A Study on Transient Combustion of Fuel Spray*

— On Diesel Combustion —

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In heat engines, i.e., boilers, gas turbines, diesel engines etc., the burning of the injected fuel spray is a very important problem. Since an intermittent injection is adopted especially in a diesel combustion, it is important to solve its transient combustion of spray in order to improve the performances of engines.

In this paper, the calculatons of the combustion rate of fuel spray were carried out by assuming that the fuel spray was simply a gathering of a single droplet and each droplet was exposed to the thermal reactions in the presence of the oxygen. As the calculations, it was cleared out that the general behavior of the transient combustion in a diesel engine could be explained qualitatively by the methods of calculations stated in this paper.

1. Introduction

In the diesel combustion, the combustion rate due to the combustion of spray affects the performances to a large extent[1),2)]. Therefore a precise solution of the transient combustions has been required. A few works of this kind have been reported hitherto[3),4)], but most of them are not theoretical owing to the complexity of the combustion phenomena.

On the other hand, the expereimental methods in which the suitable graphic patterns or certain functions are nearly allied to the practical combustion rates, are also used[4)].

In this paper, a theory which combined the evaporation theory of a single droplet and the theory of thermal reactions, was applied to the combustion of the spray. As a result of calculations, qualitative agreement could be obtained between the calculated combustion rates and the experimental ones.

2. Process of calculations

Nomenclature

Φ : Constant of reaction rate

C : Concentration

n : Order of reaction

B : Total fuel quantity injected

t, τ and z : Time respectively

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$Q(z)$: weight of fuel
M: Molecular weight
P: Pressure
T: Temperature
V: Volume
R: Gas constant
G: Weight
E: Activity energy
H: Calorific value
λ: Exces air factor or heat conductivity
k: Coefficient of reaction
β: Rate of evaporation
D_p: Coefficient of diffusion
P_s: Saturated vapor pressure of fuel
C_v: Specific heat of gas in constant volume
D: Diameter of droplet
Sh: Sherwood number
Sc: Schmidt number
Re: Reynolds number
Nu: Nusselt number
α: Heat-transfer coefficient
l: Evaporation heat
γ: Specific weight
L: Stoichiometric air
F: Area
K: Ratio of specific heat

Subscripts

f: Fuel
o_2: Oxygen
e: At combustion end
0: Standard value or initial point
d: Surface of droplets
v: Vapor
i: At injection
b: Heat released by combustion
g: Gas
c: Heat by cooling

Assuming that the thermal reactions in combustion processes occur simply between the fuel and the oxygen, the next general equation holds;

$$\frac{d[C_f]}{dz} = -\Phi[C_f]^{n_f}[Co_2]^{n_{o2}} \tag{1}$$

Defining σ' and x as the ratio of fuel injected and the ratio of fuel burned respectively

$$\sigma' = \frac{Q_i(z)}{B}, \qquad x = \frac{Q_b(z)}{B} \tag{2}$$

From eq. (2), $[C_f]$ and $[Co_2]$ become

$$\left.\begin{aligned} [C_f] &= \frac{(\sigma' - x)B}{M_f V} \\ [Co_2] &= \frac{0.232(\lambda_0 - x)LB}{Mo_2 \cdot V} \end{aligned}\right\} \tag{3}$$

Assuming n_f, $n_{o_2}=1$ in eq. (1) and substituting eq. (3) into eq. (1), eq. (1) is expressed as,

$$\frac{d[C_f]}{dz} = -\frac{0.232 \cdot \Phi \cdot L \cdot B}{M_f \cdot Mo_2} \frac{(\sigma' - x)(\lambda_0 - x)}{V^2} \tag{4}$$

From the relation $\frac{d[C_f]}{dz} = -\frac{B}{M_f V}\frac{dx}{dz}$, one has

$$\frac{dx}{dz} = \frac{0.232 \cdot L \cdot B \cdot \Phi}{Mo_2} \frac{(\sigma' - x)(\lambda_0 - x)}{V} \tag{5}$$

Defining Y as

$$Y = \frac{0.232 \cdot L \cdot B \cdot \Phi}{Mo_2 \cdot V}$$

eq. (5) becomes

$$\frac{dx}{dz} = Y(\sigma' - x)(\lambda_0 - x) \tag{6}$$

Applying the relation $\xi = \lambda_0 - x$ to eq. (6), the next Bernoulli's equation holds ;

$$\frac{d\xi}{dz} + Y(\sigma' - \lambda_0) = -Y\xi^2 \tag{6$'$}$$

Accordingly, a general analytical solution for eq. (6)$'$ is obtained as follows ;

$$x = \lambda_0 - \frac{\psi}{\int Y\psi dz + \zeta} \tag{7}$$

$$\text{where,} \quad \psi = \exp\left(-\int Y(\sigma' - \lambda_0)\, dz\right)$$

As stated above, when assuming that the injection fuel is directly exposed to the thermal reactions, an analytical solution (7) can be obtained. But the satisfactory explanations for the behavior of diesel combustions were impossible by using the solution (7).

Thus, an assumption that the injected fuel is not directly exposed to the thermal reactions but the evaporated part of injected fuel is exposed to the thermal reactions with oxygen, seems to be more suitable to a considerable extent for solving combustion problems of this kind. Under this assumption, the following calculations were carried out.

Defining σ as the ratio of evaporated fuel to B, σ becomes,

$$\sigma = \frac{Q_v(z)}{B} \tag{8}$$

On the other hand, though Φ is expressed in general by the relation of Arrhenius, the next relation of Φ is obtained considering the effects of higher pressure in the combustion chamber[3);

$$\Phi = \mathrm{k}\left(\frac{P}{P_o}\right)^2 \exp\left(-\frac{E}{RT}\right) \tag{9}$$

Substitution of eq. (8) and (9) into eq. (5) yields;

$$\frac{dx}{dz} = \frac{0.232\cdot k\cdot L\cdot B}{Mo_2}\left(\frac{P}{P_o}\right)^2 \exp\left(-\frac{E}{RT}\right)\frac{(\sigma - x)(\lambda_0 - x)}{V} \tag{10}$$

In order to calculate the variable σ in eq. (10), the following calculations of the evaporation process of single droplet are first carried out.

$$dG_f = \beta\cdot F_d(P_s - P_o)\, dt$$

Upon putting $\beta = \dfrac{D_p\cdot Sh}{D}$, $F_d = \pi\cdot D^2$ and $P_o = 0$ into the above eq., the next equation is obtained;

$$dG_f = \pi\cdot D\cdot D_p\cdot P_s\cdot Sh\cdot dt \tag{11}$$

$$\begin{aligned} \text{where,}\quad & Sh = 2(1 + Sc^{1/3} Re^{1/2}) \\ & D_p = 0.121\times 10^{-4}\left(\frac{T}{273}\right)\left(\frac{P_o}{P}\right) \\ & P_s = 6.0\times 10^7\cdot \exp\left(\frac{-4.15\times 10^3}{T_d}\right) \end{aligned} \tag{12}$$

Using the relation $dG_f = -d(\pi/6\cdot\gamma_f\cdot D^3)$, eq. (11) becomes

$$-\int_{D0}^{D} D dD = \int_0^{\tau} \frac{2\cdot D_p\cdot P_s\cdot Sh}{\gamma_f}\, dt$$

Defining $\Omega = \dfrac{4\cdot D_p\cdot P_s\cdot Sh}{\gamma_f}$, the next equation holds;

$$D^2 = D_o^2 - \int_0^{\tau} \Omega dt \tag{13}$$

Further, by using eq. (13), dG_f can be expressed as follows,

$$\begin{aligned} dG_f &= -\frac{\pi}{2}\cdot\gamma_f\cdot D^2\cdot dD \\ &= \frac{\pi}{4}\cdot\Omega\cdot\gamma_f\sqrt{D_o^2 - \int_0^t \Omega\cdot dt}\; dt \end{aligned}$$

Defining $\theta(\tau)=\frac{G_f(\tau)}{G_o}$, $\theta(\tau)$ will be expressed as

$$\theta(\tau)=\frac{3}{2}\cdot\frac{1}{D_o^2}\int_0^{\tau}\Omega\cdot\sqrt{D^2-\int_0^t\Omega\cdot dt}\,dt \tag{14}$$

$$\text{where,}\quad \theta(\tau)=1 \text{ when} \int_0^t \Omega dt \geqq D_o^2$$

By using eq. (14) and the relation $\tau=z-t$, the following relationship between $Q_v(z)$ and $Q_i(t)$ can be obtained;

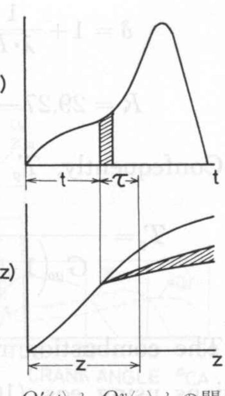

図1 $Q_i'(t)$ と $Q^v(z)$ との関係

Fig. 1. Relation between $Q_i'(t)$ and $Q_v(z)$

$$Q_v(z)=\int_0^z Q'_i(t)\cdot\theta(z-t)\,dt \tag{15}$$

$$\text{where,}\quad Q'_i(t)=dQ_i(t)/dt.$$

Namely, $\sigma(z)$ in eq. (10) will be expressed as

$$\sigma(z)=\frac{3}{2B}\frac{1}{D_o^3}\int_0^z Q_i'(t)\left[\int_0^{z-t}\sqrt{D_o^2-\int_0^t\Omega\cdot dt}\,dt\right]dt \tag{16}$$

On the other hand, as for the surface temperature of droplet T_d in eq. (12), the next equation of heat balance holds;

$$C_f\cdot G_f\cdot dT_d=\pi\cdot D^2\cdot\alpha(T-T_d)\,dt-1_f\cdot dG_f$$

Using the relation $\alpha=\frac{\lambda_g}{D}Nu$, the next equation is obtained;

$$C_f\cdot G_f\cdot dT_d=\pi\cdot D\cdot\lambda_g\cdot Nu\cdot(T-T_d)\,dt-l_f\cdot dG_f \tag{17}$$

$$\text{where,}\quad Nu=2(1+Pr^{1/3}\,Re^{1/2})$$

$$l_f=60+0.56\,T_d+0.3(T-T_d)-0.5\,T_{do}$$

Since the combustion occurs under non-steady conditions especially in diesel combustion, the following equations hold for the variables $P_g(z)$ and $T_g(z)$.

From the equation of heat balance in the cylinder, eq. (18) will be obtained.

$$H\cdot B\cdot\frac{dx}{dz}-\frac{dQ_c}{dz}=\frac{d}{dz}(G_g\cdot C_v\cdot T_g)+A\cdot P_g\cdot\frac{dV}{dz} \tag{18}$$

Applying $P_g\cdot V=G_g\cdot R\cdot T_g$ to eq. (18), the following differential equation will be derived.

$$\frac{dP_g}{dz}+K\cdot P_g\cdot\frac{1}{V}\cdot\frac{dV}{dz}=\frac{K-1}{AV}\left(H\cdot B\cdot\frac{dx}{dz}-\frac{dQ_c}{dz}\right) \tag{19}$$

Further, the following relations will hold;

$$\lambda(z)=\frac{G_{go}\cdot H}{B\cdot x\cdot L}$$

$$\delta = 1 + \frac{1}{\lambda \cdot L}$$

$$R = 29.27 - \frac{0.14}{\lambda}$$

Confequently, T_g can be expressed approximately as

$$T_g = \frac{P_g \cdot V}{G_{go}\left(1 + \frac{B \cdot x}{G_{go} \cdot H}\right)\left(29.27 - \frac{0.14 \cdot B \cdot x \cdot L}{G_{go} \cdot H}\right)} \tag{20}$$

The combustion rate $\frac{dx}{dz}$ can be obtained by means of the numerical calculations using eqs. (10), (16), (17), (19) and (20).

Besides, main values of the constants used in the calculations were as follows; $k = 2.5 \times 10^{11}\ \mathrm{sec}^{-1}$, $E = 4 \times 10^4$ kcal/kmol and $\lambda_e = 1.5$.

And the the calculations were carried out under the assumption of $Re = 0$, *i.e.*, $Sh = Nu = 2$.

3. Results of calculation and discussions

The simplest model of the evaporation process was first treated. The calculations were carried out under such assumptions that the pressure P_g and the temperature T_g of the circumstances around the droplets did not change, and these were the values at the beginning of the fuel injection. Besides, the surface temperature of droplets T_d was also assumed not to change, so that the heat quantity supplied to the droplets was consumed only for the evaporation of fuel droplets, namely $C_f \cdot G_f \cdot dT_d = 0$ in eq. (17).

The relations between the diameter of the droplets and the combusion rates calculated under the conditions stated above, are shown in Fig. 2. In this figure, the combustion rates were expressed with the heat released per unit crank angle at 1200 rpm of engine speed instead of the heat released per unit time, since the expressions using the heat released per unit crank angle for combustion rates might help understand the combustion rate more easily than that using the heat released per unit time in the diesel combustions. It is seen in Fig. 2 that as decreasing the diameter of the droplets the maximum value of combusion rate increases and the combustion duration shortens. In this case, the position where the maximum rate of diffusion combustion was seen, coincided with the one where the fuel injection ended.

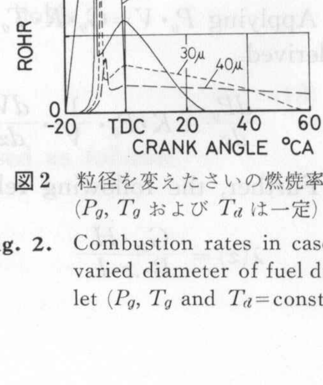

図 2 粒径を変えたさいの燃焼率 (P_g, T_g および T_d は一定)

Fig. 2. Combustion rates in case of varied diameter of fuel droplet (P_g, T_g and T_d=const.)

And this tendency corresponded with the results of experiments reported by Prof. Matsuoka[4]. As stated above, the combustion rate was calculated using the simplest model for the combustion of fuel spray. And as a results of calculations, the tendency of two stage combustion which consisted of the pre-mixed combustion following the diffusion combustion and which was usually used in diesel combustions, could be also explained.

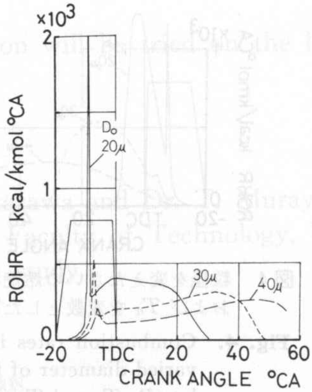

図3 粒径を変えたさいの燃焼率 (P_g, T_gを変数としT_dは一定)

Fig. 3. Combustion rates in case of varied diameter of fuel droplet (P_g, T_g=variables and T_d=const.)

Next, the calculations were carried out with the assumptions that the surface temperature of droplets was unvaried and that the ambient temperature T_g and pressure P_g around the droplets were the variables which affected the evaporation process of droplets.

Fig. 3. shows the relationships between the combustion rates and the diameter of droplets. In this case, the changes of the combustion rates by varying the diameter of droplets resembled comparatively the case shown in Fig. 2. But, the combustion rates at earlier stages of combustion decreased and that at the later stages of combustion tended to increase as compared with the case shown in Fig. 2.

This tendency can be surmised from the relation of $D_p = f(T_g/P_g)$.

Namely, the combustion rate at the early stage of combustion depends rather on P_g than T_g owing to comparatively lower lower temperature, so that D_p seems to decrease and the combustion rate at earlier stages of combustion decreases. On the other hand, since the D_p seems to increase owing to increasing temperature caused by the combustion, the combustion rate at later stages of combustion is considered to increase. Further, in case where the diameter of droplets was large, the combustion duration tended to shorten at a larger extent as compared with the combustion duration obtained under the assumption that P_g and T_g were unvaried.

The further calculations for obtaining the combustion rates were carried out with the assumptions that the surface temperature of droplets T_d was a variable which was given in eq. (17) and that P_g and T_g were also treated as variables.

Fig. 4. shows the relations between the diameter of droplets and the combustion rates calculated under the assumption stated above. As compared with a case where T_d was not treated as a variable, the combustion rates at earlier stages of combustion decreased furthermore and simultaneously the one at later stages of combustion tended to increase in consideration of T_d. And the part of combustion due to the pre-mixed combustion was hardly recognized in the case when the diameter of droplets was 40 μ.

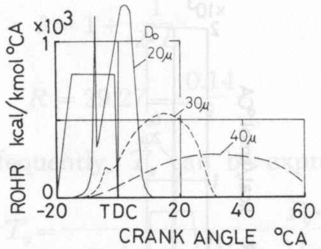

図4 粒径を変えたさいの燃焼率（P_g, T_g および T_a を変数とした）

Fig. 4. Combustion rates in case of varied diameter of fuel droplet (P_g, T_g and T_a = variables)

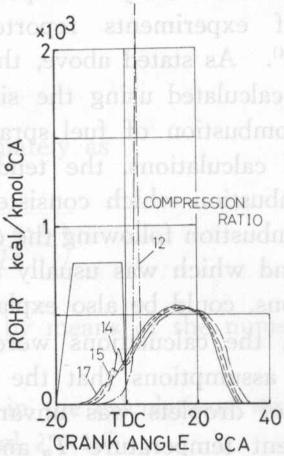

図5 圧縮比を変えたさいの燃焼率

Fig. 5. Combustion ratios in case of varied compression ratios

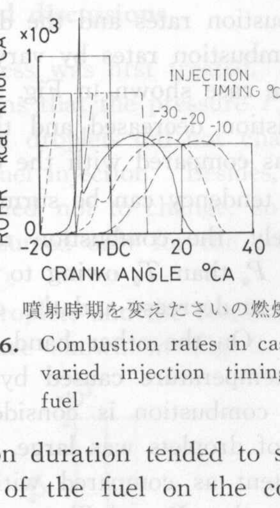

図6 噴射時期を変えたさいの燃焼率

Fig. 6. Combustion rates in case of varied injection timing of fuel

The effects of the compression ratios on combustion rates calculated under the same conditions, are shown in Fig. 5.

As decreasing the compression ratio, the combustion rate at earlier stages of combustion increased and the space between the beginning of injection and the position where the initial maximum rate of combustion appeared, also increased.

Namely, as the compression ratio decreased, the so-called ignition lag and the initial maximum rate of combustion increased. As for this, the experimental tendencies of combustion rates which showed almost the same tendency with calculated one, were obtained as reported in a previous report[5]. In addition, as decreasing the compression ratio, the combustion duration tended to shorten.

Further the effects of the injection timing of the fuel on the combustion rates are shown in Fig. 6.

As advancing the injection timing, the combustion rate owing to the premixed combustion increases. And this tendency could be observed usually in diesel combustion[5].

4. Conclusions

In this paper, an attempt to calculate the combustion rates for transient combustion of the spray especially for the diesel combustion, was carried out by using a theory that combined the evaporation theory of a single droplet and the theory of thermal reactions.

As the results of calculations, it was made clear that the general tendencies which appeared in the transient combustion of diesel engines, could be explained qualitatively.

Further calculations for the transient combustion will be tried on the basis of these calculations in the future.

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