FAST ACA ALGORITHM BASED ON CENTROID CUTTING METHOD FOR SOLVING ELECTROMAGNETIC SCATTERING

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This paper proposes a fast algorithm of the moment method based on the centroid cutting method. We use the centroid cutting method to solve the singularity problem which has always plagued the matrix behavior. At the same time this paper uses the Adaptive Cross Approximation algorithm (ACA) to compress the matrix. Numerical result demonstrates this new method owns a high accuracy. Compared with the traditional method of moments, this method has higher efficiency without reducing the accuracy.

Key Words : *Method of moments(MoM);Adaptive Cross Approximation algorithm(ACA); Radar Cross Section (RCS);*

1. INTRODUCTION

Electromagnetic scattering problem has always been a focus issue of calculation in computational electromagnetic[1-2]. Method of moments (MoM) is a representative method that has a high accuracy and it used to solve the electromagnetic scattering problem.

However, the calculation of the method of moments will have the singularity problem caused by the Green function[3]. And the calculated matrix is a huge matrix with a very dense feature, which directly causes our calculations to become very slow.

The centroid method of moments can be used to solve the electromagnetic scattering of electrically large objects. This method does not need to solve the complex basis function and Green's function directly and it can convert the integral operation equation in the impedance matrix elements into a summation operation. Further more, it will sparse the matrix and improve the calculation efficiency while keeping the accuracy at a high standrad. The adaptive cross approximation (ACA)[4] method is fully algebraic, and it can be easily inserted into existing MoM codes. Finally the calculation time will be reduced effectively by using this method[5].

This paper introduces a kind of method which uses the centroid method of moments to solve the singularity problem of Green's function, and then uses the adaptive cross approximation algorithm to speed up the calculation of the matrix, and finally obtain the accurate calculation results and it is confirmed that it can greatly reduce the calculation time.

2. FORMULATION

The MoM is based on Stratton-Chu formulations. When using the method of moments to calculate electromagnetic scattering, we usually use electric field integral equation to analyze the problem. And we use to apply the RWG basis function to generate the impedance matrix by using the electric field integral equation[6], the formulation will be obtained:

$$Z_{mn} = l_m [j\omega (\frac{A_{mn}^{+} \cdot \rho_m^{c+}}{2} + \frac{A_{mn}^{-} \cdot \rho_m^{c-}}{2}) + \phi_{mn}^{-} - \phi_{mn}^{+}] \quad (1)$$

In the above formulation, m and n correspond to two side elements. "$\cdot$" represents dot product, l_m is the side length of m . The $\rho_m^{c\pm}$ is the vector between the free vertex $v_m^{c\pm}$ and the center point $r_m^{c\pm}$ of the two triangles $T_m^{\pm}$ of the edge element m .Where A is the magnetic vector potential and ϕ is the scalar potential:

$$A_{mn}^{\pm}=\frac{\mu}{4\pi}\left[\frac{l_n}{2A_n^+}\int_{T_n^+}\rho_n^+(r')g_m^{\pm}(r')dS'+\frac{l_n}{2A_n^-}\int_{T_n^-}\rho_n^-(r')g_m^{\pm}(r')dS'\right] \quad (2a)$$

$$\phi_{mn}^{\pm}=-\frac{1}{4\pi j\omega\varepsilon}\left[\frac{l_n}{A_n^+}\int_{T_n^+}g_{mn}^{\pm}(r')dS'-\frac{l_n}{A_n^-}\int_{T_n^-}g_{mn}^{\pm}(r')dS'\right] \quad (2b)$$

Both of the above two formulations contain Green function, which is defined as follows:

$$g_m^{\pm}(r')=\frac{e^{-jk\left|r_m^{c\pm}-r'\right|}}{4\pi\left|r_m^{c\pm}-r'\right|} \quad (3)$$

When $r_m^{c\pm}$ and r' are almost equal, the singularity problem will arise.

Here, we use a centroid cutting method to solve the singularity problem which is a very efficient and convenient way. The principle diagram of the centroid cutting method is shown in the figure 1 as below:

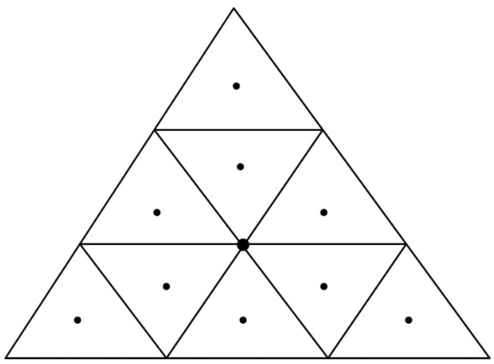

Fig.1 Schematic diagram of centroid cutting method

The principle of the centroid cutting method is: use the RWG basis function to triangulate the surface of the conductor, and the integral of the function $g(r)$ on the original triangle can be expressed as:

$$\int_{T_m}g(r)dS=\frac{A_m}{9}\sum_{k=1}^{9}g(r_k^c) \quad (4)$$

r_k^c represents the center of each sub angle, A_m represents the area of the original triangle, the magnetic vector position and electrical standard position can be simply rewritten as:

$$A(r)=\mu\cdot\frac{A}{9}\cdot G(R)\sum_{k=1}^{9}J(r_k^c) \quad (5a)$$

$$\phi(r)=\frac{j}{\omega\varepsilon}\cdot G(R)\cdot\frac{A}{9}\sum_{k=1}^{9}\nabla'\cdot J(r_k^c) \quad (5b)$$

Finally, the matrix equation can be expressed in the following formulation[7]:

$$Z_{mn}=jk\eta\int_{T_m^++T_m^-}f_m(r)\cdot\frac{A}{9}\left[G(R)\sum_{k=1}^{9}f_n(r_k^c)+\frac{1}{k^2}\nabla G(R)\cdot\sum_{k=1}^{9}\nabla\cdot f_n(r_k^c)\right]ds \quad (6)$$

In this way, the impedance matrix is divided and reformed by the centroid cutting method. The matrix filled by this method has no singularity problem. However, how to quickly solve the matrix is still a problem[8]. Next, we will use a fast algorithm to solve the matrix problem[9].

The adaptive cross-approximation algorithm is a fast and effective method. It is generally used by people for fast calculation of the matrix. It has a great effect on the fast calculation of the matrix.

Using this algorithm can quickly decompose the dense impedance matrix and speed up the final calculation[10-11].

The process of this algorithm is given as follows:

Initialization:

1) Initialize the index value of the first row $I_1=1$, $\tilde{Z}=0$.

2) Define an error matrix $\tilde{R}$ of intermediate variables and initialize the first row of the approximate error matrix:

$$\tilde{R}(I_1,:)=Z(I_1,:)$$

3) Find the maximum value in the first row to determine the first column index J_1:

$$|\tilde{R}(I_1,J_1)|=\max_j(|\tilde{R}(I_1,j)|)$$

4) Get the first row of the matrix:

$$v_1=\tilde{R}(I_1,:)/\tilde{R}(I_1,J_1)$$

5) Initialize the first column of the approximate error matrix:

$$\tilde{R}(:,J_1)=Z(:,J_1)$$

6) Get the first column of the matrix:

$$u_1=\tilde{R}(:,J_1)$$

$$\|\tilde{Z}^{(1)}\|^2=\|\tilde{Z}^{(0)}\|^2+\|u_1\|^2\|v_1\|^2$$

7) Find the maximum value in the first column to determine the second row index I_2:

$$|\tilde{R}(I_2,J_1)|=\max_i(|\tilde{R}(i,j_1)|),i\neq I_1$$

The kth cycle:

1) Update the I_k row of the approximate error matrix:

$$\tilde{R}(I_k,:) = Z(I_k,:) - \sum_{l=1}^{k-1} (u_l)_{I_k} v_l$$

2) Find the maximum value in the I_k row to determine the kth column index J_k:

$$|\tilde{R}(I_k,J_k)| = \max_j (|\tilde{R}(I_k,j)|), j \neq J_{1,....,}J_{k-1}$$

3) Get the kth row of the V matrix:

$$v_k = \tilde{R}(I_k,:) / \tilde{R}(I_k,J_k)$$

4) Update the J_k column of the approximate error matrix:

$$\tilde{R}(:,J_k) = Z(:,J_k) - \sum_{l=1}^{k-1} (v_l)_{J_k} u_l$$

5) Get the k column of the U matrix:

$$u_k = \tilde{R}(:,J_k)$$

$$|\tilde{Z}^{(k)}\|^2 = \|\tilde{Z}^{(k-1)}\|^2 + 2\sum_{j=1}^{k-1} |u_j^T u_k| \cdot |v_j^T v_k| + \|u_k\|^2 \|v_k\|^2$$

6) Judge convergence error:

if $\|u_k\|\|\|v_k\| \le \varepsilon \|\tilde{Z}^{(k)}\|$,and the iteration ends.

Otherwise, it starts to find the next line.

7) Find the next row index I_{k+1}:

$$|\tilde{R}(I_{k+1},J_k)| = \max_i (|\tilde{R}(i,J_k)|), i \neq I_1,.....,I_k$$

After the algorithm is completed, the principle is to construct two low-order matrices $U^{m\times k}$ and $V^{k\times n}$ so that their product $\tilde{Z}^{m\times n}$ is approximately equal to the original matrix:

$$Z^{m\times n} \approx \tilde{Z}^{m\times n} = U^{m\times k} V^{k\times n} \quad (7)$$

3. NUMERICAL RESULTS

In this paper, all the computations were finished on a computer of 2.60GHz and 8 GB for Inter(R) Core(TM) i7-4720HQ. The terminating tolerance of ACA is selected as ε=10^{-3}. This paper simulates and gives the results. We have considered two examples. The first example is a metal ball made of perfect electrical conductor(PEC),and the second one is a Seam metal ball cone which Made of PEC.

In the first example, we set the sphere with a radius of 2m, the incident wave is incident vertically from top to the bottom with 300MHz. And finally there are 17590 metallic triangles in free space.

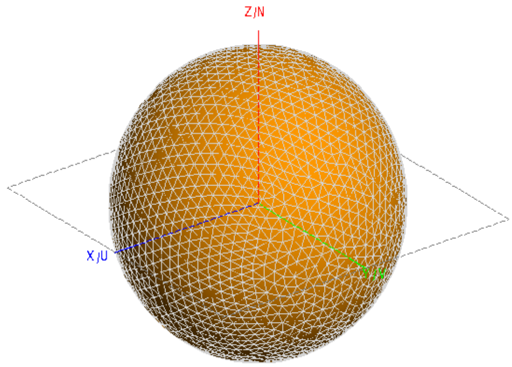

Fig.2 Conductor Metal Ball

As shown in Figure 2 above, it is a schematic diagram of the modeling of the sphere.

The feko software is a professional software for calculating electromagnetics with powerful functions. The data compared in this article are all compared with the results of feko. The following figure 3 shows the comparison of the radar cross section (RCS) calculated by the sphere. As shown in the following schematic diagram, the comparison of the algorithm in this article is the simulation result of feko software.

"Theta" represented by the abscissa is the observation angle, and the ordinate is RCS.

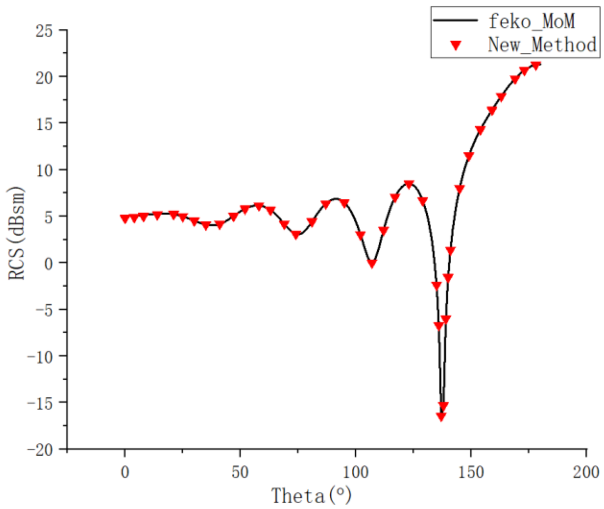

Fig.3 RCS of the metal ball

In the second example, due to a gap, our parameters have become more. In order to facilitate simulation and testing, the radius of each of our objects is set to be much smaller. The radius of the sphere is 0.08m and the length of the cone is about 0.6m. In order to ensure accuracy, we set the frequency to 3000MHz and obtain 17600 metal triangles in free space by setting the grid size.

Similar to Figure 2 and Figure 3, Figure 4 is the modeling diagram of the metal cone with seams, and Figure 5 is the RCS calculation result.

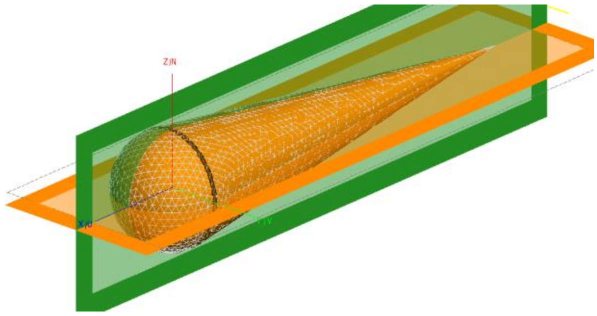

Fig.4 Seam metal ball cone

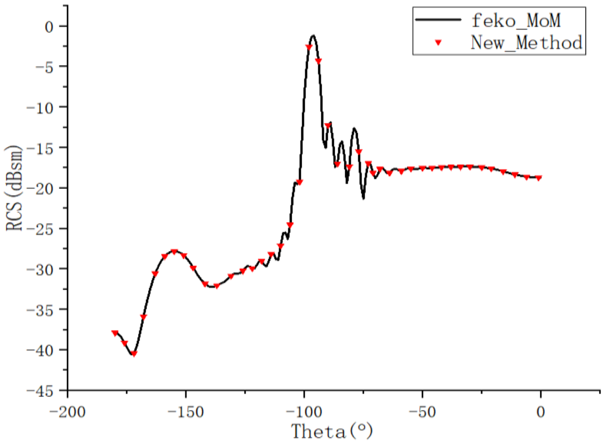

Fig.5 RCS of the Seam metal ball cone

Figures 3 and 5 show that the new method has a high degree of fit with the traditional method of moments, indicating the accuracy of the method, and the obtained data are shown below. The unknowns in the table are the number of unknown triangle pairs obtained when modeling the object. All data are accurate to single digits.

Table 1 Memory

Example	Unknowns	MoM	New method
sphere	17590	2739MB	873MB
Seam metal ball cone	17600	2342MB	679MB

Table 2 Total time(s)

Example	Unknowns	MoM	New method
sphere	17590	1388s	481s
Seam metal ball cone	17600	2568s	925s

It can be seen from the above two tables and simulation examples that the new method greatly reduces the memory and analysis calculation time compared to the moment method while ensuring the calculation accuracy.

CONCLUSION

The new method proposed in this article is relatively effective. Based on the method of moments, the new method in this paper further uses the centroid cutting method to solve the singularity problem of electrically large sizes., and accelerates the calculation of the matrix through ACA. The final result shows that this method greatly reduces the calculation time and CPU usage of memory.

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