−13.8	0.895	5.094
−128.0	0.112	1.579	−85.0	0.631	8.710	−57.2	0.943	5.105
0.7	0.173	1.561	−124.2	0.596	9.702	−108.6	1.050	5.124
−40.0	0.145	1.567	−114.1	0.612	7.588	−126.8	1.041	5.127
			−155.6	0.618	10.429	−166.5	1.601	5.146
			−21.4	0.598	4.663	−38.4	1.006	6.205
			−39.2	0.626	4.706	−70.3	0.999	7.943
			−78.6	0.595	6.926			
			−105.7	0.650	8.424			
			5.9	0.465	4.419			

Sample 4			Sample 5			Sample 6		
θ_i(°C)	$K(W\cdot m^{-1}\cdot K^{-1})$	$Q(W\cdot m^{-1})$	θ_i(°C)	$K(W\cdot m^{-1}\cdot K^{-1})$	$Q(W\cdot m^{-1})$	θ_i(°C)	$K(W\cdot (m^{-1}\cdot K^{-1}$	$Q(W\cdot m^{-1})$
9.6	0.735	5.304	18.8	0.908	7.465	9.8	0.929	4.016
2.7	0.750	5.273	9.3	0.867	5.424	24.2	0.959	5.743
−15.0	1.260	8.339	8.4	0.841	5.900	0.4	0.939	5.757
−10.3	1.227	8.091	1.3	0.835	5.891	−25.6	1.961	8.044
−27.8	1.295	9.091	−23.3	1.810	8.052	−55.7	2.278	9.344
−55.2	1.307	8.665	−44.6	1.735	10.096	−27.3	1.896	8.337
−81.9	1.511	9.058	−104.7	2.149	10.295	−163.5	3.274	11.492
−17.5	1.353	5.888	−143.8	2.467	11.289	−23.5	2.189	8.089
−105.2	1.626	6.394	−21.6	1.747	8.344	−55.0	2.301	9.352
−140.0	1.659	8.484	−74.7	1.762	10.479	−110.7	2.751	10.366
−24.9	1.253	6.059	−131.7	2.311	10.933	−24.6	1.972	6.993
−75.9	1.482	7.718	−179.9	2.912	12.576	−111.5	2.843	9.347
−18.8	1.321	7.624	−19.0	1.879	7.523	−177.9	3.525	8.336
−155.2	1.683	8.361	−123.3	2.264	10.985			
−150.4	1.751	10.633	−34.2	1.648	8.319			
−148.1	1.686	12.731	−79.2	1.860	11.918			
8.9	0.707	5.798	−110.9	2.263	12.700			
			19.8	1.012	6.913			
			16.6	0.954	6.870			
			0.3	0.957	6.679			
			−20.5	1.909	7.679			
			−18.8	1.821	7.665			
			−40.5	1.970	7.695			
			−73.1	2.151	10.647			
			−67.6	2.115	10.670			
			−121.0	2.314	11.879			
			−24.9	1.820	6.827			
			−25.6	1.729	6.725			

Table 2. Conditions of samples for thermal conductivity tests

		$S1$	$S2$	$S3$	$S4$	$S5$	$S6$
Moisture Content w (%)		0.0	27.0	37.0	46.9	54.8	55.9
Dry Density ρ_d(g/cm³)		0.893	0.893	0.893	0.943	0.956	0.990
Degree of Saturation (%)		0.0	38.2	52.3	72.5	86.6	93.6
Volu-metric Ratios	ϕ_s	0.369	0.369	0.369	0.390	0.395	0.409
	ϕ_a	0.631	0.390	0.301	0.168	0.081	0.038
	ϕ_w or ϕ_i	0.00	0.241	0.330	0.442	0.524	0.553

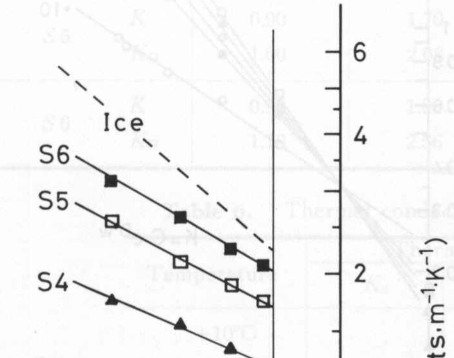

Fig. 7. Relation between thermal conductivity and absolute temperature.

6 Conclusion

The foregoing results are plotted in Fig. 7 in bilog coordinates making the moisture content as a parameter: the longitudinal axis presents the thermal conductivity and the transverse axis presents the absolute temperature. It shows that line are almost straight in the frozen state. The following formula (7) was derived as the relation that is held between K, the thermal conductivity of frozen soil, and T, the absolute temperature, in the region of about 100 to 270 K.

$$K = A \cdot T^B , \qquad (7)$$

where A and B are constants. Table 3 presents the calculated values of A and B for ice and frozen soil samples with different moisture contents. It is considered most likely that values of B, namely gradients of straight line in Fig. 7, change continuously from the value 0.176 of dry soil to the value -0.85 of ice as moisture contained increases, although the present study lacks data of samples with moisture contents near 0 percent.

Figure 8 presents plots of log thermal conductivity against moisture

Table 3. Values of constants A and B of formula (7)

Constant	Sample						
	$S1$	$S2$	$S3$	$S4$	$S5$	$S6$	Ice
A	0.0032	0.85	6.70	14.6	33.0	54.0	270
B	0.716	−0.065	−0.37	−0.44	−0.53	−0.59	−0.85

content, with temperatures of samples as a parameter. It shows that lines are almost straight. In other words, the relation between thermal conductivity and moisture content is exponential, which confirms the validity of Higashi's empirical formula (8) in the region from 10 to −160°C:

$$K = C \cdot e^{D \cdot w}, \qquad (8)$$

where w is the moisture ratio, C and D are constants. The numerical values of C and D are given in Table 4.

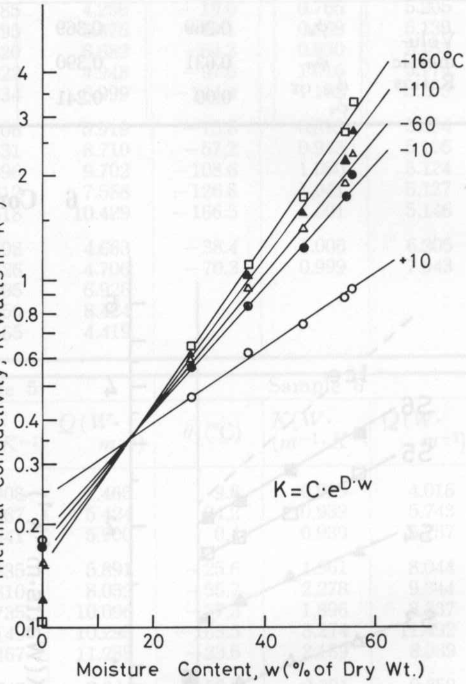

Fig. 8. Plots of log thermal conductivity vs. moisture content.

A comparison is made of thermal conductivity of soil in Table 5 between values K obtained from the experimental results and values K_G calculated from formula (9).

$$K_G = K_s^{\phi_s} \cdot K_a^{\phi_a} \cdot K_w^{\phi_w} \cdot K_i^{\phi_i}, \qquad (9)$$

where K_G is the effective conductivity of an aggregate calculated from the geometric mean of conductivities of four components (soil particle, air, water, and ice), based on a hypothetical model in which they are distributed at random. In this formula K_s, K_a, K_w, K_i are the thermal conductivities of each component, where the values $4.5\ W \cdot m^{-1} \cdot K^{-1}$ was used for K_s; ϕ_s, ϕ_a, ϕ_w, ϕ_i are the volumetric ratios of each component, where $\phi_s + \phi_a + \phi_w + \phi_i = 1$ and it was regarded that $\phi_i = 0$ for unfrozen samples and $\phi_w = 0$ for frozen samples. The presence

Table 4. Values of constants C and D of formula (8)

Constant	Temperature				
	+10°C	−10°C	−60°C	−110°C	−160°C
C	0.241	0.178	0.172	0.163	0.150
D	2.47	4.18	4.51	4.85	5.36

Table 5. Comparison of thermal conductivity between values measured and values calculated from formula (9)

		Temperature (°C)				
		+10	−10	−60	−110	−160
$S1$	K	0.17	0.16	0.14	0.12	0.10
	K_G	0.170	0.162	0.143	0.122	0.099
$S2$	K	0.46	0.58	0.60	0.61	0.63
	K_G	0.362	0.492	0.470	0.454	0.433
$S3$	K	0.62	0.83	0.94	1.04	1.13
	K_G	0.478	0.742	0.730	0.738	0.745
$S4$	K	0.73	1.23	1.40	1.58	1.75
	K_G	0.757	1.388	1.424	1.533	1.676
$S5$	K	0.90	1.70	1.90	2.20	2.65
	K_G	1.00	2.08	2.19	2.47	2.85
$S6$	K	0.94	1.90	2.30	2.80	3.30
	K_G	1.18	2.56	2.73	3.13	3.70

Table 6. Thermal conductivities of components of samples

Temperature	Thermal Conductivity ($W \cdot m^{-1} \cdot K^{-1}$)			
	K_s	K_a	K_w	K_i
+10°C		0.0250	0.574	
−10		0.0232		2.33
−60	4.50	0.0190		2.66
−110		0.0148		3.46
−160		0.0107		4.79

of unfrozen water even at subfreezing temperature does not allow $\phi_w=0$, but the volumetric ratio is so small that this approximation is considered to hold. In the calculation of the effective conductivities K_G of the aggregate it was assumed that the average thermal conductivity of soil particles remains unchanged by temperature, whereas for the thermal conductivities of other three components the values in Table 6 were used, in which the thermal conductivities of air and water at each temperature were based on "Handbook of Heat Transfer"[23] and the thermal conductivities of ice were derived from the formula of Jakob et al.

Examination of Table 5 discloses a good agreement between the calculated values from formula (9) and the experimental values obtained. In other words, an important approximation of effective thermal conductivity of a three-component aggregate (soil particle, air and either water or ice) is given by the geometric mean of formula (9). However, it remains questionable whether formulas (7), (8), and (9) give a good agreement with experimental results, in cases where the dry density or particle size distribution of a sample is changed. (These conditions were kept invariable in the present study.) Similar experiments will be pursued to clarify this point.

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