# Efficiency Optimized Speed Control

# of Field Oriented Induction Motor Including Core Loss

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**Abstract: -** This paper presents an efficiency optimized speed control scheme of field oriented induction motor. In order to take core loss into account, we insert a core loss resistance in parallel to internal induced voltage branch into the d-q axes equivalent circuit. The optimum efficiency controller is to realize the optimum slip angular frequency to minimize the controllable losses composed of copper losses and core loss by rotor flux weakening under field oriented control. We construct the speed control system with efficiency optimization applied on optimal regulator theory. The effectiveness of this control scheme is confirmed by simulation results.

## 1. Introduction

The field oriented control theory of the induction motor drive contributes a special control method. With this control method, the induction motor drives are very fascinating and challenging subjects to research and development. Recently, high performance control and estimation techniques of field oriented induction motor have received wide attention to the researchers. Therefore, the induction motors can successfully replace expensive DC motors due to their simple robust construction, low inertia, high output power to weight ratios, immunity to heavy overloading and high performance at high speed of rotation. The orthogonal-axis physical model of the core losses and the magnetic saturation of the main flux path of electric machine have been suggested [1]. In consideration with core loss, decoupled flux and torque control do not take place in field oriented induction machines [2]. Due to the perturbation of field oriented control with taking core loss into account, some compensation should be considered which has already been investigated [2,3,4]. A direct-field-oriented induction motor with deadbeat rotor flux controller has been developed [4] with consideration of core loss and an adaptive control of the rotor flux variable frequency induction motor drive [5] has been examined in which flux provides the maximum efficiency. Also, an indirect field oriented induction motor with optimal control theory [6] has been constructed without considering core loss in which slip angular frequency provides maximum efficiency. The decoupled flux and toque controls have been confirmed under the assumption that core loss may be neglected [6,7,8].

In this paper, an orthogonal-axis model of induction motor taking core loss into account is derived with a parallel resistance connected to the internal induced voltage branch for representing the core loss and attempted to provide an excellent performance. This type of approach to core losses modeling was suggested in [1] and used in [2,3] for the analysis of steady state condition.

To overcome the parasitic effect of core loss, here an efficiency optimized speed control scheme of field oriented induction motor is proposed using the derived model. In the proposed control system, multi-input and multi-output optimal regulator theory was used to achieve simultaneously field oriented control, speed control and maximum efficiency control. Induction machine losses can be reduced substantially for any speed and load operating point by controlling the slip angular frequency in an indirect field oriented control system.

To achieve maximum efficiency, we concentrate on optimal slip angular frequency with the concept of field weakening and indirect field oriented control. Based on the theoretical knowledge and control scheme theory, simulation results were carried out.

The validity of the control scheme is verified by simulation results. The proposed control method can be implemented for speed control, which minimizes controllable losses.

## 2. Model of Induction Motor Taking Core Loss into Account

The stator and rotor voltage equations together with the equation of motion can be used to express system dynamics. In the model of field oriented control of induction motor considering core loss, a resistance $R_m$ is connected in parallel to internal induced voltage branch in d-q axes equivalent circuit. The d-q axis equivalent circuit of an induction motor in a synchronously rotating reference frame is shown in **Fig.1**. The conventional d-q axis model, which considers core loss of an induction motor in a synchronously rotating reference frame (called arbitrary reference frame), is represented by the following equations (1)- (4).

$$v_1 = r_1 i_1 + \ell_1 \frac{di_1}{dt} + j\omega_e \ell_1 i_1 + e_i \tag{1}$$

$$0 = r_2 i_2 + \frac{d\Phi_2}{dt} + j(\omega_e - \omega_m)\Phi_2 \tag{2}$$

$$e_i = R_m i_i = \frac{d\Phi_g}{dt} + j\omega_e \Phi_g \tag{3}$$

$$\begin{gathered} \tau_e = p\mathrm{Im}(i_2^{*}\Phi_2) \\ \frac{d\omega_m}{dt} = -\frac{D}{J}\omega_m + \frac{p}{J}(\tau_e - \tau_L) \end{gathered} \tag{4}$$

With the set of flux linkage equations (5)-(7) in order to consider core loss.

$$\Phi_1 = \ell_1 i_1 + \Phi_g \tag{5}$$

$$\Phi_2 = \ell_2 i_2 + \Phi_g \tag{6}$$

Where

$$\begin{gathered} \Phi_g = M i_m \\ i_i + i_m = i_1 + i_2 \end{gathered} \tag{7}$$

Here, $v_1$, $e_i$, $i_1$, $i_2$, $i_m$, $i_i$, $\Phi_1$, $\Phi_2$ and $\Phi_g$ vector can be expressed as $F = F_d + jF_q$ and the projection of F space vector is shown in Fig. A in Appendix. Here $e_i$ is internal induced voltage.

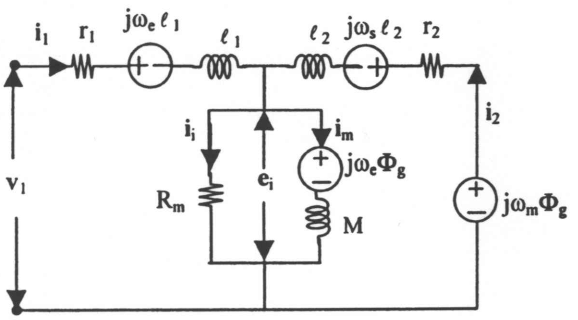



**Fig. 1.** Equivalent d-q axes Dc model circuit with core loss into consideration.

The foregoing equations are derived with assumption that main flux saturation is neglected. As a result, the relation between magnetizing current and magnetizing flux space vector is linear. Equations (1) through (7) provide available information for the analysis of efficiency optimized speed control and field oriented control of induction motor taking core loss into account.

## 3. State Equations

To represent the dynamic behavior of induction machines, the state equations are derived based on an orthogonal d-q axis co-ordinate, with the d-axis as real and the q-axis as imaginary. The state equations can be expressed as

$$\frac{d\omega_m}{dt} = -\frac{D}{J}\omega_m + \frac{p^2}{J}\tau_e - \frac{p}{J}\tau_L \tag{8}$$

$$\begin{aligned} \frac{di_1}{dt} &= -\frac{r_1 + R_m}{\ell_1} i_1 - \frac{R_m L_r}{M\ell_1} i_2 \\ &\quad + \frac{R_m}{M\ell_1}\Phi_2 - j\omega_e i_1 + \frac{1}{\ell_1} v_1 \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{di_2}{dt} &= -\frac{R_m}{\ell_2} i_1 - \frac{R}{\ell_2} i_2 + \frac{R_m}{M\ell_2}\Phi_2 \\ &\quad + j\frac{1}{\ell_2}\omega_m \Phi_2 - j\omega_e i_2 \end{aligned} \tag{10}$$

$$\frac{d\Phi_2}{dt} = -r_2 i_2 - j\omega_e \Phi_2 + j\omega_m \Phi_2 \tag{11}$$

The dynamic behavior of an induction motor can be expressed without considering equation (10). With core loss into consideration, this equation takes place in state equation to eliminate the degradation of dynamic response. From the view-point of control, here defined three inputs from state equations are the primary angular frequency $\omega_e$, applied orthogonal axis voltage in stator $v_{1d}$ and $v_{1q}$. Also, in general case, three outputs are the rotational angular frequency $\omega_m$, d-axis rotor flux $\Phi_{2d}$ and q-axis rotor flux $\Phi_{2q}$. But when we consider the case of efficiency optimized speed control, we take only two outputs except rotor d-axis flux and concentrate on slip angular frequency. The rotor d-axis flux and current are assumed as estimated or observed values.

## 4. Control System

To achieve desired output, it is needed to pay attention to the following control systems.

1. Field oriented control
2. Efficiency optimized speed control and
3. Application of optimal regulator theory.

### 4.1 Field Oriented Control

Considering core loss, the model of induction motor is shown in Fig. 1 and the resistance $R_m$ represents the core loss.

From Fig.1, the voltage equations of an induction motor with consideration of core loss are given by (1) to (3). The basic theoretical knowledge of field oriented method is illustrated in **Fig.2**. To achieve field oriented control, the q-axis component of the rotor flux should inevitably be set to zero (i.e. $\Phi_{2q} = 0$). It is shown that the rotor d-axis current is zero also (i.e. $i_{2d} = 0$)

To satisfy the above condition, the following equations are obtained from (1) through (3).

$$\Phi_{2d} = Mi_{1d} - \frac{M\ell_2}{R_m}\omega_e i_{2q} \tag{12}$$

$$\omega_s = -r_2\frac{i_{2q}}{\Phi_{2d}} \tag{13}$$

$$\tau_e = -p i_{2q}\Phi_{2d} \tag{14}$$

$$R_m = \frac{\omega_e^2}{P_i}(\ell_2^2 i_{2q}^2 + \Phi_{2d}^2) \tag{15}$$

Where $P_i$ is core loss.

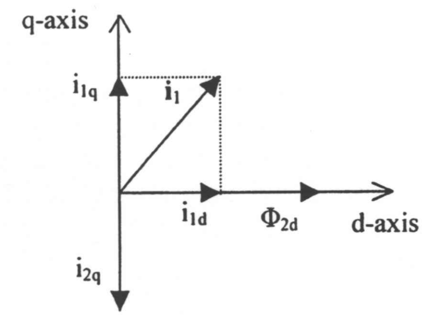



**Fig. 2.** Vector diagram in d-q axes of basic field oriented system.

From equation (15), it can easily be understood that the core loss resistance is a function of frequency and flux level, which is explained in detail in [2]. Equations (12) and (14) express the rotor flux and torque, respectively.

The d-axis flux is affected by the q-axis current flowing through core loss resistance and violates the basic conception of Fig. 2. The q-axis rotor flux $\Phi_{2q}$ may be complex to force to be zero. But we can realize this condition by adjusting control inputs from the view of optimal control strategy.

**4. 2 Efficiency Optimized Speed Control**

The improvement of the efficiency can be achieved by controlling slip angular frequency in a constant speed and load torque. The slip frequency for maximum efficiency is independent of the load torque and speed [5] and the flux level must be reduced with field weakening.

In order to obtain optimized efficiency speed control the optimal slip angular frequency is adjusted automatically in accordance with the speed command.

The overall efficiency depends on the losses of induction motor. The controllable losses of induction motor are expressed by

$$P_{loss} = r_1|i_1|^2 + r_2|i_2|^2 + R_m|i_1|^2 \tag{16}$$

The equation (16) becomes equation (17) as a function of slip angular frequency at constant speed and load torque with consideration of all parameters to be constant.

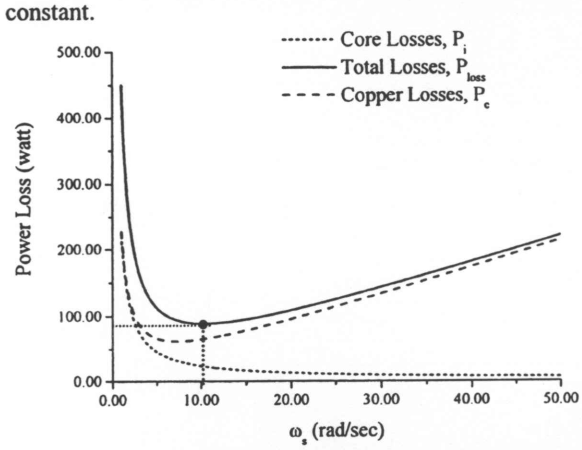



**Fig. 3.** Power losses vs. slip angular frequency.

$$P_{loss} = k_m\{k_1\omega_s^3 + k_2\omega_s^2 + k_3\omega_s + k_4 + \frac{k_5}{\omega_s}\} \tag{17}$$

Where $k_m = \dfrac{\tau_L}{p r_2 R_m^2 M^2}$

$k_1 = r_1 M^2 \ell_2^2 + R_m M^2 \ell_2^2$

$k_2 = 2r_1 M^2 \ell_2^2 \omega_m + 2R_m \ell_2^2 M^2 \omega_m$

$k_3 = r_1 M^2 \ell_2^2 \omega_m^2 - 2r_1 r_2 R_m \ell_2 M + r_1 r_2^2 M^2 + 2r_1 r_2 R_m L_r M$
$\quad + r_1 R_m^2 L_r^2 + R_m \ell_2^2 M^2 \omega_m^2 + r_2^2 R_m M^2 + r_2 R_m^2 M^2$

$k_4 = 2r_1 r_2^2 M^2 \omega_m + 2r_1 r_2 R_m L_r M \omega_m + 2r_2^2 R_m M^2 \omega_m$
$\quad - 2r_1 r_2 R_m \ell_2 M \omega_m$

$k_5 = r_1 r_2^2 R_m^2 + r_1 r_2^2 M^2 \omega_m^2 + r_2^2 R_m M^2 \omega_m^2$

We can obtain the slip angular frequency providing the minimum loss by setting the derivative of $P_{loss}$ with respect to $\omega_s$ to zero

$$\frac{dP_{loss}}{d\omega_s} = 0 \tag{18}$$

Since the equation (18) becomes a more complex form in terms of $\omega_s$, we used Newton Raphson Method to find out the minimal value of $\omega_s$ i.e. $\tilde{\omega}_s$.

By using equation (13), we can write the value of minimal slip angular frequency in terms of the ratio of rotor current to flux as shown in equation (19).

$$\tilde{k} = \frac{1}{r_2}\tilde{\omega}_s = -\frac{i_{2q}}{\Phi_{2d}} \tag{19}$$

**Fig. 3** shows the power losses vs. slip angular frequency curve at 900 r/min. Fig. 3 confirmed us that the maximum efficiency can be controlled by optimal slip angular frequency.

### 4.3 Application of Optimal Regulator Theory

The mathematical model of an induction motor has no linear relationship between the stator current and either torque or the flux. The state equations are become nonlinear due to the product of one state to another state or control input. Using the equations (8) through (11), we carried out linearization at the steady state operating point. Performing the lenearization, let us introduce the state variables, input, output and disturbance as follows:

$$\begin{aligned} \mathbf{x} &= [\omega_m \ \ i_{1d} \ \ i_{1q} \ \ i_{2d} \ \ i_{2q} \ \ \Phi_{2d} \ \ \Phi_{2q}]^T \\ \mathbf{u} &= [\omega_e \ \ v_{1d} \ \ v_{1q}]^T \\ \mathbf{y} &= [\omega_m \ \ \Phi_{2q}]^T, \ \mathbf{d} = \tau_L \end{aligned} \tag{20}$$

Where, T is the transposition.

Equations (21) and (22) are obtained by transforming (8) to (11) into the discrete time form with a sampling period $T_s$. For the sake of simplicity, the notation of sampling period $T_s$ is neglected in the following equations.

Controlled object

$$\mathbf{x}(k+1) = \mathbf{Ax}(k) + \mathbf{Bu}(k+1) + \mathbf{Ed}(k) \tag{21}$$

Output

$$\mathbf{y}(k) = \mathbf{Cx}(k) \tag{22}$$

Where k means sampling time $kT_s$ and **A**: 7×7 matrix, **B**: 3×7 matrix, **E**: 7×1 matrix, **C**: 3×7 matrix and k is sampling time.

The error signal **e**(k) can be defined as

$$\mathbf{e}(k) = \mathbf{R}(k) - \mathbf{y}(k) \tag{23}$$

Where

$$\mathbf{e}(k) = [e_{\omega m} \ \ e_{\Phi 2q}]^T \qquad \mathbf{R}(k) = [\omega_m^{\ R} \ \ \Phi_{2q}^{\ R}]^T \tag{24}$$

Here, **R**(k) is desired signal for the controlled variable. The error for efficiency optimized control $\mathbf{e}_\eta$ is defined from (19) as follows:

$$\mathbf{e}_\eta(k) = i_{2q}(k) + \tilde{k}\Phi_{2d}(k) \tag{25}$$

We assumed that all states of equation (21) are controllable and observable and that the equation shows the actual controlled object. An augmented system called the error system can be written as

$$\begin{bmatrix} \mathbf{e}(k+1) \\ \mathbf{e}_\eta(k) \\ \Delta\mathbf{x}(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{I}_2 & 0 & -\mathbf{CA} \\ 0 & 1 & \mathbf{K} \\ 0 & 0 & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{e}(k) \\ \mathbf{e}_\eta(k-1) \\ \Delta\mathbf{x}(k) \end{bmatrix} + \begin{bmatrix} -\mathbf{CB} \\ 0 \\ \mathbf{B} \end{bmatrix} \begin{bmatrix} \omega_e(k) \\ v_{1d}(k) \\ v_{1q}(k) \end{bmatrix} \tag{26}$$

Now the new state variable **X**(k) is expressed by (27)

$$\mathbf{X}(k) = [\mathbf{e}(k) \quad \mathbf{e}_\eta(k-1) \quad \Delta\mathbf{x}(k)]^T \tag{27}$$

Where Δ is the first difference value, for example

$$\Delta\mathbf{x}(k) = \mathbf{x}(k) - \mathbf{x}(k-1) \tag{28}$$

Let us assume that the reference signal for speed control is a step function, therefore the state equation of the error system is as follows:

$$\mathbf{X}(k+1) = \mathbf{\Psi X}(k) + \mathbf{G}\Delta\mathbf{u}(k) \tag{29}$$

Where, **Ψ**: 10×10 matrix, **G**: 3×10 matrix. It is clear that the sensitivity to the parameter variation can be attained as long as the error system (29) exists. The system (29) is controlled to be stable in spite of the parameter variations of the control object (21).

The performance index or cost function at each sampling time k(k=1,2,..), **J** is defined as

$$\mathbf{J}(k) = \sum_{k=1}^{\infty}[\mathbf{X}(k)^T\mathbf{QX}(k) + \mathbf{u}(k)^T\mathbf{Hu}(k)]$$

(30)

The matrices **Q** and **H** are weighting matrices and chosen to be symmetrical. The matrix **H** is assumed to be positive definite, whereas the matrix **Q** is assumed to be positive semi-definite.

By using optimal control theory, the control input can be derived easily as

$$\Delta\mathbf{u}(k) = \mathbf{F}_B\mathbf{X}(k) \tag{31}$$

$$\mathbf{F}_B = -[\mathbf{H} + \mathbf{G}^T\mathbf{PG}]^{-1}\mathbf{G}^T\mathbf{P\Psi} \tag{32}$$

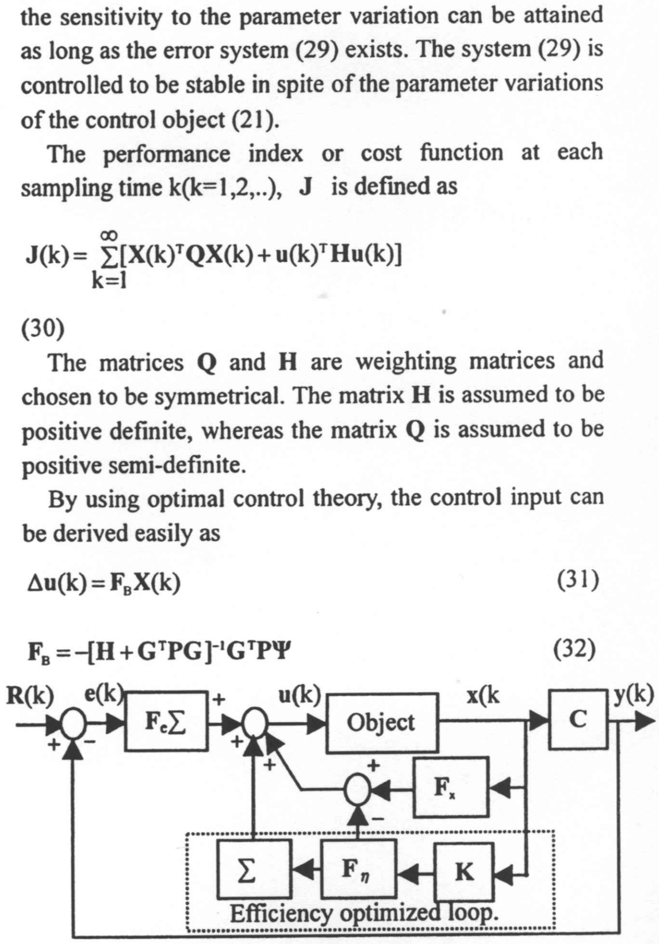



**Fig. 4.** Control system structure with an efficiency optimization.

To evaluate the feedback gain matrix, it is necessary to solve the Riccati equations in steady state. Then the matrix **P** becomes

$$\mathbf{P} = \mathbf{Q} + \mathbf{\Psi}^T\mathbf{P\Psi} - \mathbf{\Psi}^T\mathbf{PG}[\mathbf{H} + \mathbf{G}^T\mathbf{PG}]^{-1}\mathbf{G}^T\mathbf{P\Psi} \tag{33}$$

To find out the control system structure, it is needed the solution of equation (31) as

$$\begin{aligned} \mathbf{u}(i) &= \mathbf{F}_e\sum_{i=1}^{k}\mathbf{e}(i) + \mathbf{F}_\eta\sum_{i=1}^{k}\mathbf{e}_\eta(i) + (\mathbf{F}_x - \mathbf{F}_\eta\mathbf{K})\mathbf{x}(k) \\ &+ \mathbf{F}_\eta\mathbf{Kx}(0) - \mathbf{F}_x\mathbf{x}(0) + \mathbf{u}(0) \end{aligned} \tag{34}$$

In above equation the effects of the desired signal and the disturbance during the transient state are neglected. From the above equation we can define that first and second term represent integral, third term proportional and other terms as compensating action. Also, from above equation and proposed control system, we derive a block diagram as shown in **Fig. 4**.

## 5. Simulation Results

In order to show the validity of the proposed control scheme, we have carried out some simulation results by using the equivalent d-q axis d.c model on a synchronously rotating reference frame. The induction motor has the rated values of 1.1 KW, stator impressed voltage 200/√3 volts/phase, rotational speed 1000 r/min, 6 pole, 50 Hz. The values of induction motor parameters for simulation are given below:

$r_1 = 0.2842\ \Omega$, $r_2 = 0.2878\ \Omega$, $J = 0.0179$ Kg-m$^2$

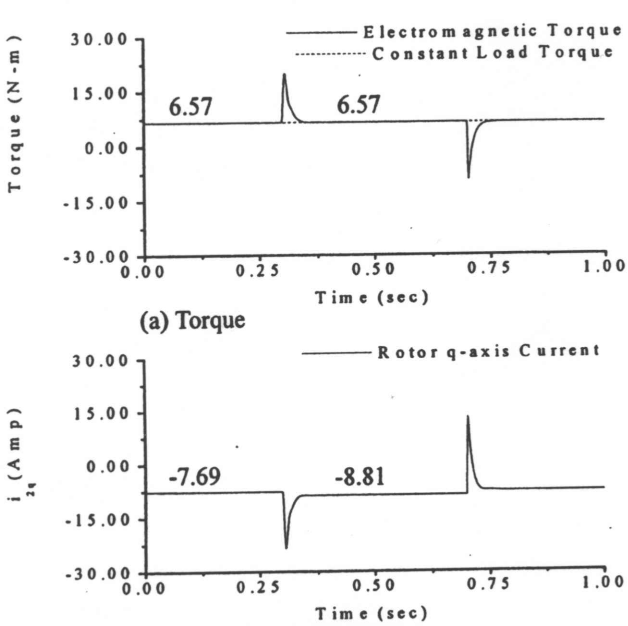
$L_s = 28.3$ mH, $L_r = 28.8$ mH, $M = 26.8$ mH, $D = 0$.

The core loss 27 watt at 50 Hz, sampling period 100 μsec and constant 50% load torque were used for simulation. The values of weighting factor matrices **H** and **Q** are determined by trial and error method to obtain excellent performance of proposed control scheme.

The values of the weighting factors used for quick response are as follows:

$H_1 = 200.0$ $H_2 = 2000.0$ $H_3 = 1000.0$

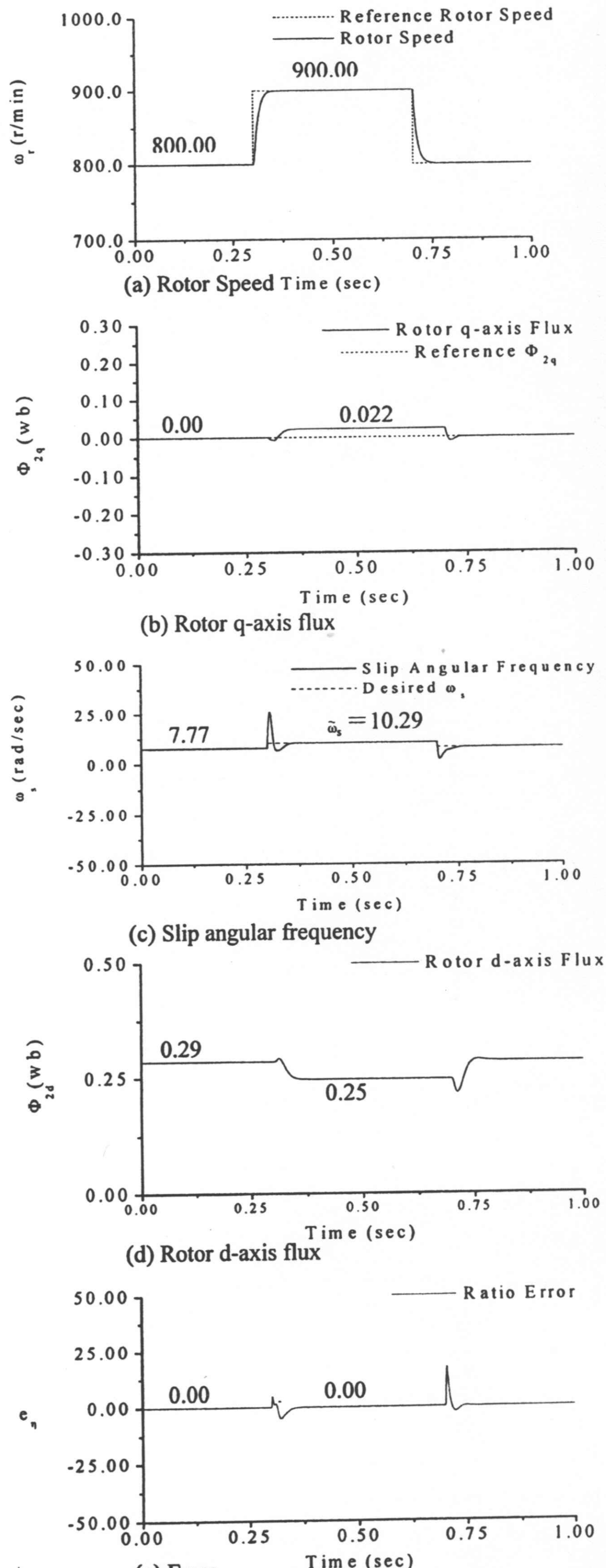
$Q_1 = 2.0$ $Q_3 = 0.15875$ and $Q_4 = 20000.0$




(a) Torque

(b) Rotor q-axis current

**Fig. 5.** Step response for desired signal.




(a) Rotor Speed

(b) Rotor q-axis flux

(c) Slip angular frequency

(d) Rotor d-axis flux

(e) Error

**Fig. 6.** Step response for desired signal.

The theoretical value of minimal slip angular frequency ($\tilde{\omega}_s$ = 10.29 rad/sec) and $\tilde{k}$ were calculated at 900 r/min by using Newton Raphson Method.

**Fig. 5** and **Fig. 6** show the response for a step change in the desired rotor speed with $\Phi_{2q}$ held at zero and hence the ideal field oriented control system is obtained. Fig. 6(e) shows that the ratio of rotor current and rotor flux is kept at the desired value of the error response. Consequently, the value of $Q_3$ was kept very small and the error $e_\eta$ quickly converges to zero. The Fig. 6(d) also proved that the flux level must be reduced for efficiency optimization in terms of slip angular frequency. Also, Fig. 5 shows the performance of torque and effects of rotor q-axis current.

## 6. Conclusion

The efficiency optimized speed control scheme for minimizing field oriented induction motor losses has been described. The achievement of the proposed control synthesis is investigated by simulation results. The optimal efficiency controller realizes on the operation of minimal slip angular frequency with field weakening.

The proposed control system is robust and adaptive to the parameter variation. In spite of core loss taking into account, simulation results of an induction motor show excellent efficiency optimized speed control system characteristics.

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**Appendix:**

The space vector F can be expressed as

$$\mathbf{F} = \mathbf{F_A} + a\mathbf{F_B} + a^2\mathbf{F_C}$$

Where, $a = e^{j\frac{2\pi}{3}}$

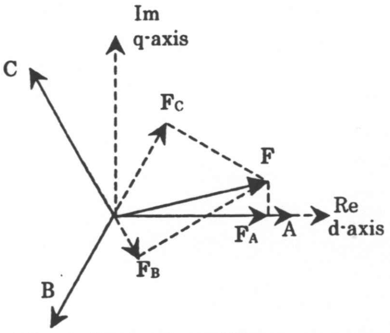


**Fig. A.** Projection of **F** space vector.