AN IMPROVED BLIND SOURCE SEPARATION ALGORITHM IN COMPLEX DOMAIN

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In this paper, based on the linear instantaneous mixed blind source separation mathematical model, we propose an improvement to the traditional complex maximization of non-Gaussianity algorithm (CMN), introduce the augmented Lagrangian multiplier to fuse FastICA's inequality constraints with the CMN algorithm, and use the Quasi-Newton algorithm to update iterations. Compared with the existing blind source separation algorithms the bounded component analysis (BCA) and the Quasi-Newton complex maximization of non-Gaussianity algorithm (NCMN), the experimental simulation results show that the improved NCMN algorithm (n-NCMN) has a better separation effect, especially under the condition of lower signal-to-noise ratio (SNR).

Key Words : *blind source separation, the complex maximization of non-Gaussianity, Lagrangian multiplier, the Quasi-Newton*

1. INTRODUCTION

Blind source separation algorithms have been widely used in various fields such as communication countermeasures[1)], speech recognition[2)], and Electroencephalogram (EEG)[3-5)]. The so-called blind source separation is to recover the source signal from the multi-channel received signal without knowing or less knowing the prior information of the source signal. According to different mathematical models, blind source separation problems are divided into two categories, one is the linear blind source separation problem in which the source signal is linearly mixed, and the other is the nonlinear blind source separation problem. The linear blind source separation problem is divided into linear instantaneous blind source separation and convolution blind source separation.

Bounded Component Analysis (BCA) and Independent Component Analysis (ICA) algorithms are two types of unsupervised learning methods to solve the problem of blind source separation. The BCA algorithm[6)] uses the observation vector to project into a one-dimensional linear space to estimate the source signal. The algorithm does not have strong requirements on the source signal, and only needs to meet the bounds of the source signal, but its convergence speed is very slow. There are two main categories of commonly used ICA algorithms. One is based on high-order cumulants. For example, the Fourth-order Blind Indentification algorithm (FOBI)[7)], although this method is very effective, it requires different kurtosis values; the Joint Approximate Diagonalization of Eigenmatrices algorithm (JADE)[8)], because the algorithm uses a fourth-order cumulant, it is very simple to expand the complex situation, but when the number of sources increases, the performance will be affected. The other is based on non-linear functions. In the framework of maximum likelihood (ML)[9)], information maximization (Infomax)[10)], mutual information maximization or non-Gaussian (negative entropy) frame maximization[11)], the nonlinear function is used to implicitly generate high-order statistics. And in the case of real values, the maximum convergence and accuracy of the non-Gaussian method are better, such as FastICA, but the algorithm is not ideal for separation in the complex domain.

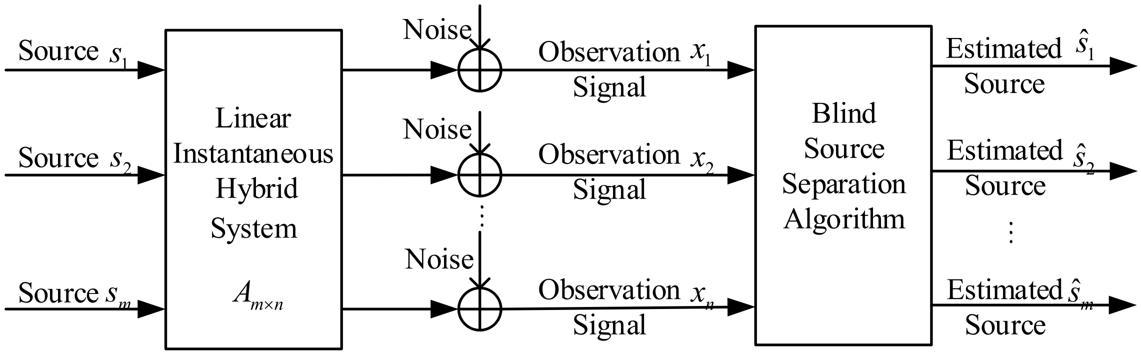

Fig.1 Mathematical Model of Blind Source Separation.

In order to overcome the shortcomings of the above blind source separation algorithm, the Complex Maximization of Non-Gaussianity (CMN) algorithm came into being[12)]. CMN algorithm mainly uses the complex analytic function under the non-Gaussian framework, and introduces the composite maximization of two non-Gaussian algorithms. It realizes the blind source separation in the complex domain by connecting the maximization of non-Gaussian (negative entropy) with mutual information. Although the classic CMN algorithm is a very effective method in complex ICA, it is based on a noiseless linear instantaneous mixture model. When there is noise in the channel, the separation performance will deteriorate. However, in many practical situations, the mixed signal will be contaminated by noise. In response to the above problems, we add Lagrangian constraints to remove the influence of noise on the separation effect, and use the Quasi-Newton method to update iteratively. After a lot of experimental simulations, the results show that the separation performance of improved NCMN algorithm (n-NCMN) is better in various scenarios, especially in the environment of low signal-to-noise ratio.

The rest of this article is organized as follows: Section 2 introduces the blind source separation model and blind source separation algorithm, Section 3 verifies the superiority of the n-NCMN algorithm through simulation, and Section 4 gives the conclusion.

2. THEORETICAL FRAMEWORK

The linear instantaneous mixed blind source separation model is shown in **Fig.1**. m source signals passes through the instantaneous linear mixing system, the signals which is interfered by noise is transmitted to the receiving end to obtain n observation signals. Then the BSS algorithm is used to separate the observed signal, and finally we can obtain the estimation of the source signal.

According to **Fig.1**, build a mathematical model of blind source separation, the observations meet the familiar linear mixing model

$$X = AS + N \tag{1}$$

where $S=[s_1,s_2,\dots,s_m]^T \in F^{m\times L}$ denotes m source signals, each source signal includes L sample points, $s_i=[s_{i1},s_{i2},\dots,s_{iL}]\in F^L$, $i=1,2,\dots,m$. $[\cdot]^{\mathrm{T}}$ is transpose, $F=R$ denotes the set of real domain, $F=C$ denotes the set of complex domain.

$A\in F^{m\times n}$ is the hybrid matrix, which denotes the linear instantaneous hybrid system in the **Fig.1**.

$X=[x_1,x_2,\dots,x_n]^T \in F^{n\times L}$ is the observation matrix, which represents n observation signals, and the number of sample points in each signal is L .

$N\in F^{n\times L}$ is additive white Gaussian noise with the same dimension as X .

(1) Whitening pretreatment

In most blind source separation algorithms, the source signal must first be whitened. For the sake of generality, the source signals need to meet the conditions of zero mean, unit variance, and non-correlation. The whitening pretreatment process is spatial decorrelation, which can further simplify the problem of blind source separation. The observed signal after whitening preprocessing is defined as:

$$Y = BX \tag{2}$$

where, B is the pre-whitening matrix satisfying $B=\Lambda^{-1/2}U^H$, Y is the observation matrix after pre-whitening, Λ is the matrix composed of m the largest eigenvalues of $R_{xx}=E\left\{(X-\mu_x)(X-\mu_x)^H\right\}$, $\mu_x=E\{X\}$ is the expectation of the source signal, and U is the matrix composed of the corresponding eigenvectors.

Therefore, the estimate of the source signal can be defined as:

$$\hat{S} = W^H Y \quad (3)$$

where $W \in F^{n\times m}$ is the separator matrix.

(2) Objective function

The objective function of the classic blind source separation algorithm (ICA) is:

$$J(W) = E\left\{G\left(\left|W^H Y\right|^2\right)\right\} \quad (4)$$

where $G(v)$ is a smooth non-linear function. You can choose from the following three expressions:

$$G_1(v) = \log(0.1+v) \quad (5)$$

$$G_2(v) = \tanh(v) \quad (6)$$

$$G_3(v) = v^2 \quad (7)$$

We update the traditional objective function by introducing the Lagrangian factor, and use the inequality constraints to maximize the application of known prior knowledge, thereby improving the effect and the stability of blind source separation. The updated objective function is defined as:

$$\begin{aligned} J(W) &= E\left\{G\left(\left|W^H X\right|^2\right)\right\} \\ &+ \frac{1}{2\alpha}\left(\left(\max\left\{\theta h(W,c)+\alpha,0\right\}\right)^2 - \alpha^2\right) \end{aligned} \quad (8)$$

among them, $h(W,c) = \beta - \varepsilon(W,c) \le 0$, and $f(W) = WW^H - 1 = 0$, they are respectively inequality constraints and equality constraints for the blind source separation model. α is th $g(v)$ e learning rate, θ augmented Lagrangian multiplier, $\varepsilon(W,c)=\left|W^H c\right|^2$, c is the reference column vector, β is the threshold of the inequality.

(3) Algorithm iteration

By using the Quasi-Newton methods, we derive the oneunit iteration of n-NCMN as:

$$\alpha \leftarrow \max\left(\theta h(W,c)+\alpha,0\right) \quad (9)$$

$$\begin{aligned} W^{i+1} = &-E\left\{G^*(v)g(v)x\right\} \\ &+E\left\{g(v)g^*(v)\right\}W^i \\ &+E\left\{YY^H\right\}E\left\{G^*(v)g'(v)\right\}W^* \\ &+sign(\alpha)*2\theta\left|\left(W^i\right)^H c\right|^2\left(\left(W^i\right)^H c\right)^* * c \end{aligned} \quad (10)$$

where, $v = W^H Y$, we select the non-linear function $G(v) = v^2$, $g(v)$ and $g'(v)$ denote the first-order and second-order derivative of $G(v)$ and i represent the number of iterations. Schmidt orthogonalization is performed on the obtained separation matrix W, namely $W \leftarrow \left(WW^H\right)^{1/2} W$.

3. EXPERIMENTAL RESULT

In order to quantify the separation performance of the blind source separation algorithm, the similarity coefficient is selected as the algorithm performance evaluation index. The similarity coefficient is defined as:

$$\xi_{ij} = \frac{\left|\sum_k^L \hat{s}_i(k)s_j(k)\right|}{\sqrt{\sum_k^L \hat{s}_i^2(k)\sum_k^L s_j^2(k)}} \quad (11)$$

where, $\hat{s}_i$ is the i_{th} estimated source, $i = 1,2,\ldots,m$, s_j is the i_{th} original source, $j = 1,2,\ldots,m$, L represents the number of sample points of the source, ξ_{ij} is any constant within the range $[0,1]$. When $\xi_{ij} = 1$, the separated signal $\hat{s}_i$ is consistent with the original signal s_j or satisfies a constant multiple relationship in amplitude; if $\xi_{ij} = 0$, the separated signal $\hat{s}_i$ and the original signal s_j are independent of each other. The closer the similarity coefficient is 1, the better the separation performance.

The matrix composed of similarity coefficients is recorded as the similarity coefficient matrix Γ:

$$\Gamma \in R^{m\times m},\ [\Gamma]_{ij} = \xi_{ij} \quad (12)$$

In order to remove the influence of the permutation uncertainty caused by the algorithm, this paper chooses the improved average similarity coefficient $\bar{\xi}$ as the evaluation index of the separation performance. The similarity coefficients of all signals and the corresponding estimated signals are added and averaged to obtain the average similarity coefficient $\bar{\xi}$:

$$\bar{\xi} = E\left(\sum_i^m max\left(\Gamma_j\right)\right),\ i = 1,\ldots,m \quad (13)$$

where, Γ_j represents the i_{th} column of the matrix Γ.

After many Monte Carlo simulations, it is found that the source power ratio and the number of sample points have little effect on the separation effect of the separation algorithm, which can be almost ignored(as shown in **Fig.2** and **Fig.3**). Therefore, the source power ratio and the number of sample points are fixed during the comparison experiment. This article mainly compares three algorithms (BCA, n-NCMN, NCMN) in different domains (real domain and complex number domain) and related source signals in Gaussian distribution and non-Gaussian distribution (binomial distribution) scenarios, and proves the improved separation effect of n-NCMN algorithm is better. It is worth noting that in the algorithm comparison experiment of related signal sources $SNR = 20dB$.

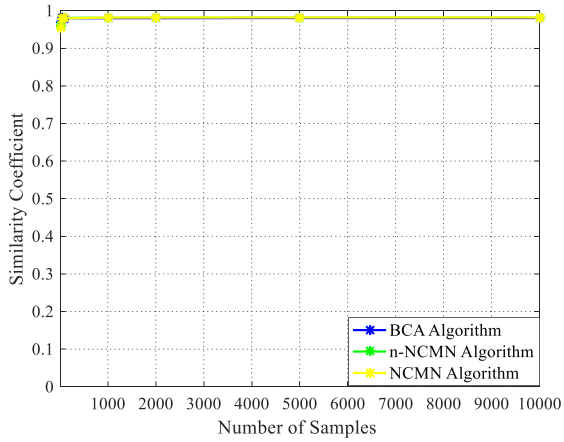

Fig.2 number of symbol comparison of three algorithms

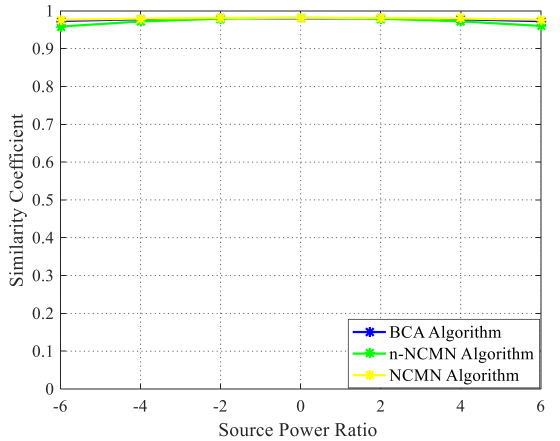

Fig.3 source power ratio comparison of three algorithms

Table 1 Gaussian distributed signal simulation parameters

Parameter	Value
Monte Carlo simulation times	100
Number of sources	2
Sample points	1000
Source power ratio	0dB
Signal-to-noise ratio	-8dB~30dB
Source type	real/complex/correlated Gaussian signals

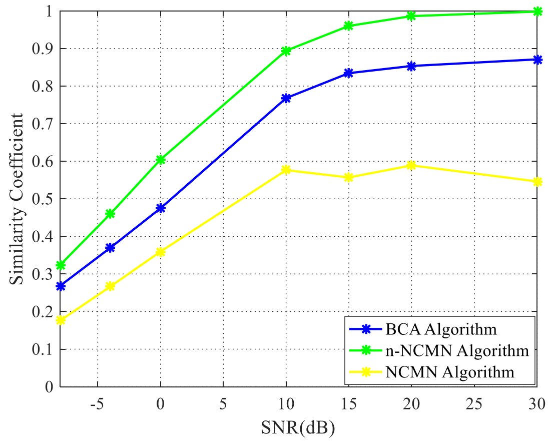

Fig.5 Complex Gaussian signal comparison of three algorithms

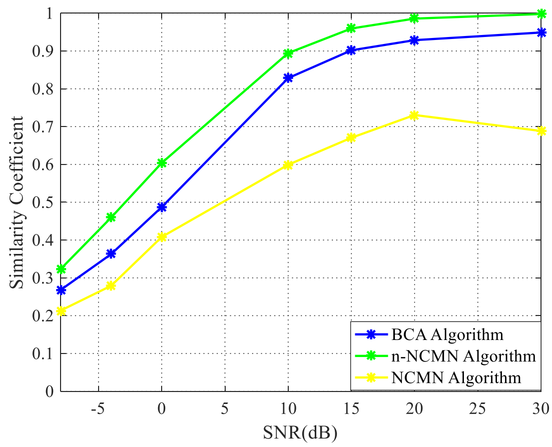

Fig.4 Real Gaussian signal comparison of three algorithms

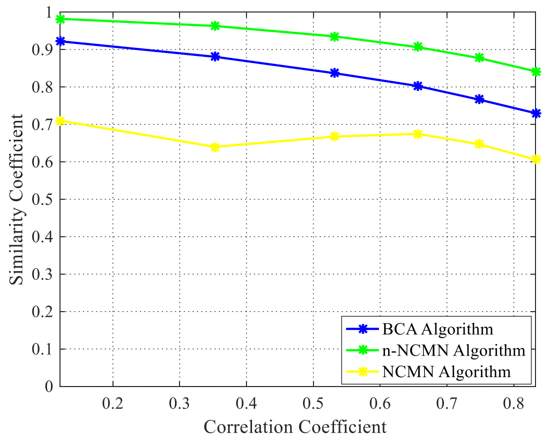

Fig.6 Correlated Gaussian signal comparison of three algorithms

(1) Algorithm simulation comparison of subject to Gaussian distribution

The experimental simulation parameters of Gaussian distributed signals are shown in **Table 1**, and the experimental results are shown in **Fig.4**, **Fig.5**, and **Fig.6**. The experimental results show that the effect of the n-NCMN algorithm is obviously more advantageous than the other two algorithms for separating source signals that obey the Gaussian distribution.

(2) Algorithm simulation comparison of signals subject to binomial distribution

The experimental simulation parameters of signals subject to non-Gaussian distribution (binomial distribution) are shown in **Table 2**. The experimental results are shown in **Fig.7**, **Fig.8**, and **Fig.9**. In the experiment of this section, the source signal satisfies S~B(1,0.5). The experimental results show that under the condition of low signal-to-noise ratio, the n-NCMN algorithm is better than the other two algorithms in separating signals that obey the binomial distribution.

4. CONCLUSION

According to the comparison results of experimental simulations, it is concluded that no matter what distribution the source signal obeys, whether it is in the real domain or the complex domain, or whether it is correlated, the separation effect of the n-NCMN algorithm is better than that of the NCMN and BCA algorithms in a noisy environment. Especially under the condition of low signal-to-noise ratio, compared with the traditional NCMN algorithm, the performance of n-NCMN is improved by at least 10%, and since n-NCMN is based on the FastICA framework, its convergence speed is very fast.

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Table 2 Binomial distributed signal simulation parameters

Parameter	Value
Monte Carlo simulation times	100
Number of sources	2
Sample points	1000
Source power ratio	0dB
Signal-to-noise ratio	-8dB~30dB
Source type	real/complex/correlated binomial signals

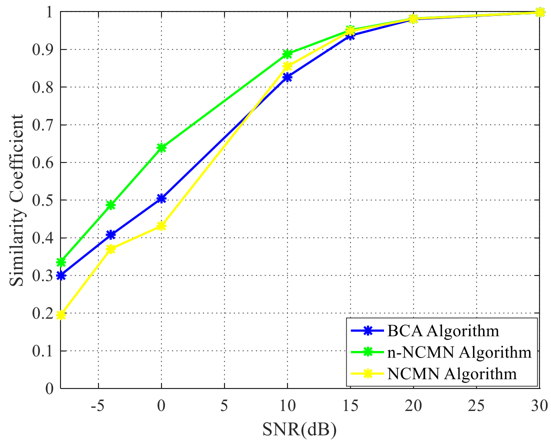

Fig.8 Complex Binomial distributed signal comparison of three algorithms

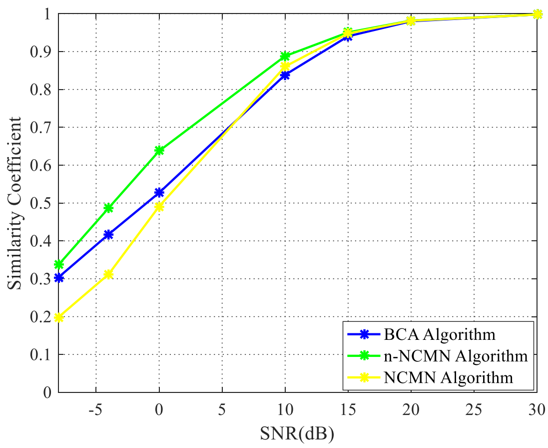

Fig.7 Real Binomial distributed signal comparison of three algorithms

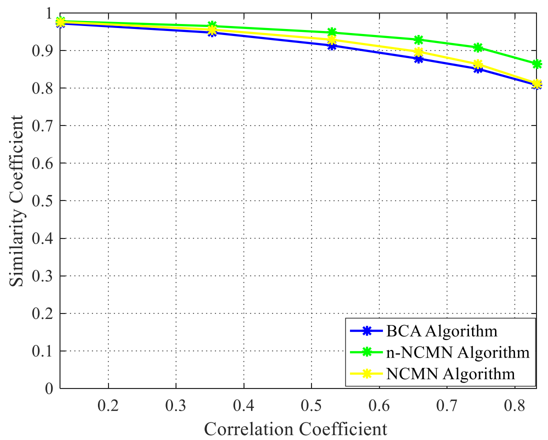

Fig.9 Correlated Binomial distributed signal comparison of three algorithms

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