

# Supplier selection based on process yield for contact lens manufacturing industry

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In this paper, a case study on the supplier selection problem for the contact lens manufacturing industry is investigated. Generally, when people attempt to compare the performance between different suppliers, quality level of the goods is the most popular and convincing criterion for customers. Contact lens belongs to the medical device on the market and it has been widely used for myopia. Since the poor-quality medical device might directly cause the potential hazard to the users, therefore the required quality of products is very strict which makes the conventional attributes method invalid. The author applied a variables method based on process yield index  $S_{pk}$  to conduct hypothesis testing for the supplier selection problem.

**Key Words:** *supplier selection problem, process yield index, contact lens, hypotheses testing.*

## 1. INTRODUCTION

Supplier selection is an important issue. Ho et al. (2010) pointed out that the popular criteria for supplier selection including quality, delivery, price/cost, manufacturing capability, service, management, technology, research and development (R&D), financial status, the flexibility of cooperation, company's reputation, business relationship, risk and safety and environment.

Among those mentioned factors, "quality" plays a main criterion in the purchases. Especially in today's competitive business environment, it is very essential to work with right suppliers in the supply chain which ensures the received submission has a high-quality level. Since the preliminary study on supplier selection problem by Dickson in 1966 (Dickson (1966)), various decision-making approaches have been developed to deal with the problems. The relevant investigations can be found in Degraeve (2000), De Boer et al. (2001), Aissaoui et al. (2007) and Tai and Wu (2012).

In this paper, the author utilized an exact process-yield index  $S_{pk}$  for the supplier selection problem in the contact lens industry. The

hypothesis testing between two given suppliers is presented based on the test statistic  $\Omega$ . For practical purposes, the practitioners from other industries can follow the operating procedure introduced in the illustration to implement the method.

## 2. PROCESS YIELD INDEX

The most commonly used process capability indices,  $C_p$ ,  $C_a$  and  $C_{pk}$  are well discussed in the past decades (Pearn et al. (1992); Kotz and Johnson (2002); Wu et al. (2009)). Since  $C_p$  can only measure the variation of process,  $C_a$  can only reflect the degree of centering, and  $C_{pk}$  can only provide a lower bound for evaluating process yield, Boyles (1994) proposed a yield-based index, namely  $S_{pk}$  for normally distributed processes and it is an exact measurement that has a one-to-one relationship between index value and process yield. The definition of  $S_{pk}$  can be seen as below:

$$S_{pk} = \frac{1}{3} \Phi^{-1} \left[ \frac{1}{2} \Phi \left( \frac{USL - \mu}{\sigma} \right) + \frac{1}{2} \Phi \left( \frac{\mu - LSL}{\sigma} \right) \right] \quad (1)$$

Table 1 shows various quality level based on

index value and the corresponding process yield as well as nonconformities in PPM (NCPMM) as follows:

**Table 1** Corresponding values among various index value, process yield and NCPMM.

$S_{pk}$	Process yield	NCPMM
1.00	0.997300204	2699.796
1.20	0.999681783	318.217
1.40	0.999973309	26.691
1.60	0.999998413	1.587
1.80	0.999999933	0.067
2.00	0.999999998	0.002

Note that in the real application, process parameters  $\mu$  and  $\sigma$  are usually unknown. Lee *et al.* (2002) proposed the following natural estimator of  $S_{pk}$ :

$$\hat{S}_{pk} = \frac{1}{3} \Phi^{-1} \left[ \frac{1}{2} \Phi \left( \frac{USL - \bar{x}}{S} \right) + \frac{1}{2} \Phi \left( \frac{\bar{x} - LSL}{S} \right) \right] \quad (2)$$

where  $\bar{x}$  and  $S$  are the sample mean and the sample standard deviation, respectively, which can be calculated by the collected sample items from a well-controlled production system, USL is the upper specification limit and LSL is the lower specification limit and  $\Phi$  is the cumulative probability function (CDF) of standard normal distribution. The exact sampling distribution is mathematically intractable. Lee *et al.* (2002) then used the Taylor expansion technique and take the first order of the expansion to obtain the approximate distribution of  $\hat{S}_{pk}$  as

$$\hat{S}_{pk} \xrightarrow{a} N \left( S_{pk}, \frac{(a^2 + b^2)}{36n [\phi(3S_{pk})]^2} \right) \quad (3)$$

where  $n$  is the sample size,  $a$  and  $b$  are the functions of  $C_p$  and  $C_a$  which can be calculated by:

$$a = \frac{1}{\sqrt{2}} \left\{ 3C_p (2 - C_a) \phi [3C_p (2 - C_a)] + 3C_p C_a \phi (3C_p C_a) \right\} \quad (4)$$

$$b = \phi [3C_p (2 - C_a)] - \phi [3C_p C_a] \quad (5)$$

To compare two given suppliers' performance (supplier A and supplier B), we consider the

hypotheses testing for contrasting the two  $S_{pk}$  values, which are calculated from the sample items of A and B:  $H_0 : S_{pk}^A \geq S_{pk}^B$  versus  $H_1 : S_{pk}^A < S_{pk}^B$  (or equivalent to  $H_0 : S_{pk}^B / S_{pk}^A \leq 1$  versus  $H_1 : S_{pk}^B / S_{pk}^A > 1$ ). Here, the process quality characteristic of two suppliers,  $X_A$  and  $X_B$  are independent normally distributed,  $X_A \sim N(\mu_A, \sigma_A^2)$  and  $X_B \sim N(\mu_B, \sigma_B^2)$ , then  $\bar{x}_A \sim N(\mu_A, \sigma_A^2/n)$  and  $\bar{x}_B \sim N(\mu_B, \sigma_B^2/n)$ . Consequently, the sample estimator of two suppliers can be defined as:

$$\hat{S}_{pk}^A = \frac{1}{3} \Phi^{-1} \left[ \frac{1}{2} \Phi \left( \frac{USL - \bar{x}_A}{S_A} \right) + \frac{1}{2} \Phi \left( \frac{\bar{x}_A - LSL}{S_A} \right) \right] \quad (5)$$

$$\hat{S}_{pk}^B = \frac{1}{3} \Phi^{-1} \left[ \frac{1}{2} \Phi \left( \frac{USL - \bar{x}_B}{S_B} \right) + \frac{1}{2} \Phi \left( \frac{\bar{x}_B - LSL}{S_B} \right) \right] \quad (6)$$

and the test statistic  $\Omega = \hat{S}_{pk}^B / \hat{S}_{pk}^A$  can be expressed as:

$$\Omega = \frac{\hat{S}_{pk}^A}{\hat{S}_{pk}^B} \sim \frac{N(\mu_A, \sigma_{A,S}^2)}{N(\mu_B, \sigma_{B,S}^2)} \quad (7)$$

Accordingly, the distribution of the test statistic  $\Omega$  is the convolution of two normal distributions, and its probability density function can be defined as follows (also see Lin and Pearn (2009) for the detailed derivation):

$$\begin{aligned} f_{\Omega}(r) &= \frac{1}{\sqrt{2\pi}} \left[ \sigma_{B,S}^2 + (\sigma_{A,S}^2 \cdot r^2) \right]^{-\frac{1}{2}} \\ &\times \left[ (\sigma_{B,S}^2 \cdot S_{pk}^A) + (\sigma_{A,S}^2 \cdot S_{pk}^B \cdot r) \right] \\ &\times \exp \left\{ \frac{1}{2} \left[ \left( \frac{S_{pk}^A}{\sigma_{A,S}^2} + \frac{S_{pk}^B \cdot r}{\sigma_{B,S}^2} \right) \right. \right. \\ &\quad \times \left. \left. \frac{(\sigma_{B,S}^2 \cdot S_{pk}^A) + (\sigma_{A,S}^2 \cdot S_{pk}^B \cdot r)}{\sigma_{B,S}^2 + \sigma_{A,S}^2 \cdot r^2} \right) \right. \\ &\quad \left. \left. - \frac{1}{2} \left( \frac{(S_{pk}^A)^2}{\sigma_{A,S}^2} + \frac{(S_{pk}^B)^2}{\sigma_{B,S}^2} \right) \right] \right\} \\ &\times \left[ 2\Phi \left( \frac{(\sigma_{B,S}^2 \cdot S_{pk}^A) + (\sigma_{A,S}^2 \cdot S_{pk}^B \cdot r)}{\sigma_{A,S} \cdot \sigma_{B,S} \sqrt{\sigma_{B,S}^2 + \sigma_{A,S}^2 \cdot r^2}} \right) - 1 \right] \\ &+ \left\{ \frac{\sigma_{A,S} \cdot \sigma_{B,S}}{\pi (\sigma_{B,S}^2 + \sigma_{A,S}^2 \cdot r)} \right. \\ &\quad \left. \times \exp \left[ -\frac{1}{2} \left( \frac{(S_{pk}^A)^2}{\sigma_{A,S}^2} + \frac{(S_{pk}^B)^2}{\sigma_{B,S}^2} \right) \right] \right\} \end{aligned} \quad (8)$$

where  $-\infty < r < \infty$ . Lin and Pearn (2009) have tabulated the corresponding critical value under the given sample size for making the decision (see Table 2).

**Table 2** Critical values for rejecting  $S_{pk}^B \leq S_{pk}^A$  with  $n_1 = n_2 = 30(10)200$  and  $\alpha = 0.05$  (Lin and Pearn (2009))

$n$	$c_0$	$n$	$c_0$
30	1.358	120	1.163
40	1.302	130	1.156
50	1.265	140	1.150
60	1.239	150	1.144
70	1.219	160	1.139
80	1.203	170	1.135
90	1.191	180	1.131
100	1.180	190	1.127
110	1.171	200	1.124

### 3. APPLICATION

#### (1) CONTACT LENS INDUSTRY

A Contact lens, a thin lens worn directly on the surface of the eyes which can correct vision or for cosmetic or therapeutic reasons. Contact lens belongs to a contact-type medical device that might cause potential hazards to the users. Therefore, before the product is ready on the market, it should pass the official verification, such as the International Standard Organization (ISO), Food and Drug Administration (FDA), Pharmaceutical and Medical Device Law (PMDL) or Taiwan FDA (TFDA).

The most well-known quality characteristic of contact lens is “diopter”. Diopter is a unit of measurement which determines the curvature degree (or called optical power) of spherical lens. A wrong diopter of lens may cause dizziness or fatigue to the users. Hence, the quality level of diopter is very important for customers in the purchases. Lensometer (see Figure 1) is commonly used to measure the lens diopter by placing the lens on the laser zone position (see Figure 2).



**Fig.1** Lensometer (Liu et al. (2020))



**Fig.2** Position (Liu et al. (2020))

#### (2) ILLUSTRATION

For a specific product, the target diopter is 1.75 with USL=2.00 and LSL=1.50. The minimal quality requirement of this product is  $S_{pk} = 1.00$  (or process yield=99.73% and NCPM=2700). Tables 2 and 3 show the collected data from two suppliers’ production line with sample size  $n_1 = n_2 = 100$ , and Figures 3 and 4 suggest that the collected data from both processes could be normally distributed under the *Anderson-Darling* test with  $p$ -value  $< 0.05$ .

To examine if the supplier B is superior to the supplier in terms of process yield, the hypothesis testing:

$$H_0 : \frac{S_{pk}^B}{S_{pk}^A} \leq 1$$

$$H_1 : \frac{S_{pk}^B}{S_{pk}^A} > 1$$

Based on the collected data, the sample means and sample standard deviations for both supplier A and supplier B are  $\bar{x}_A = 1.7581$  and  $S_A = 0.0771$ , and  $\bar{x}_B = 1.7475$  and  $S_B = 0.0637$ , respectively. The corresponding sample estimators are  $\hat{S}_{pk}^A = 1.0731$  and  $\hat{S}_{pk}^B = 1.3057$ . Thus, the test statistic is  $\Omega = \hat{S}_{pk}^B / \hat{S}_{pk}^A = 1.3057 / 1.0731 = 1.2167$ . By checking Table 2, the critical value is  $c_0 = 1.1180$  when  $n=100$ . Since the test statistic  $\Omega = 1.2167 > 1.1180$ , the comparison result concludes that supplier B performs better than supplier A with a 95% confidence level.

Please note that the data (or the production line) should ensure the existence of normality assumption and the process is in-control before applying the proposed method. The misestimate might occur if the abovementioned two assumptions are invalid.

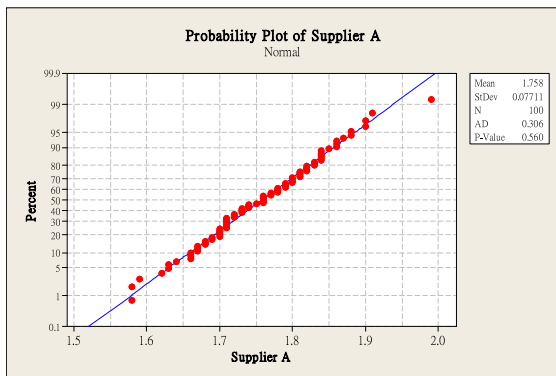
**Table 3** The collected sample data form supplier A (unit: diopter)

1.73	1.80	1.70	1.67	1.68	1.88	1.71	1.81	1.87	1.74
1.76	1.78	1.73	1.70	1.72	1.86	1.71	1.84	1.86	1.73
1.83	1.72	1.73	1.80	1.81	1.71	1.59	1.68	1.76	1.79
1.69	1.81	1.71	1.58	1.77	1.76	1.71	1.76	1.72	1.86
1.71	1.82	1.84	1.77	1.74	1.66	1.63	1.67	1.79	1.62
1.99	1.83	1.79	1.69	1.80	1.78	1.82	1.78	1.70	1.91
1.70	1.78	1.90	1.84	1.88	1.81	1.82	1.78	1.74	1.81
1.90	1.80	1.73	1.68	1.70	1.82	1.76	1.85	1.71	1.71
1.83	1.70	1.84	1.75	1.64	1.63	1.72	1.84	1.76	1.74
1.71	1.66	1.58	1.67	1.66	1.79	1.76	1.80	1.84	1.77

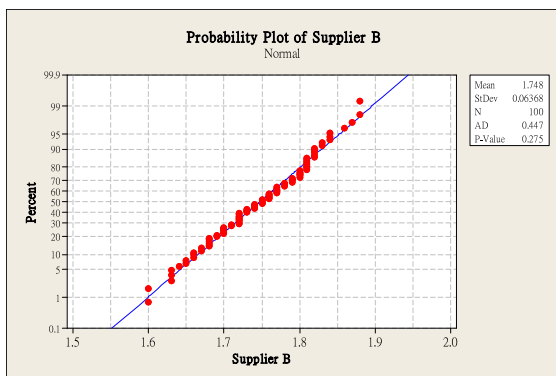
**Table 4** The collected sample data form supplier B (unit: diopter)

1.87	1.79	1.69	1.77	1.81	1.63	1.74	1.74	1.68	1.70
1.70	1.73	1.72	1.72	1.74	1.68	1.80	1.77	1.81	1.76
1.63	1.72	1.82	1.75	1.68	1.70	1.66	1.70	1.77	1.65
1.76	1.78	1.73	1.80	1.77	1.78	1.88	1.76	1.84	1.63
1.74	1.82	1.72	1.69	1.79	1.79	1.77	1.73	1.77	1.76
1.72	1.82	1.68	1.81	1.65	1.70	1.71	1.72	1.81	1.66
1.81	1.60	1.67	1.80	1.88	1.81	1.82	1.80	1.83	1.72
1.79	1.75	1.75	1.75	1.60	1.80	1.78	1.81	1.72	1.80
1.78	1.68	1.82	1.76	1.64	1.74	1.76	1.83	1.72	1.81
1.73	1.72	1.84	1.71	1.84	1.68	1.70	1.67	1.66	1.86

**Fig. 3** Normal probability plot of collected data from supplier A.



**Fig. 4** Normal probability plot of collected data from supplier B.



**4. CONCLUSION**

Supplier selection is the work of choosing a superior supplier to obtain the high-quality materials or components to support the outputs of organizations. In this paper, the author proposed a case study to demonstrate the supplier selection method based on process yield index  $S_{pk}$  to ensure the exact measure on process yield under a well-control and normally distributed production system. For practical use in the real application, the

user can follow the operating procedure introduced in the illustration to implement the method. Please note that the data should ensure the existence of normality assumption and the process is in-control or the decision might be misestimated.

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