Addendum and corrigendum to the paper "On a uniqueness theorem for the differential equation $u' = f(t, u)$ in a Banach space"

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In the paper mentioned above the statement and the proof of the example need to be modified as follows:

$$u' = f(t, u) = \begin{cases} 1 + \dfrac{1}{1+\sqrt{u}} & (0 \leqq t \leqq T,\ u \geqq 0) \\ 2 & (0 \leqq t \leqq T,\ u < 0), \end{cases}$$

$$u(0) = 0 .$$

The assertion $V(t, u) = \dfrac{-1}{2\sqrt{u}(1+u)^2}$ should be replaced by

$$V(t, u) = \frac{-1}{2\sqrt{u}(1+\sqrt{u})^2}\left(1 + \frac{1}{1+\sqrt{u}}\right).$$

Hence, for $(t, u), (t, v) \in (0, T) \times U = (0, T) \times \{u\,;\, u > 0\}$,

$$\big(f(t, u) - f(t, v),\ V(t, u) - V(t, v)\big)$$

$$= -\frac{(\sqrt{u}-\sqrt{v})^2}{2\sqrt{uv}\,[(1+\sqrt{u})(1+\sqrt{v})]^3}\left\{1 + 2(\sqrt{u}+\sqrt{v}) + u + \sqrt{uv} + v + \frac{1+3(u+v+\sqrt{u}+\sqrt{v}+\sqrt{uv}) + (u+v)(\sqrt{u}+\sqrt{v})}{(1+\sqrt{u})(1+\sqrt{v})}\right\} \leqq 0 .$$

This example, however, obviously satisfies

$$\big(u - v,\ f(t, u) - f(t, v)\big) \leqq 0 \ \text{ for } (t, u),\ (t, v) \in (0, T) \times U .$$

Thus the uniqueness of solutions to the above equation follows from the monotonicity arguments. We therefore give here another example. Consider the following scalar differential equation

$$u' = f(t, u) = \begin{cases} a(t) + \dfrac{\sqrt{u}}{1+\sqrt{u}} & (0 \leqq t \leqq T,\ u \geqq 0) \\ a(t) & (0 \leqq t \leqq T,\ u < 0), \end{cases}$$

$$u(0) = 0 ,$$

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where $a\in C[0, T]\cap C^1(0, T)$ and $a(t)\geqq 0$ for $t\in[0, T]$. In this case

$$V(t, u)=a'(t)+\frac{1}{2\sqrt{u}(1+\sqrt{u})^2}\left(a(t)+\frac{\sqrt{u}}{1+\sqrt{u}}\right)$$

for $(t, u)\in(0, T)\times U$, where $\{U=u\,;\,u>0\}$.

Hence, for (t, u), $(t, v)\in(0, T)\times U$, we have

$$\big(f(t, u)-f(t, v),\ V(t, u)-V(t, v)\big)$$

$$=-\frac{(\sqrt{u}-\sqrt{v})^2}{2[(1+\sqrt{u})(1+\sqrt{v})]^3}\left\{\frac{a(t)\,[1+2(\sqrt{u}+\sqrt{v})+\sqrt{uv}+v+u]}{\sqrt{uv}}\right.$$

$$\left.+\frac{3(1+\sqrt{u}+\sqrt{v})+u+\sqrt{uv}+v}{(1+\sqrt{u})(1+\sqrt{v})}\right\}\leqq 0\,.$$