Analysis of One-way Ribbed Plates

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The stress distributions and deflections of steel plate bridge decks with closed from longitudinial ribs which are built up with thin rectangular strip elements, and are convinient to apply folded plate theory, are disccussed.

Making use of finite Fourier transform in the longitudinal direction and finite Fourier integration transform in the transverse direction, the solution of the theoretical analysis of the decks is obtained in the terms of double Fourier series.

As numerical examples, the simply supported steel plate bridge decks which are subjected to partially distributed uniform tire load, are taken into account.

Introduction

The structure on which we are going to study, is a plate stiffened one-way by closed form longitudinal ribs connected with it.

The structure of this kind has a good design efficiency, and is used for ship, airplane and bridge structures.

The researches so far made, can roughly be grouped in two theories. The first one stands on the base emphasizing the nature of the grid work, and the plate is replaced by the beam of its thickness in the transverse direction[1)].

The second one is the bending theory of the orthotropic plate. In this case, the characteristics of the ribs which have discrete properties, may be averaged and the ribbed plate is replaced by a model of continuous media[2)].

It is, however, of a practical importance to investigate more detailed elastic behaviour.

Taking the out-of-plane deformations of plate and rib wall and the folded plate actions into account, we can express the problem by simultaneous differential and difference equations with respect to the components of displacement along the nodal line[1,3)].

By means of finite Fourier transform in the longitudinal direction and finite Fourier integration transform[5)] in the transverse direction, the solution of this problem is given by the summation of double Fourier series.

Basic Formulas and Simbolic Notations

1) Displacement Shear Equations[4)]

The displacement shear equation expresses the relationship between the

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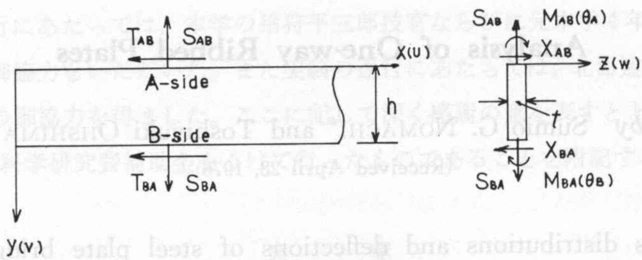

Fig. 1. Folded Plate Element.

nodal forces and nodal displacements of the folded plate element, neglecting the out-of-plane bending rigidity in the longitudinal direction as follows:

$$\begin{Bmatrix} \dot{T}_{AB} \\ T_{BA} \end{Bmatrix} = \frac{N}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_A \\ \ddot{u}_B \end{Bmatrix} + \frac{1}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} S_{AB} \\ S_{BA} \end{Bmatrix} \tag{1}$$

where $N = Eth, \quad \nu = 0, \quad \dot{f} = \dfrac{df}{dx}.$

$$\begin{Bmatrix} S_{AB} \\ S_{BA} \end{Bmatrix} = \frac{h^2}{N} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{Bmatrix} v_A \\ v_B \end{Bmatrix} + \frac{Gt}{2} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{Bmatrix} \dot{u}_A \\ \dot{u}_B \end{Bmatrix} + \frac{Gth}{6} \begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix} \begin{Bmatrix} \ddot{v}_A \\ \ddot{v}_B \end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \end{Bmatrix} = 2K \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix} - 6K \begin{bmatrix} 1 \\ 1 \end{bmatrix} R + \begin{bmatrix} C_{AB} \\ C_{BA} \end{bmatrix} \tag{3}$$

where $R = \dfrac{1}{h}(w_B - w_A), \quad C_{AB}, C_{BA} =$ load term

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \end{Bmatrix} = 2K \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix} + \frac{6K}{h} \begin{bmatrix} 1 & - \\ 1 & -1 \end{bmatrix} \begin{Bmatrix} w_A \\ w_B \end{Bmatrix} + \begin{bmatrix} C_{AB} \\ C_{BA} \end{bmatrix} \tag{4}$$

$$X_{AB} = X_{BA} = \frac{M_{AB} + M_{BA}}{h}, \quad K = \frac{Et^3}{12h} \tag{5}$$

2) Finite Fourier Integration Transform and their Inverse Transform[5)]

Let us introduce the symbolic notations

$$\boldsymbol{S}_i\big[f(x)\big] = \sum_{x=1}^{n-1} f(x) \sin \frac{i\pi x}{n} \tag{6}$$

$$\boldsymbol{C}_i\big[f(x)\big] = \sum_{x=1}^{n-1} f(x) \cos \frac{i\pi x}{n} \tag{7}$$

which are coupled with

$$f(x) = \frac{2}{n} \sum_{i=1}^{n-1} \boldsymbol{S}_i\big[f(x)\big] \sin \frac{i\pi x}{n} \tag{8}$$

$$f(x) = \frac{2}{n} \sum_{i=0}^{n} \boldsymbol{R}_i\big[f(x)\big] \cos \frac{i\pi x}{n} \tag{9}$$

where

$$\left.\begin{aligned}
&\boldsymbol{R}_0\Big[f(x)\Big]=\frac{1}{2}\Big\{\boldsymbol{C}_0\Big[f(x)\Big]+\frac{1}{2}f(u)+\frac{1}{2}f(0)\Big\}\\
&\boldsymbol{R}_i\Big[f(x)\Big]=\boldsymbol{C}_i\Big[f(x)\Big]+\frac{1}{2}(-1)^i f(u)+\frac{1}{2}f(0)\\
&\boldsymbol{R}_n\Big[f(x)\Big]=\frac{1}{2}\Big\{\boldsymbol{C}_n\Big[f(x)\Big]+\frac{1}{2}(-1)^x f(n)+\frac{1}{2}f(0)\Big\}\\
&\qquad(x,\ i=0,\ 1,\ 2,\ \cdots\ n.)
\end{aligned}\right\}\tag{10}$$

For convenience sake, let us define the second difference and the modified difference as follows:

$$\Delta^2 f(x-1)=f(x+1)-2\cdot f(x)+f(x-1)\tag{11}$$

$$\Delta f(x)=f(x+1)-f(x-1)\tag{12}$$

Applying the above formulas to the sine and cosine transforms, we have

$$\boldsymbol{S}_i\Big[\Delta^2 f(x-1)\Big]=-\sin\frac{i\pi}{n}\Big\{(-1)^i f(n)-f(0)\Big\}-D_i\,\boldsymbol{S}_i\Big[f(x)\Big]\tag{13}$$

$$\boldsymbol{S}_i\Big[\Delta f(x)\Big]=-2\sin\frac{i\pi}{n}\cdot\boldsymbol{R}_i\Big[f(x)\Big]\tag{14}$$

$$\boldsymbol{C}_i\Big[\Delta^2 f(x-1)\Big]=(-1)^i\,\Delta f(n-1)-\Delta f(0)-D_i\,\boldsymbol{R}_i\Big[f(x)\Big]\tag{15}$$

$$\begin{aligned}\boldsymbol{C}_i\Big[\Delta f(x)\Big]&=-(-1)^i\,\Delta f(n-1)-\Delta f(0)\\
&\quad+\Big(1+\cos\frac{i\pi}{n}\Big)\Big\{(-1)^i f(n)+f(0)\Big\}+2\sin\frac{i\pi}{n}\cdot\boldsymbol{S}_i\Big[f(x)\Big]\end{aligned}\tag{16}$$

where

$$D_i=2\Big(1-\cos\frac{i\pi}{n}\Big)$$

3) **Finite Fourier Transform and their Inverse Transform**[6)]

If $f(x)$ satisfy the Dirichlet's condition in the continuous coodinate $0<x<l$, the finite Fourier transform and their inverse transform are found out as follows:

$$\boldsymbol{S}_m\Big[f(x)\Big]=\int_0^l f(x)\sin\frac{m\pi x}{l}\,dx\qquad(m=1,\ 2,\ 3,\ \cdots)\tag{17}$$

$$f(x)=\frac{2}{l}\sum_{m=1}^{\infty}\boldsymbol{S}_m\Big[f(x)\Big]\sin\frac{m\pi x}{l}\tag{18}$$

The Analysis of the One-way Ribbed Plate

Taking the equilibrium of transverse bending moments and forces ($\sum M=0$, $\sum T=0$, $\sum H=0$, $\sum V=0$) along the nodal line $2r$ and $2r+1$ on the deck plate, and $2r'$ and $2r'+1'$ at the bottom of rib, according to Fig. 3, we can obtain the following equations of equilibrium:

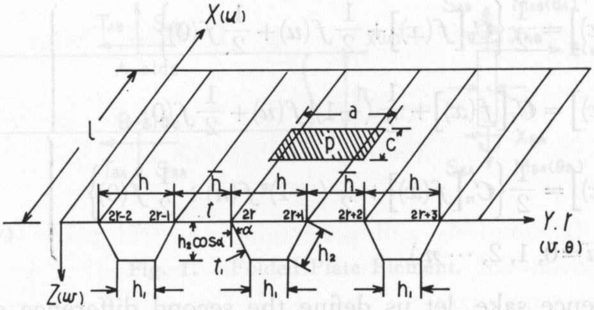

Fig. 2. One-way Ribbed Plate.

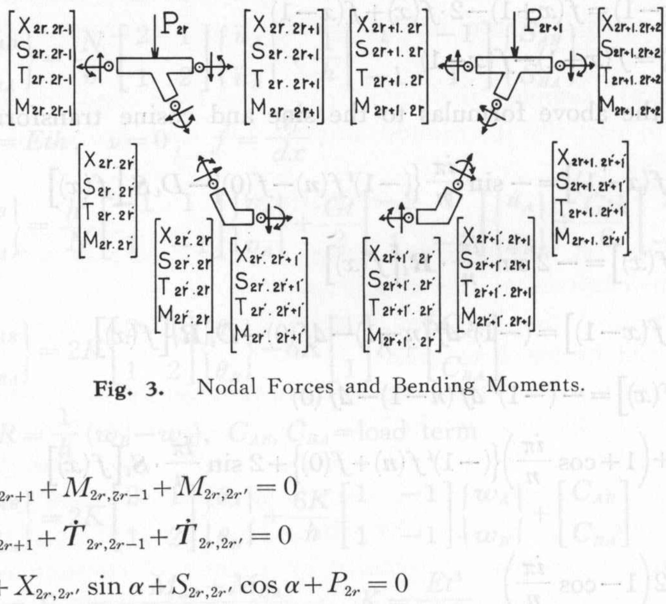

Fig. 3. Nodal Forces and Bending Moments.

at $2r$:

$$M_{2r,2r+1}+M_{2r,2r-1}+M_{2r,2r'}=0 \tag{19}$$

$$\dot{T}_{2r,2r+1}+\dot{T}_{2r,2r-1}+\dot{T}_{2r,2r'}=0 \tag{20}$$

$$V_{2r}+X_{2r,2r'}\sin\alpha+S_{2r,2r'}\cos\alpha+P_{2r}=0 \tag{21}$$

$$H_{2r}+S_{2r,2r'}\,\text{sni}\,\alpha-X_{2r,2r'}\cos\alpha=0 \tag{22}$$

at $2r+1$:

$$M_{2r+1,2r}+M_{2r+1,2r+2}+M_{2r+1,2r'+1'}=0 \tag{23}$$

$$\dot{T}_{2r+1,2r}+\dot{T}_{2r+1,2r+2}+\dot{T}_{2r+1,2r'+1'}=0 \tag{24}$$

$$V_{2r+1}-X_{2r+1,2r'+1'}\sin\alpha+S_{2r+1,2r'+1'}\cos\alpha+P_{2r+1}=0 \tag{25}$$

$$H_{2r+1}-X_{2r+1,2r'+1'}\cos\alpha-S_{2r+1,2r'+1'}\sin\alpha=0 \tag{26}$$

at $2r'$:

$$M_{2r',2r}+M_{2r',2r'+1'}=0 \tag{27}$$

$$\dot{T}_{2r',2r}+\dot{T}_{2r',2r'+1'}=0 \tag{28}$$

$$S_{2r',2r'+1'}-S_{2r',2r}\sin\alpha+X_{2r',2r}\cos\alpha=0 \tag{29}$$

$$X_{2r',2r'+1'}-S_{2r',2r}\cos\alpha-X_{2r',2r}\sin\alpha=0 \tag{30}$$

at $2r'+1'$:

$$M_{2r'+1',2r+1}+M_{2r'+1',2r}=0 \tag{31}$$

$$\dot{T}_{2r'+1',2r+1}+\dot{T}_{2r'+1',2r}=0 \tag{32}$$

$$S_{2r'+1',2r'}-S_{2r'+1',2r+1}\sin\alpha-X_{2r'+1',2r+1}\cos\alpha=0 \tag{33}$$

$$X_{2r'+1',2r'}+S_{2r'+1',2r+1}\cos\alpha-X_{2r'+1',2r+1}\sin\alpha=0 \tag{34}$$

where

$$V_{2r}=X_{2r,2r+1}-X_{2r,2r-1}$$

$$V_{2r+1}=X_{2r+1,2r+2}-X_{2r+1,2r}$$

$$H_{2r}=S_{2r,2r+1}-S_{2r,2r-1}$$

$$H_{2r+1}=S_{2r+1,2r+2}-S_{2r+1,2r}$$

By virtue of the Eqs. (1), (2), (3), (4) and (5), we may transform the above equations into the same number of the nodal displacement equations, namely four simultaneous difference equations with respect to the node designator $2r$.

Let us introduce the following expression for convenience sake,

$$\left.\begin{aligned}\nabla f_r&=f_{2r+1}+f_{2r}\\ \Delta f_r&=f_{2r+1}-f_{2r}\end{aligned}\right\} \tag{35}$$

where

$$\left.\begin{aligned}f_{2r+1}&=\frac{1}{2}(\nabla f_r+\Delta f_r)\\ f_{2r}&=\frac{1}{2}(\nabla f_r-\Delta f_r)\end{aligned}\right\} \tag{36}$$

Then the addition and subtraction of the four equations with respect to $2r+1$ and $2r$ yield

for $\nabla\dot{u}r$:

$$\begin{aligned}&\frac{h_2}{2}(N+N_2)\nabla\ddot{u}_r+\frac{\overline{N}h_2}{12}(\Delta_r^2\nabla\ddot{u}_{r-1}+6\nabla\ddot{u}_r-\Delta_r\Delta\ddot{u}_r)-\frac{N_2h_2^2}{6}\cos\alpha\nabla\ddot{w}_r\\ &+\frac{Gth_2}{4}(\Delta_r\nabla\ddot{v}_r-\Delta_r^2\Delta\ddot{v}_{r-1})+\frac{Gth_2}{2\overline{h}}(\Delta_r^2\nabla\dot{u}_{r-1}-\Delta_r\Delta\dot{u}_r)\\ &+\frac{3\overline{K}}{\overline{h}\cos\alpha}(\Delta_r\nabla\theta_r-\Delta_r^2\Delta\theta_{r-1})-\frac{6\overline{K}}{\overline{h}^2\cos\alpha}(\Delta_r^2\nabla w_{r-1}-\Delta_r\Delta w_r)\\ &=\sec\alpha\cdot\nabla P_r-\frac{\sec\alpha}{\overline{h}}(C_{2r+1,2r+2}+C_{2r+2,2r+1}-C_{2r,2r-1}-C_{2r-1,2r}),\end{aligned} \tag{37}$$

for $\Delta\dot{u}$:

$$h_2\left(\frac{N}{6}+\frac{N_2}{2}\right)\Delta\ddot{u}_r+\frac{\overline{N}h_2}{12}(\Delta_r\nabla\ddot{u}_r-\Delta_r^2\Delta\ddot{u}_{r-1}+2\Delta\ddot{u}_r)-\frac{N_2h_2^2}{6}\cos\alpha\Delta\ddot{w}_r$$

$$
\begin{aligned}
&+\frac{Gth_2}{4}(\Delta_r^2\nabla\ddot{v}_{r-1}-\Delta_r\Delta\ddot{v}_r)-2\frac{Gth_2}{h}\Delta\dot{u}_r+\frac{Gth_2}{2\bar{h}}(\Delta_r\nabla\dot{u}_r-\Delta_r^2\Delta\dot{u}_{r-1}-4\Delta\dot{u}_r)\\
&+\frac{3\bar{K}}{\bar{h}\cos\alpha}(\Delta_r^2\nabla\theta_{r-1}+4\nabla\theta_r-\Delta_r\Delta\theta_r)-\frac{6\bar{K}}{\bar{h}^2\cos\alpha}(\Delta_r\nabla w_r-\Delta_r^2\Delta w_{r-1}-4\Delta w_r)\\
&-\frac{12K}{h\cos\alpha}\left(\nabla\theta_r-\frac{2}{h}\Delta w_r\right)-\frac{12K_1}{h_1\cos\alpha}(\gamma_1-2\beta_1\gamma_1-2\beta_2)\nabla\theta_r\\
&-\frac{12K_1}{h_1\cos\alpha}(\gamma_2-2\beta_1\gamma_2-2\beta_3)\Delta w_r=\sec\alpha\cdot\Delta P_r\\
&-\frac{\sec\alpha}{\bar{h}}(C_{2r+1,2r+2}+C_{2r+2,2r+1}+C_{2r,2r-1}+C_{2r-1,2r})\\
&+\frac{2\cdot\sec\alpha}{h}(C_{2r,2r+1}+C_{2r+1,2r}),
\end{aligned}
\tag{38}
$$

for ∇v:

$$
\begin{aligned}
&\frac{\bar{N}}{2\bar{h}^2}(\Delta_r^2\nabla v_{r-1}-\Delta_r\Delta v_r)+\frac{Gt}{4}(\Delta_r\nabla\dot{u}_r-\Delta_r^2\Delta\dot{u}_{r-1})+\frac{Gt\bar{h}}{12}(\Delta_r^2\nabla\ddot{v}_{r-1}+6\nabla\ddot{v}_r-\Delta_r\Delta\ddot{v}_r)\\
&+\frac{Gth}{2}\nabla\ddot{v}_r+\frac{6K_2}{h_2}\sec\alpha(1+\gamma_1-4\alpha_1\gamma_1-4\alpha_2)\nabla\theta_r\\
&+\frac{6K_2}{h_2}\sec\alpha(\gamma_2-4\alpha_1\gamma_2-4\alpha_2)\Delta w_r-\frac{3\bar{K}}{\bar{h}}(\Delta_r^2\nabla\theta_{r-1}+4\Delta\theta_r-\Delta_r\Delta\theta_r)\tan\alpha\\
&+\frac{6\bar{K}}{\bar{h}}\tan\alpha(\Delta_r\nabla w_r-\Delta_r^2\Delta w_{r-1}-4\Delta w_r)+\frac{12K}{h}\tan\alpha\left(\nabla\theta_r-\frac{2}{h}\Delta w_r\right)\\
&=-\tan\alpha\cdot\Delta P_r+\frac{\tan\alpha}{\bar{h}}(C_{2r+1,2r+2}+C_{2r+2,2r+1}+C_{2r,2r-1}+C_{2r-1,2r})\\
&-\frac{2\tan\alpha}{h}C_{2r,2r+1}+C_{2r+1,2r}),
\end{aligned}
\tag{39}
$$

for Δv_r:

$$
\begin{aligned}
&\frac{\bar{N}}{2\bar{h}^2}(\Delta_r\nabla v_r-\Delta_r^2\Delta v_{r-1}-4\Delta v_r)+\frac{Gt}{4}(\Delta_r^2\nabla\dot{u}_{r-1}-\Delta_r\Delta\dot{u}_r)\\
&+\frac{Gth}{12}(\Delta_r\nabla\ddot{v}_r-\Delta_r^2\Delta\ddot{v}_{r-1}+2\Delta\ddot{v}_r)-\frac{2N}{h^2}\Delta v_r+\frac{Gth}{6}\Delta\ddot{v}_r\\
&+\frac{6K_2}{h_2}\sec\alpha(1+\delta_1)\Delta\theta_r-\frac{3\bar{K}}{\bar{h}}\tan\alpha(\Delta_r\nabla\theta_r-\Delta_r^2\Delta\theta_{r-1})\\
&+\frac{6\bar{K}}{\bar{h}^2}\tan\alpha(\Delta_r^2\nabla w_{r-1}-\Delta_r\Delta w_r)=-\tan\alpha\cdot\nabla P_r\\
&+\frac{\tan\alpha}{\bar{h}}(C_{2r+1,2r+2}+C_{2r+1,2r+2}-C_{2r,2r-1}-C_{2r-1,2r}),
\end{aligned}
\tag{40}
$$

for Δw_r:

$$
\frac{\bar{N}}{12}(\Delta_r^2\nabla\ddot{u}_{r-1}+6\nabla\ddot{u}_r-\Delta_r\Delta\ddot{u}_r)+\frac{1}{2}(N+N_1+2N_2)\nabla\ddot{u}_r
$$

$$-\frac{h_2}{2}(N_1+N_2)\cos\alpha\,\nabla\dddot{w}_r+\frac{Gt}{4}(\Delta_r\nabla\ddot{v}_r-\Delta_r^2\Delta\ddot{v}_{r-1})$$

$$+\frac{Gt}{2\bar{h}}(\Delta_r^2\nabla\dot{u}_{r-1}-\Delta_r\Delta\dot{u}_r)=0\,, \tag{41}$$

for Δw_r:

$$\frac{\bar{N}}{12}(\Delta_r\nabla\ddot{u}_r-\Delta_r^2\Delta\ddot{u}_{r-1}+2\Delta\ddot{u}_r)+\frac{1}{6}(N+N_1+6N_2)\Delta\ddot{u}_r$$

$$-h_2\left(\frac{N_2}{2}+\frac{N_1}{6}\right)\cos\alpha\Delta\dddot{w}_r+\frac{Gt}{4}(\Delta_r^2\nabla\ddot{v}_{r-1}-\Delta_r\Delta\ddot{v}_r)$$

$$-\frac{2Gt}{h}\Delta\dot{u}_r+\frac{Gt}{2\bar{h}}(\Delta_r\nabla\dot{u}_r-\Delta_r^2\Delta\dot{u}_{r-1}-4\Delta\dot{u}_r)=0\,, \tag{42}$$

for $\nabla\theta_r$:

$$\bar{K}(\Delta_r^2\nabla\theta_{r-1}+6\nabla\theta_r-\Delta_r\Delta\theta_r)+(6K+4K_2-12K_2\alpha_2+2K_2\gamma_1-12K_2\alpha_1\gamma_1)\nabla\theta_r$$

$$+\left(2K_2\gamma_2-12K_2\alpha_1\gamma_2-\frac{12K}{h}-12K_2\alpha_3\right)\Delta w_r$$

$$-\frac{3\bar{K}}{\bar{h}}(\Delta_r\Delta w_r-\Delta_r^2\Delta w_{r-1}-4\Delta w_r)+\nabla C_r=0\,, \tag{43}$$

for $\Delta\theta_r$:

$$\bar{K}(\Delta_r\nabla\theta_r-\Delta_r^2\Delta\theta_{r-1}+2\Delta\theta_r)+(2K+4K_2+2K_2\delta_1)\Delta\theta_r$$

$$-\frac{3\bar{K}}{\bar{h}}(\Delta_r^2\nabla w_{r-1}-\Delta_r\Delta w_r)+\Delta C_r=0\,, \tag{44}$$

where

$$\Delta\ddot{u}_{r'}=\nabla\ddot{u}_r-h_2\cos\alpha\nabla\dddot{w}_r\,,$$

$$\Delta\ddot{u}_{r'}=\Delta\ddot{u}_r-h_2\cos\alpha\Delta\dddot{w}_r\,,$$

$$\nabla C_r=C_{2r,2r+1}+C_{2r+1,2r}+C_{2r,2r-1}+C_{2r+1,2r+2}\,,$$

$$\Delta C_r=C_{2r+1,2r}-C_{2r,2r+1}+C_{2r+1,2r+2}-C_{2r,2r-1}\,,$$

$$\alpha_1=\frac{1}{B}\left(-\frac{12K_1}{h_1}\sin\alpha+\frac{6K_2}{h_2}\right),$$

$$\alpha_2=\frac{6K_2}{h_2B}\,,\qquad \alpha_3=\frac{24K_1\sin\alpha}{h_1^2B}\,,$$

$$B=\frac{24K_2}{h_2}+\frac{48K_1h_2\sin^2\alpha}{h_1^2}\,,$$

$$\beta_1=-\frac{2h_2\sin\alpha}{h_1B}\left(-\frac{12K_1}{h_1}\sin\alpha+\frac{6K_2}{h_2}\right),$$

$$\beta_2=-\frac{2h_2\sin\alpha}{h_1B}\cdot\frac{6K}{h_2}\,,$$

$$\beta_3=\frac{1}{h_1}-\frac{2h_2\sin\alpha}{h_1B}\cdot\frac{24K_1\sin\alpha}{h_1^2},$$

$$\gamma_1=\frac{2K_2-12K_1\beta_2-12K_2\alpha_2}{12K_1\beta_1+12K_2\alpha_1-6K_1-4K_2},$$

$$\gamma_2=\frac{-12(K_1\beta_3+K_2\alpha_3)}{12K_1\beta_1+12K_2\alpha_1-6K_1-4K_2},$$

$$\delta_1=-\frac{2K_2}{2K_1+4K_2},$$

$$N=Eth,\qquad \bar{N}=Et\bar{h},\qquad N_1=Et_1h_1,\qquad N_2=Et_2h_2,$$

$$K=\frac{Et^3}{12h},\quad \bar{K}=\frac{Et^3}{12\bar{h}},\quad K_1=\frac{Et_1^3}{12h_1},\quad K_2=\frac{Et_2^3}{12h_2}.$$

Making use of the finite Fourier transform with respect to the continuous variable in the x direction and the finite Fourier integration transform with respect to the discrete variable r in the y direction, and taking the boundary conditions as simple supports, we can transform above simultaneous differential and difference equations into following equations:

$$\boldsymbol{A}\cdot\boldsymbol{U}=\boldsymbol{P} \tag{45}$$

where

$$\boldsymbol{A}=\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18}\\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28}\\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} & A_{38}\\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & A_{48}\\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & 0 & 0 & 0\\ A_{61} & A_{62} & A_{63} & A_{64} & 0 & A_{66} & 0 & 0\\ 0 & 0 & 0 & 0 & A_{75} & A_{76} & A_{77} & A_{78}\\ 0 & 0 & 0 & 0 & A_{85} & A_{86} & A_{87} & A_{88} \end{bmatrix},$$

$$\boldsymbol{U}=\begin{bmatrix} \boldsymbol{S}_m\,\boldsymbol{S}_i\,[\nabla\dot{u}_r]\\ \boldsymbol{S}_m\,\boldsymbol{R}_i\,[\Delta\dot{u}_r]\\ \boldsymbol{S}_m\,\boldsymbol{R}_i\,[\nabla v_r]\\ \boldsymbol{S}_m\,\boldsymbol{S}_i\,[\Delta v_r]\\ \boldsymbol{S}_m\,\boldsymbol{S}_i\,[\nabla w_r]\\ \boldsymbol{S}_m\,\boldsymbol{R}_i\,[\Delta w_r]\\ \boldsymbol{S}_m\,\boldsymbol{R}_i\,[\nabla\theta_r]\\ \boldsymbol{S}_m\,\boldsymbol{S}_i\,[\Delta\theta_r] \end{bmatrix},\qquad \boldsymbol{P}=\begin{bmatrix} \boldsymbol{P}_1\\ \boldsymbol{P}_2\\ \boldsymbol{P}_3\\ \boldsymbol{P}_4\\ 0\\ 0\\ \boldsymbol{P}_7\\ \boldsymbol{P}_8 \end{bmatrix},$$

$$A_{11} = -\frac{h_2^2}{2}(N+N_2)\left(\frac{m\pi}{l}\right)^2 - \frac{\bar{N}h_2}{2}\left(1-\frac{D_i}{6}\right)\left(\frac{m\pi}{l}\right)^2 - \frac{Gth_2}{2\bar{h}} D_i\,,$$

$$A_{12} = \left\{-\frac{\bar{N}h_2}{6}\left(\frac{m\pi}{l}\right)^2 + \frac{Gth_2}{\bar{h}}\right\}\sin\frac{i\pi}{n}\,, \quad A_{13} = \frac{Gth_2}{2}\sin\frac{i\pi}{n}\left(\frac{m\pi}{l}\right)^2,$$

$$A_{14} = -\frac{Gth_2}{4} D_i\left(\frac{m\pi}{l}\right)^2, \quad A_{15} = -\frac{N_2 h_2^2}{6}\cos\alpha\left(\frac{m\pi}{l}\right)^4 + \frac{6\bar{K}}{\bar{h}^2\cos\alpha} D_i\,,$$

$$A_{16} = \frac{12\bar{K}}{\bar{h}^2\cos\alpha}\sin\frac{i\pi}{n}\,, \quad A_{17} = -\frac{6\bar{K}}{\bar{h}\cos\alpha}\sin\frac{i\pi}{n}\,, \quad A_{18} = \frac{3\bar{K}}{\bar{h}\cos\alpha} D_i$$

$$\boldsymbol{P}_1 = \sec\alpha\cdot\boldsymbol{S}_m\boldsymbol{S}_i[\nabla P_r + \Delta\bar{C}_r]\,,$$

$$A_{21} = -\frac{\bar{N}h_2}{6}\sin\frac{i\pi}{n}\left(\frac{m\pi}{l}\right)^2 + \frac{Gth_2}{h}\sin\frac{i\pi}{n}\,,$$

$$A_{22} = -h_2\left(\frac{N}{6}+\frac{N_2}{2}\right)\left(\frac{m\pi}{l}\right)^2 - \frac{\bar{N}h_2}{12}\left(\frac{m\pi}{l}\right)^2(2+D_i) - \frac{2Gth_2}{\bar{h}} - \frac{Gth_2}{2\bar{h}}(4-D_i)$$

$$A_{23} = \frac{Gth_2}{4} D_i\left(\frac{m\pi}{l}\right)^2, \quad A_{24} = -\frac{Gth_2}{2}\sin\frac{i\pi}{n}\left(\frac{m\pi}{l}\right)^2, \quad A_{25} = -\frac{12\bar{K}}{\bar{h}^2\cos\alpha}\sin\frac{i\pi}{n},$$

$$A_{26} = -\frac{N_2 h_2^2}{6}\cos\alpha\left(\frac{m\pi}{l}\right)^4 + \frac{6\bar{K}}{\bar{h}^2\cos\alpha}(4-D_i)$$

$$+\frac{24K}{h^2\cos\alpha} - \frac{12K_1}{h_1\cos\alpha}(\gamma_2 - 2\beta_1\gamma_2 - 2\beta_3)\,,$$

$$A_{27} = \frac{3\bar{K}}{\bar{h}\cos\alpha}(4-D_i) - \frac{12K}{h\cos\alpha} - \frac{12K_1}{h_1\cos\alpha}(\gamma_1 - 2\beta_1\gamma_1 - 2\beta_2)\,,$$

$$A_{28} = -\frac{6\bar{K}}{\bar{h}\cos\alpha}\sin\frac{i\pi}{n}$$

$$\boldsymbol{P}_2 = \sec\alpha\cdot\boldsymbol{S}_m\boldsymbol{R}_i[\Delta P_r + \nabla\bar{C}_r]\,,$$

$$A_{31} = \frac{Gt}{2}\sin\frac{i\pi}{n}\,, \quad A_{32} = \frac{Gt}{4} D_i\,, \quad A_{34} = \left\{-\frac{\bar{N}}{\bar{h}^2} + \frac{Gt\bar{h}}{6}\left(\frac{m\pi}{l}\right)^2\right\}\sin\frac{i\pi}{n}$$

$$A_{33} = -\frac{\bar{N}}{2\bar{h}^2} D_i - \frac{Gt\bar{h}}{2}\left(\frac{m\pi}{l}\right)^2\left(1-\frac{D_i}{6}\right) - \frac{Gth}{2}\left(\frac{m\pi}{l}\right)^2,$$

$$A_{35} = \frac{12\bar{K}}{\bar{h}^2}\tan\alpha\cdot\sin\frac{i\pi}{n}\,, \quad A_{38} = \frac{6\bar{K}}{\bar{h}}\tan\alpha\cdot\sin\frac{i\pi}{n}\,,$$

$$A_{36} = \frac{6K_2}{h_2}\sec\alpha(\gamma_2 - 4\alpha_1\gamma_2 - 4\alpha_3) - \frac{6\bar{K}}{\bar{h}_2}\tan\alpha(4-D_i) + \frac{24K}{h^2}\tan\alpha\,,$$

$$A_{37} = \frac{6K_2}{h_2}\sec\alpha(1+\gamma_1 - 4\alpha_1\gamma_1 - 4\alpha_2) - \frac{6\bar{K}}{\bar{h}}\tan\alpha(4-D_i) + \frac{12K}{h}\tan\alpha\,,$$

$$\boldsymbol{P}_3 = -\tan\alpha\cdot\boldsymbol{S}_m\boldsymbol{R}_i[\Delta P_r + \nabla C_r]\,,$$

$$A_{41}=-\frac{Gt}{4}D_i\,,\quad A_{42}=\frac{Gt}{2}\sin\frac{i\pi}{n}\,,\quad A_{43}=\left\{-\frac{\bar{N}}{\bar{h}^2}+\frac{Gt\bar{h}}{6}\left(\frac{m\pi}{l}\right)^2\right\}\sin\frac{i\pi}{n}\,,$$

$$A_{44}=-\frac{\bar{N}}{2\bar{h}^2}(4-D_i)-\frac{Gt\bar{h}}{12}\left(\frac{m\pi}{l}\right)^2(2+D_i)-\left(\frac{2N}{h^2}-\frac{Gth}{6}\left(\frac{m\pi}{l}\right)^2\right),$$

$$A_{45}=-\frac{6K}{\bar{h}^2}\tan\alpha D_i\,,\quad A_{46}=\frac{12\bar{K}}{\bar{h}^2}\tan\alpha\sin\frac{i\pi}{n}\,,$$

$$A_{47}=\frac{6\bar{K}}{\bar{h}}\tan\alpha\sin\frac{i\pi}{n}\,,\quad A_{48}=\frac{6K_2}{h_2}\sec\alpha(1+\delta_1)-\frac{3\bar{K}}{\bar{h}}\tan\alpha\cdot D_i$$

$$\boldsymbol{P}_4=-\tan\alpha\cdot\boldsymbol{S}_m\boldsymbol{S}_i[\nabla P_r+\Delta\bar{C}_r]\,,$$

$$A_{51}=-\left\{\frac{\bar{N}}{2}\left(\frac{m\pi}{l}\right)^2\left(1-\frac{D_i}{6}\right)+\frac{1}{2}(N+N_1+2N_2)\left(\frac{m\pi}{l}\right)^2+\frac{Gt}{2\bar{h}}D_i\right\},$$

$$A_{52}=-\left\{\frac{\bar{N}}{6}\left(\frac{m\pi}{l}\right)^2-\frac{Gt}{\bar{h}}\right\}\sin\frac{i\pi}{n}\,,\quad A_{53}=\frac{Gt}{2}\sin\frac{i\pi}{h}\left(\frac{m\pi}{l}\right)^2,$$

$$A_{54}=\frac{Gt}{4}D_i\left(\frac{m\pi}{l}\right)^2,\quad A_{55}=-\frac{h_2}{2}(N_1+N_2)\cos\alpha\left(\frac{m\pi}{l}\right)^4,$$

$$A_{61}=\left\{-\frac{\bar{N}}{6}\left(\frac{m\pi}{l}\right)^2+\frac{Gt}{\bar{h}}\right\}\sin\frac{i\pi}{n}\,,$$

$$A_{62}=-\left\{\frac{\bar{N}}{12}(2+D_i)\left(\frac{u\pi}{l}\right)^2+\frac{1}{6}(N+N_1+6N_2)\left(\frac{m\pi}{l}\right)^2+\frac{2Gt}{h}+\frac{Gt}{2\bar{h}}(4-D_i)\right\}$$

$$A_{63}=\frac{Gt}{4}D_i\left(\frac{m\pi}{l}\right)^2,\quad A_{64}=\frac{Gt}{2}\sin\frac{i\pi}{n}\left(\frac{m\pi}{l}\right)^2,$$

$$A_{66}=-h_2\left(\frac{N_2}{2}+\frac{N_1}{6}\right)\cos\alpha\left(\frac{m\pi}{l}\right)^4,\quad A_{75}=-\frac{6\bar{K}}{\bar{h}}\sin\frac{i\pi}{n}\,,$$

$$A_{78}=-2\bar{K}\sin\frac{i\pi}{n},\quad \boldsymbol{P}_7=\boldsymbol{S}_m\boldsymbol{R}_i[\nabla C_r]\,,$$

$$A_{76}=2K_2\gamma_2-12K_2\alpha_1\gamma_2-\frac{12K}{h}-12K_2\alpha_3+\frac{12\bar{K}}{\bar{h}}\left(1-\frac{D_i}{4}\right),$$

$$A_{77}=6\bar{K}\left(1-\frac{D_i}{6}\right)+6K+4K_2-12K_2\alpha_2+2K_2\gamma_1-12K_2\alpha_1\gamma_1\,,$$

$$A_{85}=\frac{3\bar{K}}{\bar{h}}D_i\,,\quad A_{86}=-\frac{6\bar{K}}{\bar{h}}\sin\frac{i\pi}{n}\,,\quad A_{87}=-2\bar{K}\sin\frac{i\pi}{n}\,,$$

$$A_{88}=\bar{K}(2+D_i)+2K+4K_2+2K_2\delta_1\,,\quad \boldsymbol{P}_8=\boldsymbol{S}_m\boldsymbol{S}_i[\Delta C_r]\,,$$

$$\Delta\bar{C}_r=-\frac{1}{\bar{h}}(C_{2r+1,2r+2}+C_{2r+2,2r+1}-C_{2r,2r-1}-C_{2r-1,2r})\,,$$

$$\nabla\bar{C}_r=-\frac{1}{\bar{h}}(C_{2r+1,2r+2}+C_{2r+2,2r+1}+C_{2r,2r-1}+C_{2r,2r-1})+\frac{2}{h}(C_{2r,2r+1}+C_{2r+1,2r})\,,$$

and the solution is given by double Fourier series as follows:

$$\begin{bmatrix} \nabla \dot{u}_r \\ \Delta \dot{u}_r \\ \nabla v_r \\ \Delta v_r \\ \nabla w_r \\ \Delta w_r \\ \nabla \theta_r \\ \Delta \theta_r \end{bmatrix} = \frac{4}{ln} \sum_{m=1}^{\infty} \sum_{i=0}^{n} \boldsymbol{U} \cdot [\mathrm{I}] \cdot \begin{bmatrix} \sin \frac{i\pi r}{n} \\ \cos \frac{i\pi r}{n} \\ \cos \frac{i\pi r}{n} \\ \sin \frac{i\pi r}{n} \\ \sin \frac{i\pi r}{n} \\ \cos \frac{i\pi r}{n} \\ \cos \frac{i\pi r}{n} \\ \sin \frac{i\pi r}{n} \end{bmatrix} \sin \frac{m\pi x}{l}, \tag{46}$$

where [I] is unit matrix.

Numerical Examples

In order to obtain the numerical results of presenting formulas, the simply supported one-way ribbed plate, on which a partially distributed load standing for a tire load, is considered. The following values are taken, in the calculation.

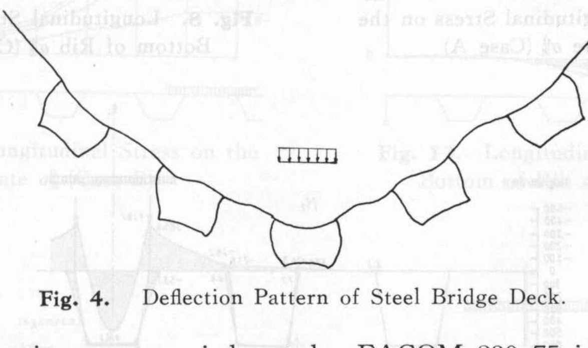

Fig. 4. Deflection Pattern of Steel Bridge Deck.

The computation was caraied out by FACOM 230–75 in Hokkaido University.

1) Symmetrical load with respect to the center of rib section (case A)

$E = 2.1 \times 10^6$ kg/cm^2, $\nu = 0.0$, $G = 8.1 \times 10^5$ kg/cm^2, $l = 287.5$c m,

$t = 12$ mm, $t_1 = 6$ mm, $h = \bar{h} = 30$ cm, $h_1 = 20$ cm, $h_2 = 22.8$ cm,

$q = 8$ kg/cm^2, $a = 50$ cm, $c = 20$ cm, $m = 45$ terms.

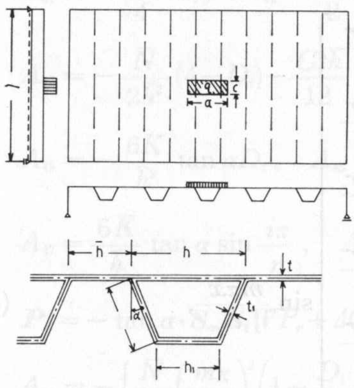

Fig. 5. Symmetrical Load (Case A)

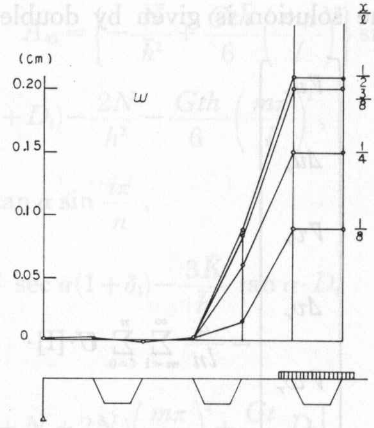

Fig. 6. Deflection w (Case A)

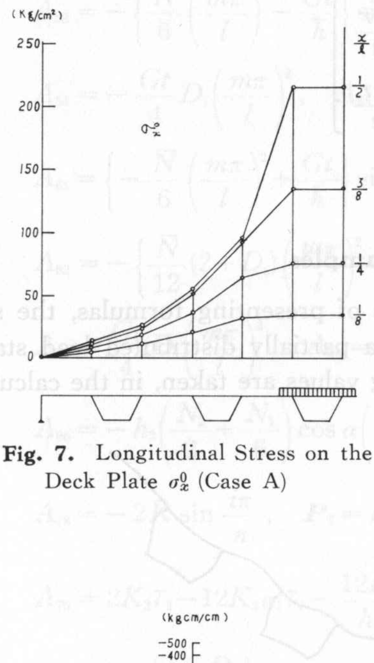

Fig. 7. Longitudinal Stress on the Deck Plate σ_x^0 (Case A)

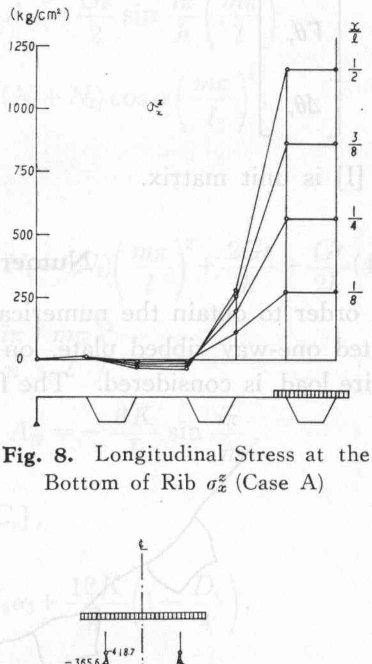

Fig. 8. Longitudinal Stress at the Bottom of Rib σ_x^z (Case A)

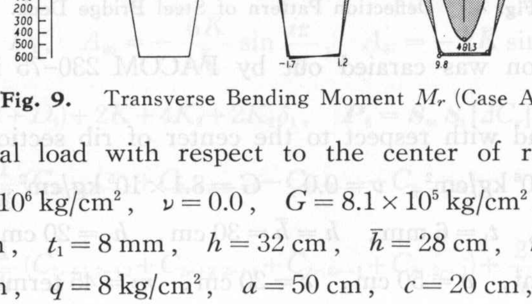

Fig. 9. Transverse Bending Moment M_r (Case A)

2) Antisymmetrical load with respect to the center of rib section (case B)

$$E = 2.1\times10^6\ \mathrm{kg/cm^2}\,,\quad \nu = 0.0\,,\quad G = 8.1\times10^5\ \mathrm{kg/cm^2}\,,\quad l = 287.5\ \mathrm{cm}\,,$$

$$t = 12\ \mathrm{mm}\,,\quad t_1 = 8\ \mathrm{mm}\,,\quad h = 32\ \mathrm{cm}\,,\quad \bar{h} = 28\ \mathrm{cm}\,,\quad h_2 = 22.8\ \mathrm{cm}\,,$$

$$h_1 = 20\ \mathrm{cm}\,,\quad q = 8\ \mathrm{kg/cm^2}\,,\quad a = 50\ \mathrm{cm}\,,\quad c = 20\ \mathrm{cm}\,,\quad m = 45\ \text{terms.}$$

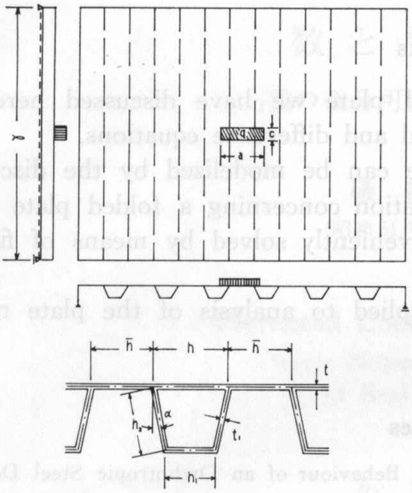

Fig. 10. Anti-symmetrical Load (Case B)

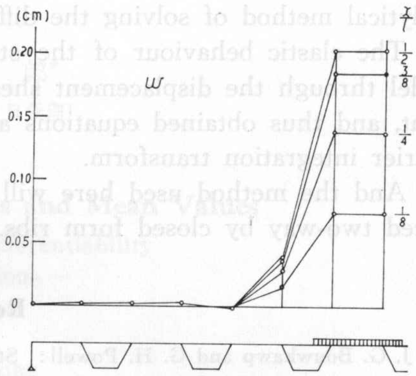

Fig. 11. Deflection w (Case B)

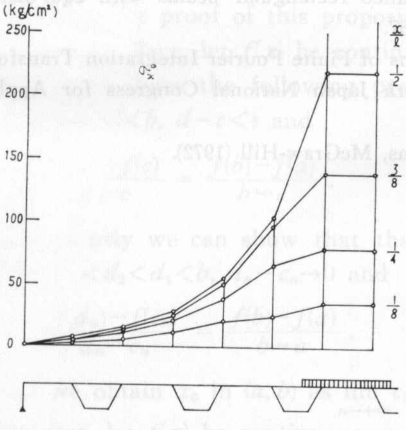

Fig. 12. Longitudinal Stress on the Deck Plate σ_x^0 (Case B)

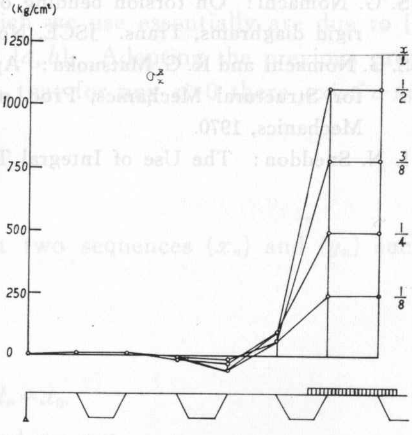

Fig. 13. Longitudinal Stress at the Bottom of Rib σ_x^z (Case B)

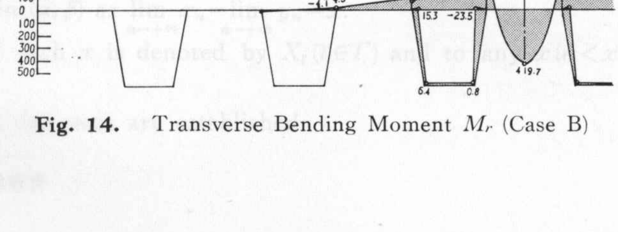

Fig. 14. Transverse Bending Moment M_r (Case B)

Remarks

The solutions of the one-way ribbed plate we have discussed here is analytical method of solving the differential and difference equations.

The elastic behaviour of the structure can be modelized by the discrete model through the displacement shear equation concerning a folded plate element, and thus obtained equations are conveniently solved by means of finite Fourier integration transform.

And the method used here will be applied to analysis of the plate reinforced two-way by closed form ribs.

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